

# Lösungen

---

## 1

```
Remove["Global`*"]

xVec[x_, y_] := {x, y};
mVec = {5, 4}; r = 2;
kreis[rVec_, r_] := rVec.rVec - r^2;
```

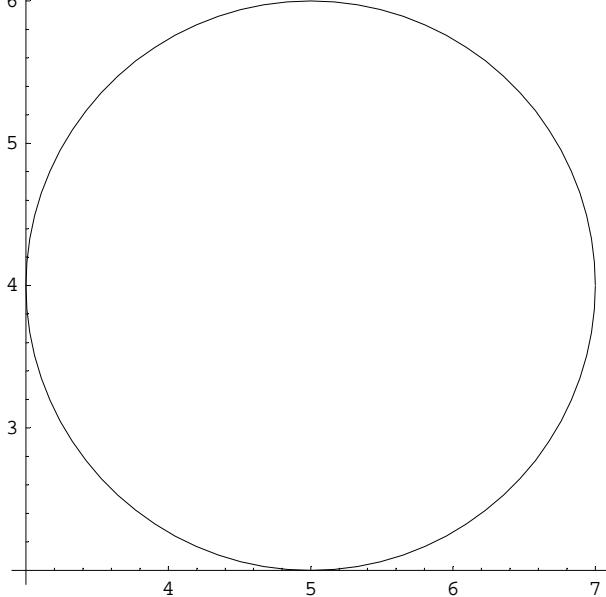
### a) Pol und Polare

```
pol = {1, 1};
kreis[(xVec[x, y] - mVec), r] == 0
-4 + (-5 + x)^2 + (-4 + y)^2 == 0

Expand[kreis[(xVec[x, y] - mVec), r]] == 0
37 - 10 x + x^2 - 8 y + y^2 == 0

<< Graphics`ImplicitPlot`
```

```
kPl = ImplicitPlot[kreis[(xVec[x, y] - mVec), r] == 0, {x, 1, 8}];
```



```
polare[xVec_, mVec_, pol_, r_] := (xVec[x, y] - mVec).(pol - mVec) - r^2;
polare[xVec, mVec, pol, r] == 0
-4 - 4 (-5 + x) - 3 (-4 + y) == 0
```

```

Simplify[polare[xVec, mVec, pol, r]] == 0
28 - 4 x - 3 y == 0

solv = Solve[{polare[xVec, mVec, pol, r] == 0, kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}]
{ {x → 1/25 (109 - 6 √21), y → 8/25 (11 + √21)}, {x → 1/25 (109 + 6 √21), y → 8/25 (11 - √21)}}

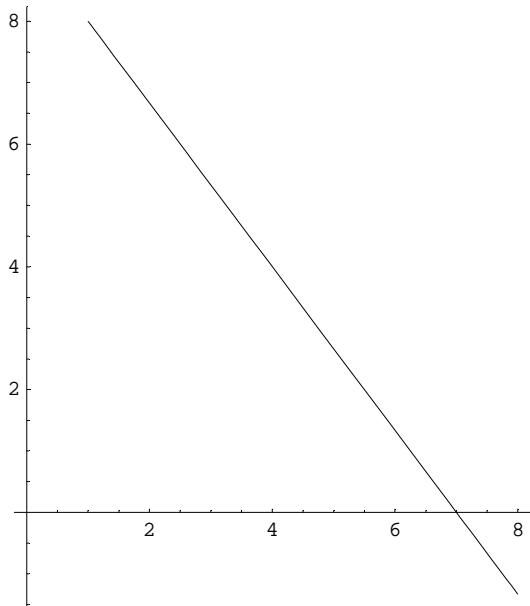
solv1 = solv // N
{{x → 3.26018, y → 4.98642}, {x → 5.45982, y → 2.05358} }

pT1 = {x, y} /. solv1[[1]]
{3.26018, 4.98642}

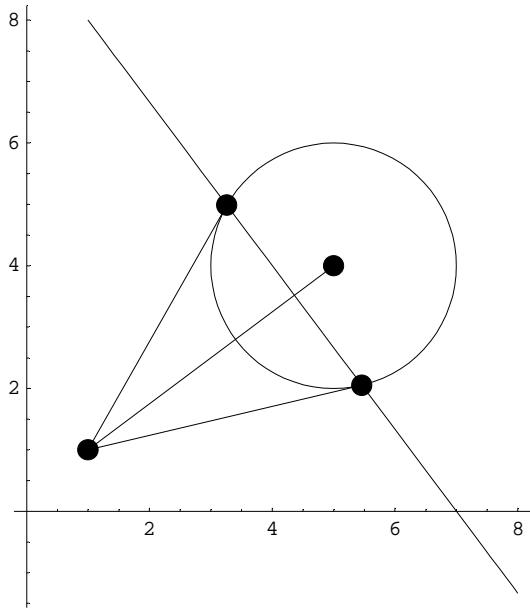
pT2 = {x, y} /. solv1[[2]]
{5.45982, 2.05358}

polarePl = ImplicitPlot[polare[xVec, mVec, pol, r] == 0, {x, 1, 8}];

```



```
Show[kPl, polarePl, Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2],
Point[pT1], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



$pT1 + t (pT2 - pT1)$   
 $\{3.26018 + 2.19964 t, 4.98642 - 2.93285 t\}$

### b) Tangente

```
tangente[xVec_, mVec_, pT_, r_] := (xVec[x, y] - mVec). (pT - mVec) - r^2;
Chop[Expand[polare[xVec, mVec, pT1, r]]] == 0
0.753394 - 1.73982 x + 0.986424 y == 0

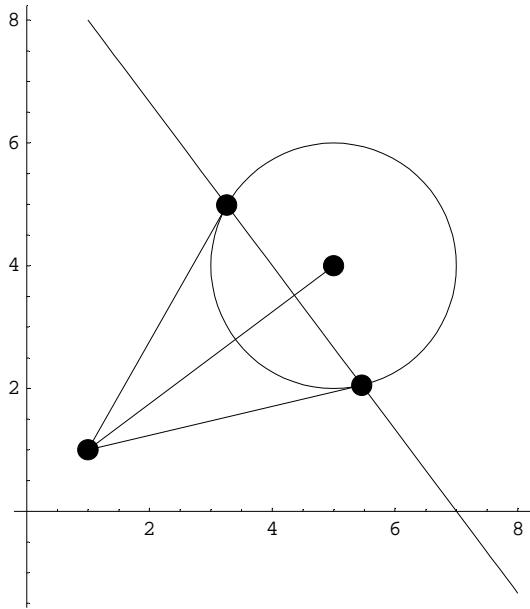
Chop[Expand[polare[xVec, mVec, pT1, r]] /. {x → 1, y → 1}] == 0
True

Chop[Expand[polare[xVec, mVec, pT2, r]]] == 0
1.48661 + 0.459818 x - 1.94642 y == 0

Chop[Expand[polare[xVec, mVec, pT2, r]] /. {x → 1, y → 1}] == 0
True
```

Die Tangentialpunkte sind identisch mit den Schnittpunkten des Kreises mit der Polaren.

```
Show[kPl, polarePl, Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2],
Point[pT1], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



```
pol + t (pT1 - pol)
{1 + 2.26018 t, 1 + 3.98642 t}

pol + t (pT2 - pol)
{1 + 4.45982 t, 1 + 1.05358 t}
```

### c) Mittelpunktsgerade zum Pol

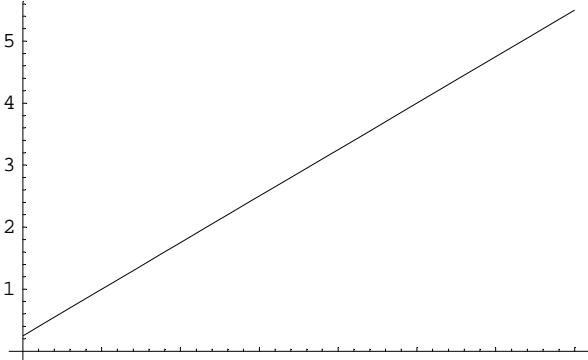
```
senkr[v_] := {-v[[2]], v[[1]]};
geradePolM[xVec_, mVec_, pol_] := (xVec - mVec).senkr[pol - mVec];

Expand[geradePolM[xVec[x, y], mVec, pol]] == 0
1 + 3 x - 4 y == 0

solvz = Solve[Expand[geradePolM[xVec[x, y], mVec, pol]] == 0, {y}] // Flatten
{y → 1/4 (1 + 3 x)}
```

```

yGer[x_] := y /. solvz;
gerPl = Plot[yGer[x], {x, 0, 7}];

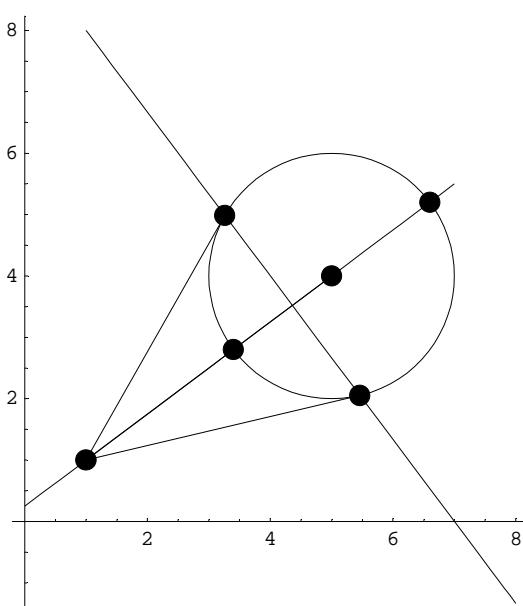
A plot showing a straight line on a Cartesian coordinate system. The x-axis is labeled from 0 to 7 with major ticks every 1 unit. The y-axis is labeled from 0 to 5 with major ticks every 1 unit. The line passes through the points (0, 0.5), (1, 1.5), (2, 2.5), (3, 3.5), (4, 4.5), (5, 5.5), and (6, 6.5). It has a positive slope of 1 and a y-intercept of 0.5.

Solve[{geradePolM[xVec[x, y], mVec, pol] == 0, kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}]
{ {x → 17/5, y → 14/5}, {x → 33/5, y → 26/5} }

solv2 = Solve[{geradePolM[xVec[x, y], mVec, pol] == 0,
    kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}] // N
{ {x → 3.4, y → 2.8}, {x → 6.6, y → 5.2} }

p1 = {x, y} /. solv2[[1]]
{3.4, 2.8}

p2 = {x, y} /. solv2[[2]]
{6.6, 5.2}

Show[kPl, polarePl, gerPl,
Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2], Point[pT1],
Point[p1], Point[p2], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];


```

```

pol + t (mVec - pol)
{1 + 4 t, 1 + 3 t}

```

#### d) Apollonius

```

Norm[p2 - pol] / Norm[p1 - pol]
2.33333

Rationalize[Norm[p2 - pol] / Norm[p1 - pol]]
7
3

Norm[p1 - pol] / Norm[p1 - {x, yGer[x]}] == Norm[p2 - pol] / Norm[p2 - {x, yGer[x]}]
3.                                     7.
----- = -----
Abs[2.8 + 1/4 (-1 - 3 x)]^2 + Abs[3.4 - x]^2   Abs[5.2 + 1/4 (-1 - 3 x)]^2 + Abs[6.6 - x]^2

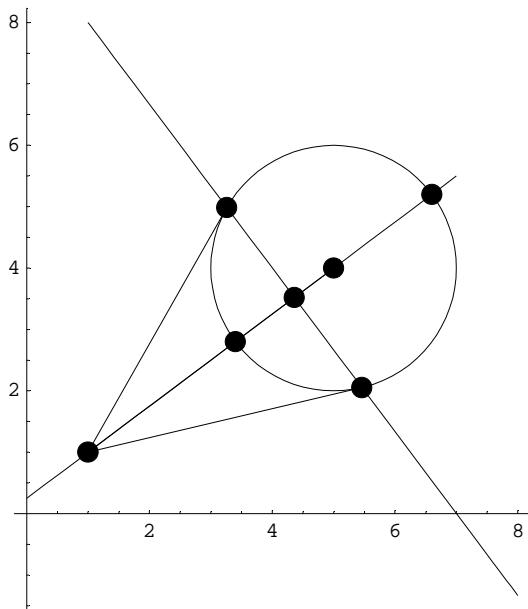
solv3 = Solve[(Norm[p1 - pol] / Norm[p1 - {x, yGer[x]}])^2 ==
(Norm[p2 - pol] / Norm[p2 - {x, yGer[x]}])^2, {x}]
{{x → 1.}, {x → 4.36}]

pol1 = {x, yGer[x]} /. solv3[[1]]
{1., 1.}

pol2 = {x, yGer[x]} /. solv3[[2]]
{4.36, 3.52}

Show[kPl, polarePl, gerPl, Graphics[
PointSize[0.04], Point[pol], Point[mVec], Point[pT2], Point[pT1], Point[p1],
Point[p2], Point[pol1], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]]];

```



### e) Potenzgerade

```

kreis1 = (kreis[(xVec[x, y] - mVec), r] == 0)
-4 + (-5 + x)^2 + (-4 + y)^2 == 0

kreis2 = (kreis[(xVec[x, y] - pol), 2 r] == 0)
-16 + (-1 + x)^2 + (-1 + y)^2 == 0

potenz1[x_, y_] := kreis[(xVec[x, y] - mVec), r];
potenz2[x_, y_] := kreis[(xVec[x, y] - pol), 2 r];

Expand[potenz1[x, y]] == potenz2[x, y]
37 - 10 x + x^2 - 8 y + y^2 == -16 + (-1 + x)^2 + (-1 + y)^2

? Reduce
Reduce[expr, vars] reduces the statement expr by solving equations or inequalities for vars
and eliminating quantifiers. Reduce[expr, vars, dom] does the reduction over
the domain dom. Common choices of dom are Reals, Integers and Complexes. Mehr...
Reduce[Expand[potenz1[x, y]] == potenz2[x, y], {x, y}]
Y ==  $\frac{17}{2} - \frac{4x}{3}$ 

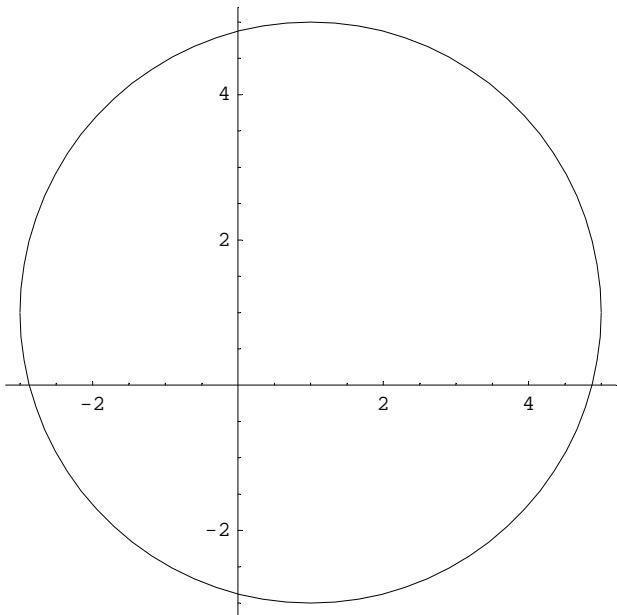
solv4 = Solve[Reduce[Expand[potenz1[x, y]] == potenz2[x, y], {x, y}], {y}] // Flatten
{Y →  $\frac{1}{6}(51 - 8x)$ }

potenzGer[x_] := y /. solv4
potenzGer[x]
 $\frac{1}{6}(51 - 8x)$ 

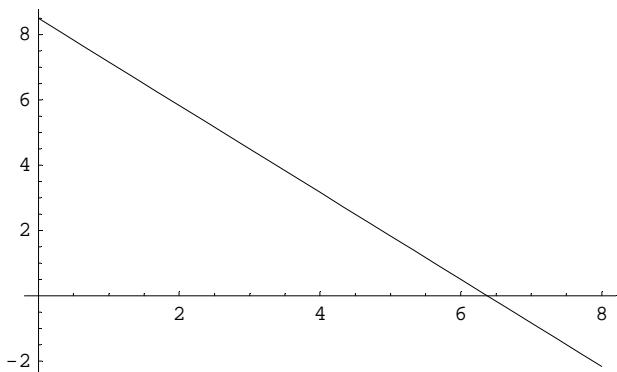
```

**f) Schnittpunkte Potenzgerade mit Kreisen**

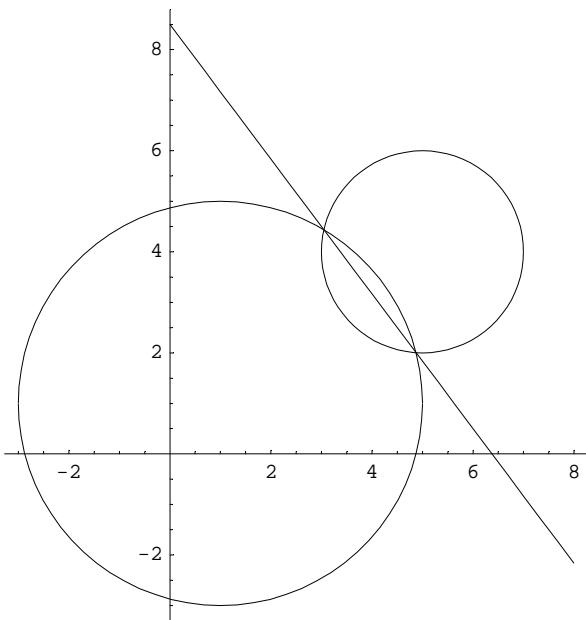
```
kP12 = ImplicitPlot[kreis2, {x, -6, 8}];
```



```
plPotGer = Plot[potenzGer[x], {x, 0, 8}];
```



```
Show[kP12, plPotGer, kP1];
```



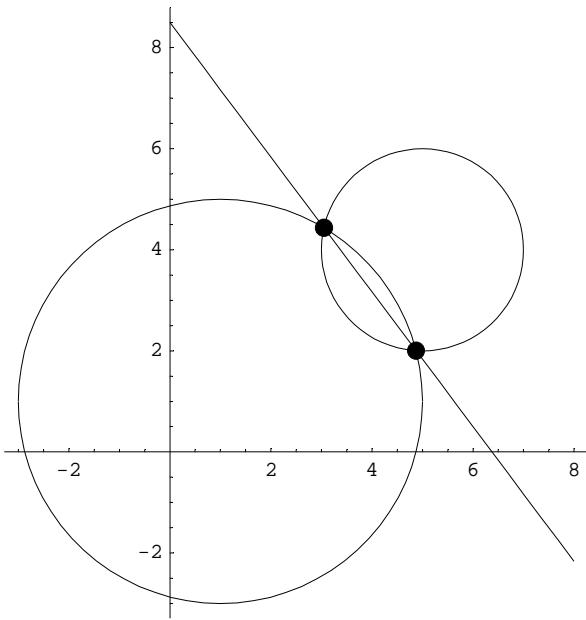
Die Potenzgerade geht durch die Schnittpunkte der beiden Kreise.

```
solv11 = Solve[{y == potenzGer[x], kreis1}, {x, y}]
{{x → 3/50 (66 - √231), y → 1/50 (161 + 4 √231)}, 
 {x → 3/50 (66 + √231), y → 1/50 (161 - 4 √231)}}

N[%]
{{x → 3.04808, y → 4.43589}, {x → 4.87192, y → 2.00411}]

ss1 = {x, y} /. solv11[[1]];
ss2 = {x, y} /. solv11[[2]];
{ss1, ss2} // N
{{3.04808, 4.43589}, {4.87192, 2.00411}}
```

```
Show[kP12, plPotGer, kP1, Graphics[{PointSize[0.03], Point[ss1], Point[ss2]}]];
```



## 2

```
Remove["Global`*"]

B = {{1, 1}, {3, 2}}; B // MatrixForm

$$\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$$


mD = {{1, 0}, {0, 2}}; mD // MatrixForm

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$


mD10 = {{1^10, 0}, {0, 2^10}}; mD10 // MatrixForm

$$\begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}$$


mD^10 // MatrixForm

$$\begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}$$


a = {1, 2}; b = {1, 3};
```

### a

```
B1 = Inverse[B];
B1 // MatrixForm

$$\begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$$

```

**b**

```
A = B.mD.Inverse[B];
A // MatrixForm
```

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}$$

**c**

```
A2 = A.A; A2 // MatrixForm
```

$$\begin{pmatrix} 10 & -3 \\ 18 & -5 \end{pmatrix}$$

```
A10 = B.(mD^10).Inverse[B];
A10 // MatrixForm
```

$$\begin{pmatrix} 3070 & -1023 \\ 6138 & -2045 \end{pmatrix}$$

**d**

```
Det[A]
```

```
2
```

```
Det[mD]
```

```
2
```

**e**

```
A.a
```

```
{2, 4}
```

```
A.b
```

```
{1, 3}
```

**f**

```
A. (λ a + μ b) // Simplify
```

```
{2 λ + μ, 4 λ + 3 μ}
```

**g**

```
flaechenProdukt[a_, b_] := Det[{a, b}];
flaechenProdukt[a, b]
```

1

**h**

```
flaechenProdukt[λ a, μ b]
```

 $\lambda \mu$ **i**

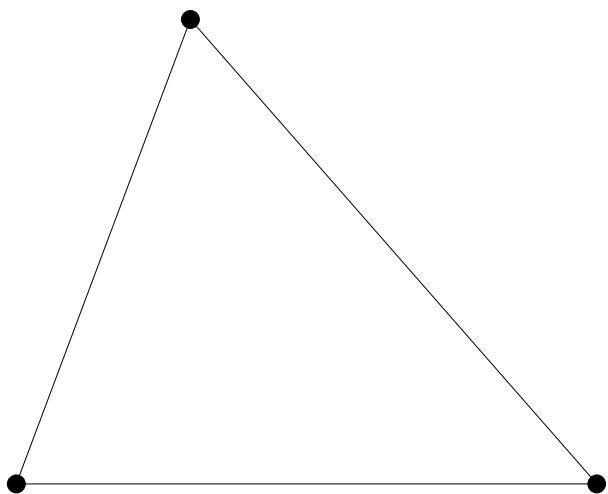
```
flaechenProdukt[λ a, μ b] / flaechenProdukt[a, b]
```

 $\lambda \mu$ **3**

```
Remove["Global`*"]

OA = {0, 0};
OB = {10, 0};
OC = {3, 8};

Show[Graphics[{PointSize[0.03], Point[OA], Point[OB],
Point[OC], Line[{OA, OB, OC, OA}]}], AspectRatio -> Automatic];
```



### a) Höhenschnittpunkt

```

AB = OB - OA;
BC = OC - OB;
CA = OA - OC;

BA = -AB; CB = -BC; AC = -CA;

normalV[x_] := {-x[[2]], x[[1]]};

nAB = normalV[AB];
nBC = normalV[BC];
nCA = normalV[CA];

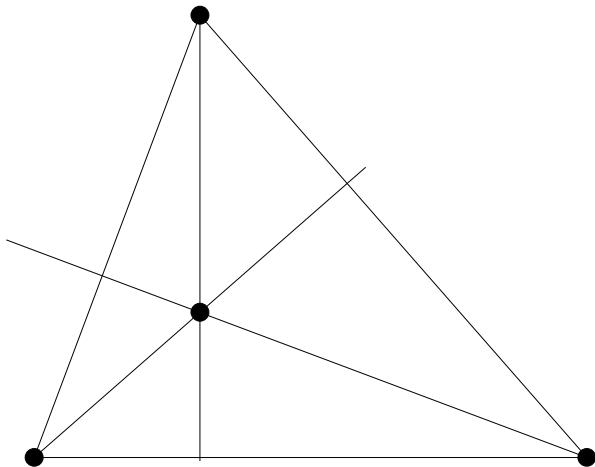
hC[t_] := OC + t nAB;
hB[s_] := OB + s nCA;
solv3 = Solve[hC[t] == hB[s], {t, s}] // Flatten
{t → - $\frac{43}{80}$ , s → - $\frac{7}{8}$ }

schnittH = hC[t] /. solv3
{3,  $\frac{21}{8}$ }

N[%]
{3., 2.625}

Show[Graphics[{PointSize[0.03], Point[OA],
Point[OB], Point[OC], Point[schnittH], Line[{OA, OB, OC, OA}],
Line[{OA, schnittH + (schnittH - OA)}], Line[{OB, schnittH + 0.5 (schnittH - OB)}],
Line[{OC, schnittH + 0.5 (schnittH - OC)}]}], AspectRatio → Automatic];

```



### b) Schwerlinienschnittpunkt

```
sC[t_] := OA + 1/2 AB + t (OC - (OA + 1/2 AB));
sB[s_] := OC + 1/2 CA + s (OB - (OC + 1/2 CA));
solve31 = Solve[sC[t] == sB[s], {t, s}] // Flatten

{t → 1/3, s → 1/3}

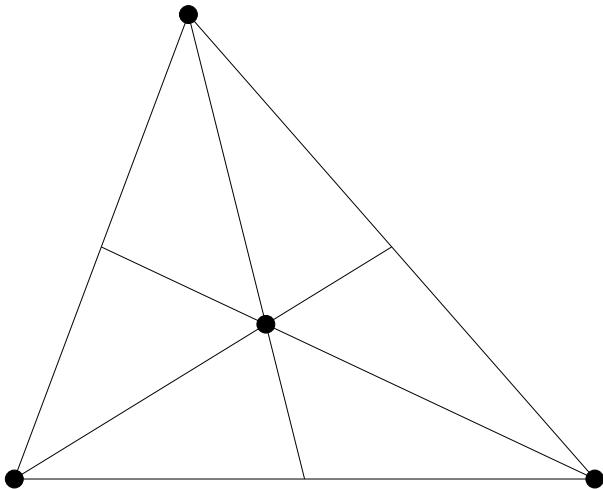
schnittS = sC[t] /. solve31

{13/3, 8/3}

N[%]

{4.33333, 2.66667}

Show[Graphics[{PointSize[0.03], Point[OA], Point[OB], Point[OC],
Point[schnittS], Line[{OA, OB, OC, OA}], Line[{OA + 1/2 AB, OC}],
Line[{OB + 1/2 BC, OA}], Line[{OC + 1/2 CA, OB}]}], AspectRatio → Automatic];
```



### c1) Umkreismittelpunkt

```
uC[t_] := OA + 1/2 AB + t nAB;
uB[s_] := OC + 1/2 CA + s nCA;
solve32 = Solve[uC[t] == uB[s], {t, s}] // Flatten

{t → 43/160, s → 7/16}

N[%]

{t → 0.26875, s → 0.4375}

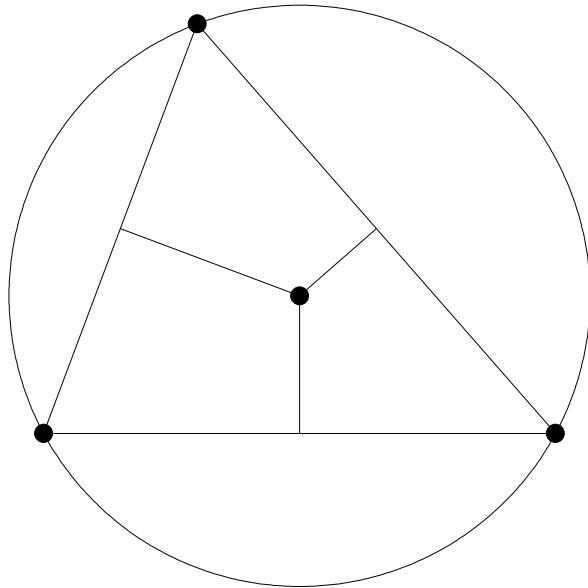
schnittU = (uC[t] /. solve32) // Simplify

{5, 43/16}
```

N[%]

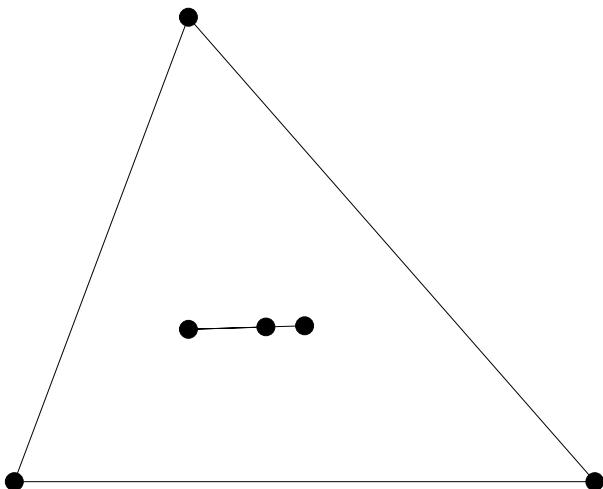
{5., 2.6875}

```
Show[Graphics[{PointSize[0.03], Point[OA], Point[OB], Point[OC],
  Point[schnittU], Line[{OA, OB, OC, OA}], Line[{OA + 1/2 AB, schnittU}],
  Line[{OB + 1/2 BC, schnittU}], Line[{OC + 1/2 CA, schnittU}], Circle[schnittU,
  Sqrt[(schnittU - OA).(schnittU - OA)]]}], AspectRatio -> Automatic];
```



#### d) Lineare Abhangigkeit

```
Show[Graphics[{PointSize[0.03], Point[OA], Point[OB], Point[OC],
  Point[schnittU], Point[schnittH], Point[schnittS], Line[{OA, OB, OC, OA}],
  Line[{schnittU, schnittH, schnittS}]}], AspectRatio -> Automatic];
```



```
Solve[schnittS - schnittH == λ (schnittU - schnittS), {λ}]
```

```
{ {λ → 2} }
```

```
Solve[schnittS - schnittH == λ (schnittU - schnittH), {λ}]
{{λ → 2/3}}
```

Linear abhängig

### e) Verhältnis

```
Solve[schnittH - schnittU == λ (schnittS - schnittU), {λ}]
{{λ → 3}}
```

#### c1) Inkreismittelpunkt

```
mC[t_] := OC + t (CA / Norm[CA] + CB / Norm[CB]);
mB[s_] := OB + s (BA / Norm[BA] + BC / Norm[BC]);
solve33 = Solve[mC[t] == mB[s], {t, s}] // Flatten
{t → 8249/(113 √73 + 73 √113 + 10 √8249), s → 1130 √73/(113 √73 + 73 √113 + 10 √8249)}
```

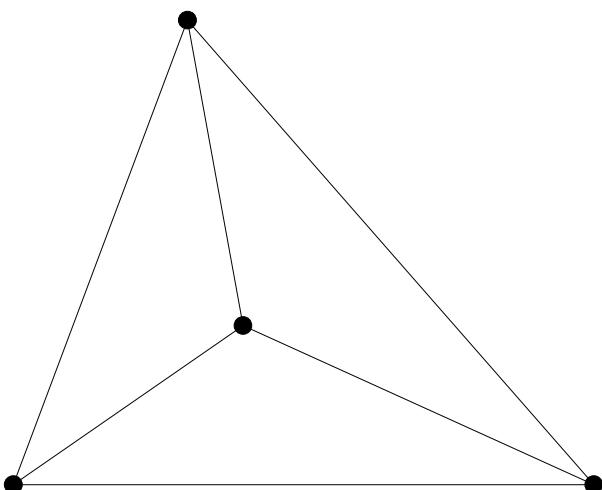
$$\left\{ t \rightarrow \frac{8249}{113 \sqrt{73} + 73 \sqrt{113} + 10 \sqrt{8249}}, s \rightarrow \frac{1130 \sqrt{73}}{113 \sqrt{73} + 73 \sqrt{113} + 10 \sqrt{8249}} \right\}$$

$$\left\{ t \rightarrow 3.11317, s \rightarrow 3.64369 \right\}$$

```
schnittI = (mC[t] /. solve33) // Simplify
{10 √113 (73 + 3 √73)/(113 √73 + 73 √113 + 10 √8249), 80 √8249/(113 √73 + 73 √113 + 10 √8249)}
```

$$\left\{ 3.95693, 2.74215 \right\}$$

```
Show[Graphics[{PointSize[0.03], Point[OA], Point[OB], Point[OC],
Point[schnittI], Line[{OA, OB, OC, OA}], Line[{OA, schnittI}],
Line[{OB, schnittI}], Line[{OC, schnittI}]}], AspectRatio → Automatic];
```



### f) Der Ausdruck h

```
a = OA - schnittU;
```

```
b = OB - schnittU;
```

```
c = OC - schnittU;
```

```
h = a + b + c
```

$$\left\{ -2, -\frac{1}{16} \right\}$$

N[%]

$$\{-2., -0.0625\}$$

```
s3 = h / 3
```

$$\left\{ -\frac{2}{3}, -\frac{1}{48} \right\}$$

N[%]

$$\{-0.666667, -0.0208333\}$$

```
(schnittH + schnittS) / 2
```

$$\left\{ \frac{11}{3}, \frac{127}{48} \right\}$$

```
(schnittH + schnittS) / 2 == schnittS + s3
```

True

### g) Der Umkreis vom Dreieck SaSbSc

```
Sc = OA + 1 / 2 AB;
```

```
Sb = OA + 1 / 2 AC;
```

```
Sa = OC + 1 / 2 CB;
```

```
uSc[t_] := Sb + 1 / 2 (Sa - Sb) + t normalV[Sa - Sb];
```

```
uSa[s_] := Sc + 1 / 2 (Sb - Sc) + s normalV[Sb - Sc];
```

```
solve35 = Solve[uSc[t] == uSa[s], {t, s}] // Flatten
```

$$\left\{ t \rightarrow -\frac{43}{160}, s \rightarrow -\frac{3}{16} \right\}$$

N[%]

$$\{t \rightarrow -0.26875, s \rightarrow -0.1875\}$$

```
schnittSU = (uSc[t] /. solve35) // Simplify
```

$$\left\{ 4, \frac{85}{32} \right\}$$

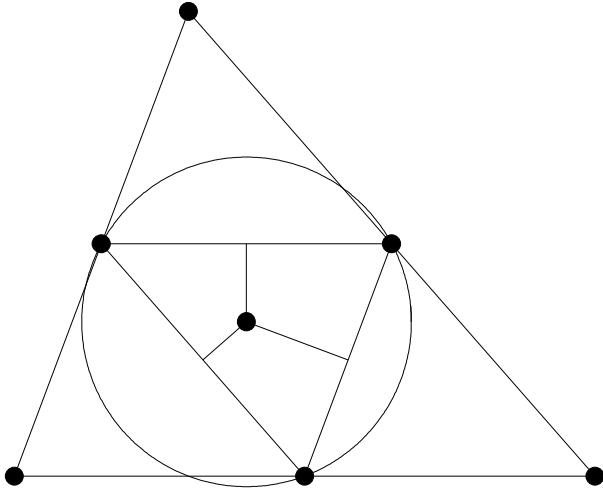
N[%]

$$\{4., 2.65625\}$$

```

sh1 = Show[Graphics[{PointSize[0.03], Point[OA], Point[OB],
Point[OC], Point[Sa], Point[Sb], Point[Sc], Line[{Sa, Sb, Sc, Sa}],
Point[schnittSU], Line[{OA, OB, OC, OA}],
Line[{Sa + 1/2 (Sb - Sa), schnittSU}], Line[{Sb + 1/2 (Sc - Sb), schnittSU}],
Line[{Sc + 1/2 (Sa - Sc), schnittSU}], Circle[schnittSU,
Sqrt[(schnittSU - Sa). (schnittSU - Sa)]]}], AspectRatio -> Automatic];

```



```
rSU = Sqrt[(schnittSU - Sa). (schnittSU - Sa)]
```

$$\frac{\sqrt{8249}}{32}$$

N[%]

2.83825

## h) Der Umkreis vom Dreieck HaHbHc

```
hC[t_] := OC + t nAB; seiteC[s_] := OA + s AB;
```

```
solv36 = Solve[hC[t] == seiteC[s], {t, s}] // Flatten;
```

```
Hc = seiteC[s] /. solv36
```

$$\{3, 0\}$$

```
hB[t_] := OB + t nCA; seiteB[s_] := OA + s AC;
```

```
solv37 = Solve[hB[t] == seiteB[s], {t, s}] // Flatten;
```

```
Hb = seiteB[s] /. solv37
```

$$\left\{ \frac{90}{73}, \frac{240}{73} \right\}$$

N[%]

{1.23288, 3.28767}

```
hA[t_] := OA + t nBC; seiteA[s_] := OB + s BC;
```

```
solv38 = Solve[hA[t] == seiteA[s], {t, s}] // Flatten;
```

```
Ha = seiteA[s] /. solv38
```

$$\left\{ \frac{640}{113}, \frac{560}{113} \right\}$$

```

uHc[t_] := Hb + 1/2 (Ha - Hb) + t normalV[Ha - Hb];
uHa[s_] := Hc + 1/2 (Hb - Hc) + s normalV[Hb - Hc];
solve39 = Solve[uHc[t] == uHa[s], {t, s}] // Flatten

{t → - $\frac{4551}{13760}$ , s → - $\frac{55}{96}$ }

N[%]

{t → -0.330741, s → -0.572917}

schnittHU = (uHc[t] /. solve39) // Simplify

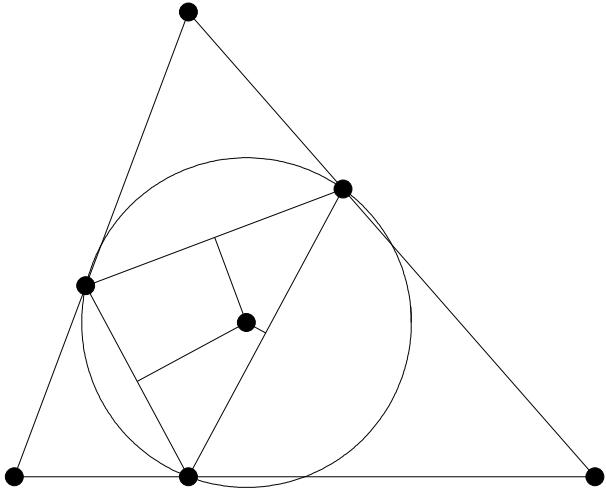
{4,  $\frac{85}{32}$ }

N[%]

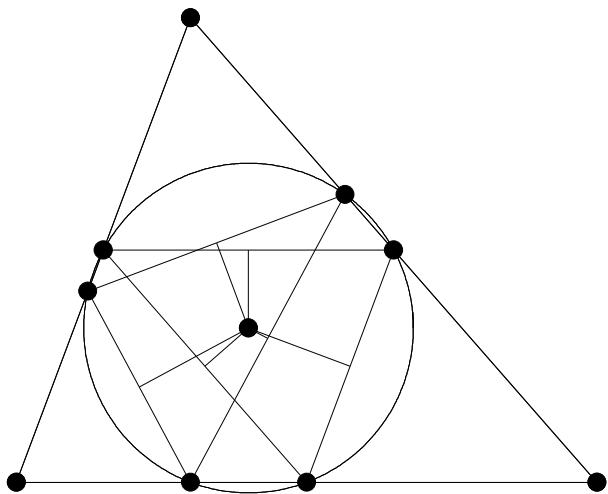
{4., 2.65625}

sh2 = Show[Graphics[{PointSize[0.03], Point[OA], Point[OB],
Point[OC], Point[Ha], Point[Hb], Point[Hc], Line[{Ha, Hb, Hc, Ha}],
Point[schnittHU], Line[{OA, OB, OC, OA}],
Line[{Ha + 1/2 (Hb - Ha), schnittHU}], Line[{Hb + 1/2 (Hc - Hb), schnittHU}],
Line[{Hc + 1/2 (Ha - Hc), schnittHU}], Circle[schnittHU,
Sqrt[(schnittHU - Ha). (schnittHU - Ha)]]}], AspectRatio → Automatic];

```



```
Show[sh1, sh2];
```



```
rHU = Sqrt[(schnittHU - Ha) . (schnittHU - Ha)]
```

$$\frac{\sqrt{8249}}{32}$$

```
N[%]
```

```
2.83825
```

```
rHU == rSU
```

```
True
```

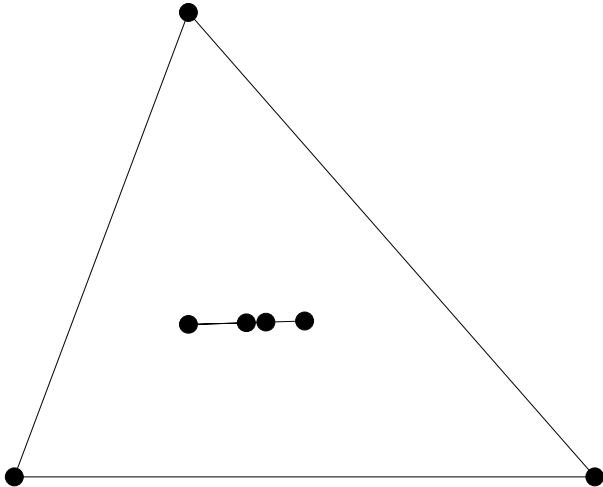
```
schnittHU == schnittSU
```

```
True
```

Gefunden ist derselbe Kreis. Dieser Kreis heisst **Feuerbachkreis**.

### i) Lineare Abhangigkeit

```
Show[Graphics[{PointSize[0.03], Point[OA], Point[OB], Point[OC], Point[schnittU],
Point[schnittH], Point[schnittS], Point[schnittHU], Line[{OA, OB, OC, OA}],
Line[{schnittU, schnittH, schnittS}]}], AspectRatio -> Automatic];
```



```
Solve[schnittU - schnittH == λ (schnittU - schnittHU), {λ}]
{{λ → 2}}
```

Linear abhangig

### j) Das Verhaltnis

```
verhaeltnis = Norm[schnittH - schnittU] / Norm[schnittHU - schnittU]
2
```

## 4

```
Remove["Global`*"]

OA = {0, 0, 1};
OB = {10, 0, 1};
OC = {3, 8, 3};
OD = {1, 2, 8};
```

### a

```
volumen = 1 / 6 Abs[Det[{OA - OD, OB - OD, OC - OD}]]
260/3
```

---

**N[%]**  
86.6667

**grundflG = Norm[Cross[OA - OC, OB - OC]] / 2**  
 $10 \sqrt{17}$

**N[%]**  
41.2311

**hD = 3 volumen / grundflG**  
 $\frac{26}{\sqrt{17}}$

**N[%]**  
6.30593

**b**

**h[t\_] := OD + t Cross[OA - OC, OB - OC];**  
**h[t]**  
 $\{1, 2 - 20t, 8 + 80t\}$

**fG[λ\_, μ\_] := OC + λ (OA - OC) + μ (OB - OC);**  
**fG[λ, μ]**  
 $\{3 - 3\lambda + 7\mu, 8 - 8\lambda - 8\mu, 3 - 2\lambda - 2\mu\}$

**solv4 = Solve[h[t] == fG[λ, μ], {t, λ, μ}] // Flatten**  
 $\left\{t \rightarrow -\frac{13}{170}, \lambda \rightarrow \frac{201}{340}, \mu \rightarrow -\frac{11}{340}\right\}$

**HD = h[t] /. solv4**  
 $\left\{1, \frac{60}{17}, \frac{32}{17}\right\}$

**N[%]**  
 $\{1., 3.52941, 1.88235\}$

**c**

**ODneu = OC + (OD - HD)**  
 $\left\{3, \frac{110}{17}, \frac{155}{17}\right\}$

**N[%]**  
 $\{3., 6.47059, 9.11765\}$