Lösungen

1

a

```
pi1[n_]:= 4 Sum[((-1)^k)/(2 k - 1),{k,1,n}];
Table[{n," ",N[pi1[n],10]},{n,1,30}]//TableForm
```

1	−4.000000000
2	−2.666666667
3	−3.466666667
4	−2.895238095
5	−3.339682540
6	−2.976046176
7	−3.283738484
8	−3.017071817
9	−3.252365935
10	−3.041839619
11	−3.232315809
12	−3.058402766
13	−3.218402766
14	−3.070254618
15	−3.208185652
16	−3.079153394
17	−3.200365515
18	−3.086079801
19	−3.194187909
20	−3.091623807
21	−3.189184782
22	−3.096161526
23	−3.185050415
24	−3.099944032
25	−3.181576685
26	−3.103145313
27	−3.178617011
28	−3.105889738
29	−3.176065177
30	−3.108268567

Schwache Konvergenz

b

```
pi2[n_]:= Abs[Sqrt[Abs[Sqrt[(90 Sum[1/(k^4),{k,1,n}])]]]];
Table[{n," ",N[pi2[n],10]},{n,1,45}]//TableForm
```

1	3.080070288
2	3.127107866
3	3.136152380
4	3.138997889
5	3.140161179
6	3.140721718
7	3.141024158
8	3.141201402
9	3.141312040
10	3.141384622
11	3.141434195
12	3.141469195
13	3.141494605
14	3.141513496
15	3.141527831
16	3.141538904
17	3.141547593
18	3.141554506
19	3.141560074
20	3.141564610
21	3.141568341
22	3.141571439
23	3.141574032
24	3.141576219
25	3.141578077
26	3.141579665
27	3.141581030
28	3.141582211
29	3.141583237
30	3.141584133
31	3.141584919
32	3.141585611
33	3.141586222
34	3.141586766
35	3.141587249
36	3.141587681
37	3.141588068
38	3.141588416
39	3.141588730
40	3.141589013
41	3.141589270
42	3.141589503
43	3.141589716
44	3.141589909
45	3.141590086

Konvergenz etwas besser

c Genauigkeit in Abhängigkeit der Anzahl Reihenglieder empirisch

```
{{"n","s(n)"},{1,1},{2,2},{3,5},{4,7},{5,15},{6,45}}//TableForm
```

n	s(n)
1	1
2	2
3	5
4	7
5	15
6	45

Kommentar: Ein empirisches Resultat wie dieses ist nicht verlässlich. Grund ist die Nichtberücksichtigung der Maschinenpräzision. Bei dem hier verwendeten Programm lässt sich die numerische Präszision voreinstellen. Im hier beobachteten Bereich ist

d Manipulation der Gehauigkeit

```
?MachinePrecision
```

```
MachinePrecision is a symbol used to indicate machine-number precision.
```
Mehr...

```
$MachinePrecision
```

```
15.9546
```

A typical value of `$MachinePrecision` is 53 $\log_{10} 2$ or approximately 16.

```
?*Precision
```

System`

InterpolationPrecision	WorkingPrecision
MachinePrecision	$MachinePrecision
Precision	$MaxExtraPrecision
PrintPrecision	$MaxPrecision
SetPrecision	$MinPrecision

Global`

```
$Precision
```

```
SetPrecision[pi2,40]
```

```
pi2
```

```
pi2[n_]:= Abs[Sqrt[Abs[Sqrt[(90 Sum[1/(k^4),{k,1,n}])]]]];
Table[{n," ",N[pi2[n],10]},{n,1,45}]//TableForm
```

1	3.080070288
2	3.127107866
3	3.136152380
4	3.138997889
5	3.140161179
6	3.140721718
7	3.141024158
8	3.141201402
9	3.141312040
10	3.141384622
11	3.141434195
12	3.141469195
13	3.141494605
14	3.141513496
15	3.141527831
16	3.141538904
17	3.141547593
18	3.141554506
19	3.141560074
20	3.141564610
21	3.141568341
22	3.141571439
23	3.141574032
24	3.141576219
25	3.141578077
26	3.141579665
27	3.141581030
28	3.141582211
29	3.141583237
30	3.141584133
31	3.141584919
32	3.141585611
33	3.141586222
34	3.141586766
35	3.141587249
36	3.141587681
37	3.141588068
38	3.141588416
39	3.141588730
40	3.141589013
41	3.141589270
42	3.141589503
43	3.141589716
44	3.141589909
45	3.141590086

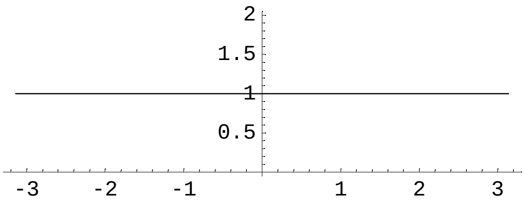
Diese Manipulation hat hier keinen Einfluss auf die Genauigkeit.

2

(10 Glieder berechnet)

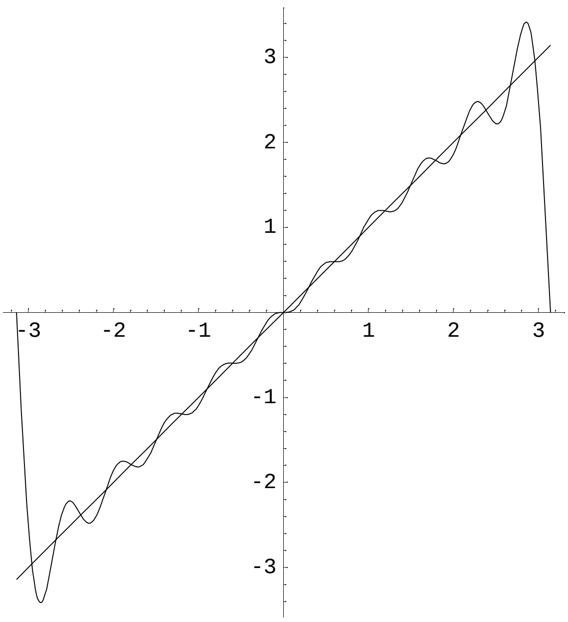
```
Remove["Global`*"]
<<Calculus`FourierTransform`;

fa0[t_]:=1;
fb0[t_]:=fa0[t 2 Pi];
fc0[t_]:=Evaluate[FourierTrigSeries[fb0[t], t, 10]];
fd0[tt_]:=Evaluate[FourierTrigSeries[fb0[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa0[t]],{t,-Pi,Pi},AspectRatio->Automatic,DisplayFunction->Identit
y];
pd=Plot[Evaluate[fd0[tt]],{tt,-Pi,Pi},AspectRatio->Automatic,DisplayFunction->Ident
ity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd0[u]
```



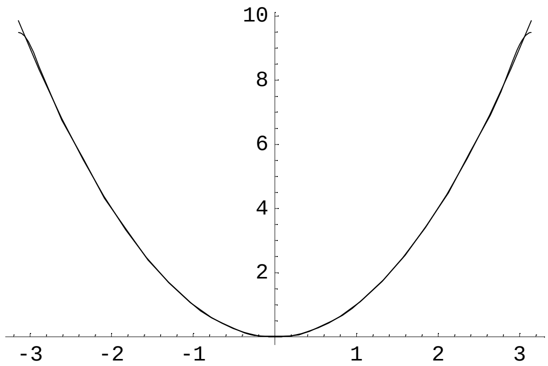
1

```
fa1[t_]:=t^1;
fb1[t_]:=fa1[t 2 Pi];
fc1[t_]:=Evaluate[FourierTrigSeries[fb1[t], t, 10]];
fd1[tt_]:=Evaluate[FourierTrigSeries[fb1[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa1[t]],{t,-Pi,Pi},AspectRatio->Automatic,DisplayFunction->Identit
y];
pd=Plot[Evaluate[fd1[tt]],{tt,-Pi,Pi},AspectRatio->Automatic,DisplayFunction->Ident
ity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd1[u]//N
```



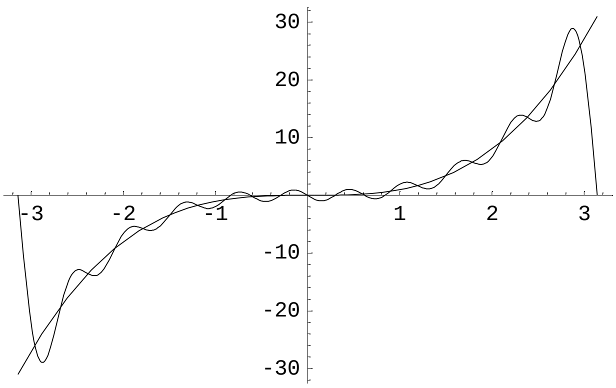
2. Sin[u] − 1. Sin[2. u] + 0.666667 Sin[3. u] − 0.5 Sin[4. u] +
0.4 Sin[5. u] − 0.333333 Sin[6. u] + 0.285714 Sin[7. u] −
0.25 Sin[8. u] + 0.222222 Sin[9. u] − 0.2 Sin[10. u]

```
fa2[t_]:=t^2;
fb2[t_]:=fa2[t 2 Pi];
fc2[t_]:=Evaluate[FourierTrigSeries[fb2[t], t, 10]];
fd2[tt_]:=Evaluate[FourierTrigSeries[fb2[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa2[t]],{t,-Pi,Pi},DisplayFunction->Identity];
pd=Plot[Evaluate[fd2[tt]],{tt,-Pi,Pi},DisplayFunction->Identity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd2[u]//N
```



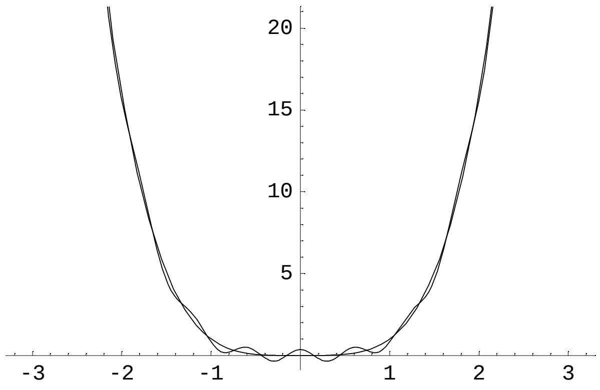

```
3.28987 - 4. Cos[u] + Cos[2. u] - 0.444444 Cos[3. u] +
  0.25 Cos[4. u] - 0.16 Cos[5. u] + 0.111111 Cos[6. u] - 0.0816327 Cos[7. u] +
  0.0625 Cos[8. u] - 0.0493827 Cos[9. u] + 0.04 Cos[10. u]
```

```
fa3[t_]:=t^3;
fb3[t_]:=fa3[t 2 Pi];
fc3[t_]:=Evaluate[FourierTrigSeries[fb3[t], t, 10]];
fd3[tt_]:=Evaluate[FourierTrigSeries[fb3[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa3[t]],{t,-Pi,Pi},DisplayFunction->Identity];
pd=Plot[Evaluate[fd3[tt]],{tt,-Pi,Pi},DisplayFunction->Identity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd3[u]//N
```



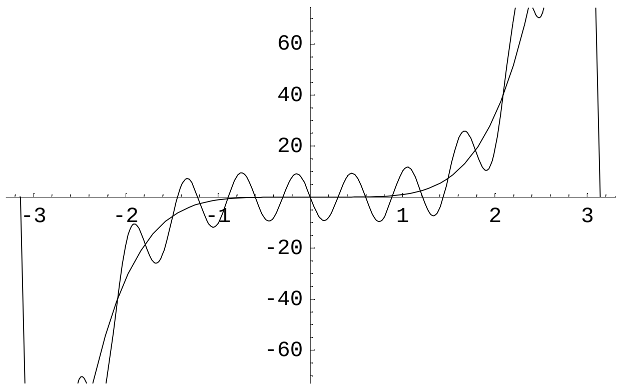

```
7.73921 Sin[u] - 8.3696 Sin[2. u] + 6.13529 Sin[3. u] -
  4.7473 Sin[4. u] + 3.85184 Sin[5. u] - 3.23431 Sin[6. u] + 2.7849 Sin[7. u] -
  2.44396 Sin[8. u] + 2.17678 Sin[9. u] - 1.96192 Sin[10. u]
```

```
fa4[t_]:=t^4;
fb4[t_]:=fa4[t 2 Pi];
fc4[t_]:=Evaluate[FourierTrigSeries[fb4[t], t, 10]];
fd4[tt_]:=Evaluate[FourierTrigSeries[fb4[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa4[t]],{t,-Pi,Pi},DisplayFunction->Identity];
pd=Plot[Evaluate[fd4[tt]],{tt,-Pi,Pi},DisplayFunction->Identity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd4[u]//N
```



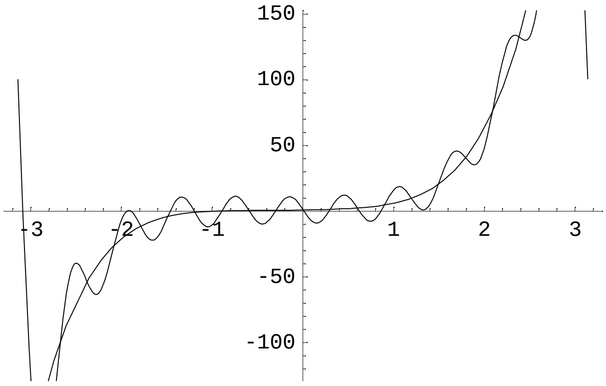

```
19.4818 - 30.9568 Cos[u] + 16.7392 Cos[2. u] - 8.18039 Cos[3. u] +
 4.7473 Cos[4. u] - 3.08147 Cos[5. u] + 2.15621 Cos[6. u] - 1.59137 Cos[7. u] +
 1.22198 Cos[8. u] - 0.96746 Cos[9. u] + 0.784768 Cos[10. u]
```

```
fa5[t_]:=t^5;
fb5[t_]:=fa5[t 2 Pi];
fc5[t_]:=Evaluate[FourierTrigSeries[fb5[t], t, 10]];
fd5[tt_]:=Evaluate[FourierTrigSeries[fb5[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa5[t]],{t,-Pi,Pi},DisplayFunction->Identity];
pd=Plot[Evaluate[fd5[tt]],{tt,-Pi,Pi},DisplayFunction->Identity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd5[u]//N
```



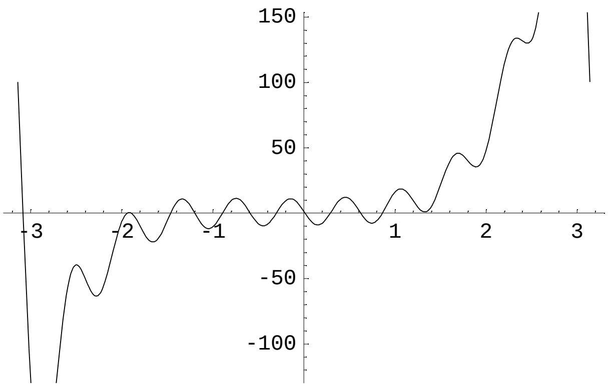

```
40.034 Sin[u] - 55.5611 Sin[2. u] + 51.3054 Sin[3. u] -
 42.7704 Sin[4. u] + 35.8822 Sin[5. u] - 30.6729 Sin[6. u] +
 26.6945 Sin[7. u] - 23.5885 Sin[8. u] + 21.109 Sin[9. u] - 19.0894 Sin[10. u]
```

```
fa6[t_]:=1+t+t^2+t^3+t^4+t^5;
fb6[t_]:=fa6[t 2 Pi];
fc6[t_]:=Evaluate[FourierTrigSeries[fb6[t], t, 10]];
fd6[tt_]:=Evaluate[FourierTrigSeries[fb6[t], t, 10]/.t->tt/(2 Pi)];
pa=Plot[Evaluate[fa6[t]],{t,-Pi,Pi},DisplayFunction->Identity];
pd=Plot[Evaluate[fd6[tt]],{tt,-Pi,Pi},DisplayFunction->Identity];
Show[pa,pd,DisplayFunction->$DisplayFunction];
fd6[u]//N
```



23.7717 − 34.9568 Cos[u] + 17.7392 Cos[2. u] − 8.62483 Cos[3. u] + 4.9973 Cos[4. u] − 3.24147 Cos[5. u] + 2.26732 Cos[6. u] − 1.673 Cos[7. u] + 1.28448 Cos[8. u] − 1.01684 Cos[9. u] + 0.824768 Cos[10. u] + 49.7732 Sin[u] − 64.9307 Sin[2. u] + 58.1074 Sin[3. u] − 48.0177 Sin[4. u] + 40.134 Sin[5. u] − 34.2405 Sin[6. u] + 29.7651 Sin[7. u] − 26.2825 Sin[8. u] + 23.508 Sin[9. u] − 21.2514 Sin[10. u]

```
Plot[Evaluate[fd0[tt]+fd1[tt]+fd2[tt]+fd3[tt]+fd4[tt]+fd5[tt]],{tt,-Pi,Pi}];
```




```
(fd0[tt]+fd1[tt]+fd2[tt]+fd3[tt]+fd4[tt]+fd5[tt])//N
```

23.7717 − 34.9568 Cos[tt] + 17.7392 Cos[2. tt] − 8.62483 Cos[3. tt] + 4.9973 Cos[4. tt] − 3.24147 Cos[5. tt] + 2.26732 Cos[6. tt] − 1.673 Cos[7. tt] + 1.28448 Cos[8. tt] − 1.01684 Cos[9. tt] + 0.824768 Cos[10. tt] + 49.7732 Sin[tt] − 64.9307 Sin[2. tt] + 58.1074 Sin[3. tt] − 48.0177 Sin[4. tt] + 40.134 Sin[5. tt] − 34.2405 Sin[6. tt] + 29.7651 Sin[7. tt] − 26.2825 Sin[8. tt] + 23.508 Sin[9. tt] − 21.2514 Sin[10. tt]

3

a

(10 Glieder berechnet)

```
Remove["Global`*"]

f[t_]:=E^(Sin[t]+Cos[3 t]);
a[0]=1/Pi NIntegrate[f[t],{t,-Pi,Pi}]
```

3.20583

```
a[k_]:=1/Pi NIntegrate[Evaluate[f[t]Cos[k t]],{t,-Pi,Pi}];
a[3]
```

1.43103

```
b[k_]:=1/Pi NIntegrate[Evaluate[f[t]Sin[k t]],{t,0,2Pi}];
b[3]
```

-0.0501147

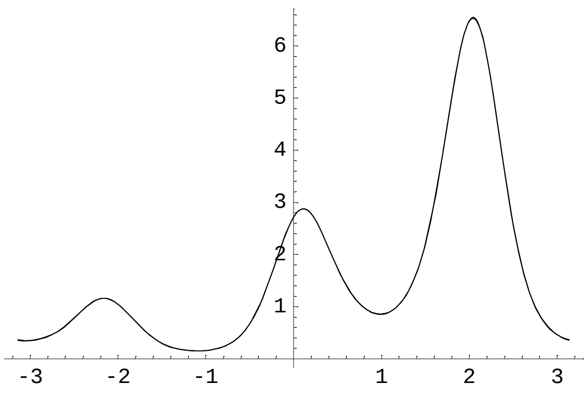
```
Table[{a[k],b[k]},{k,1,10}]
```

{{-0.150344, 1.43098}, {-0.342988, -0.638503}, {1.43103, -0.0501147}, {-0.0299241, 0.638796}, {-0.153317, -0.152751}, {0.343674, -0.0240745}, {-0.00292481, 0.153433}, {-0.0368396, -0.0247505}, {0.0561079, -0.00589725}, {-6.92603×10^{-10}, 0.0250554}}

```
g[t_]= a[0]/2+Sum[Evaluate[a[k] Cos[k t]+b[k] Sin[k t]],{k,1,10}];
g[t]
```

$1.60292 - 0.150344\,\mathrm{Cos}[t] - 0.342988\,\mathrm{Cos}[2\,t] + 1.43103\,\mathrm{Cos}[3\,t] - 0.0299241\,\mathrm{Cos}[4\,t] - 0.153317\,\mathrm{Cos}[5\,t] + 0.343674\,\mathrm{Cos}[6\,t] - 0.00292481\,\mathrm{Cos}[7\,t] - 0.0368396\,\mathrm{Cos}[8\,t] + 0.0561079\,\mathrm{Cos}[9\,t] - 6.92603\times10^{-10}\,\mathrm{Cos}[10\,t] + 1.43098\,\mathrm{Sin}[t] - 0.638503\,\mathrm{Sin}[2\,t] - 0.0501147\,\mathrm{Sin}[3\,t] + 0.638796\,\mathrm{Sin}[4\,t] - 0.152751\,\mathrm{Sin}[5\,t] - 0.0240745\,\mathrm{Sin}[6\,t] + 0.153433\,\mathrm{Sin}[7\,t] - 0.0247505\,\mathrm{Sin}[8\,t] - 0.00589725\,\mathrm{Sin}[9\,t] + 0.0250554\,\mathrm{Sin}[10\,t]$

```
Plot[Evaluate[{f[t],g[t]}],{t,-Pi,Pi}];
```



b

```
?TrigTo*
```

TrigToExp[expr] converts trigonometric functions in expr to exponentials. Mehr…

```
TrigToExp[g[t]]
```

$$1.60292 - (0.0751721 - 0.715492\, i)\, e^{-i t} - (0.0751721 + 0.715492\, i)\, e^{i t} - (0.171494 + 0.319251\, i)\, e^{-2 i t} - (0.171494 - 0.319251\, i)\, e^{2 i t} + (0.715515 - 0.0250574\, i)\, e^{-3 i t} + (0.715515 + 0.0250574\, i)\, e^{3 i t} - (0.0149621 - 0.319398\, i)\, e^{-4 i t} - (0.0149621 + 0.319398\, i)\, e^{4 i t} - (0.0766583 + 0.0763753\, i)\, e^{-5 i t} - (0.0766583 - 0.0763753\, i)\, e^{5 i t} + (0.171837 - 0.0120372\, i)\, e^{-6 i t} + (0.171837 + 0.0120372\, i)\, e^{6 i t} - (0.0014624 - 0.0767163\, i)\, e^{-7 i t} - (0.0014624 + 0.0767163\, i)\, e^{7 i t} - (0.0184198 + 0.0123753\, i)\, e^{-8 i t} - (0.0184198 - 0.0123753\, i)\, e^{8 i t} + (0.028054 - 0.00294863\, i)\, e^{-9 i t} + (0.028054 + 0.00294863\, i)\, e^{9 i t} - (3.46301 \times 10^{-10} - 0.0125277\, i)\, e^{-10 i t} - (3.46301 \times 10^{-10} + 0.0125277\, i)\, e^{10 i t}$$

4

Selbststudium nach Skript.