

Lösungen

1

```
Remove["Global`*"]

EVPNorm[Matrix_] :=
  Transpose[Table[N[Transpose[Matrix][[k]] / Norm[Transpose[Matrix][[k]]]],
    {k, 1, Length[Transpose[Matrix]]}]]

X = {{1, 2, 1}, {1, 2, 2}, {1, 1, 3}};
Dl = {{1, 0, 0}, {0, -1, 0}, {0, 0, -1}};
M = X.Dl.Inverse[X];
MatrixForm[M]


$$\begin{pmatrix} 7 & -10 & 4 \\ 8 & -11 & 4 \\ 8 & -10 & 3 \end{pmatrix}$$


v1 = {4, -3, 5}; v2 = {-3, 4, 5};
```

■ a

```
Eigenvalues[M]

{-1, -1, 1}
```

■ b Eigenvektoren zu doppelten Eigenwerten nicht eindeutig

```
Eigenvectors[M] // Transpose // MatrixForm


$$\begin{pmatrix} -1 & 5 & 1 \\ 0 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$


EVPNorm[Eigenvectors[M] // Transpose] // MatrixForm


$$\begin{pmatrix} -0.447214 & 0.780869 & 0.57735 \\ 0. & 0.624695 & 0.57735 \\ 0.894427 & 0. & 0.57735 \end{pmatrix}$$

```

■ c

```
Inverse[M] // MatrixForm


$$\begin{pmatrix} 7 & -10 & 4 \\ 8 & -11 & 4 \\ 8 & -10 & 3 \end{pmatrix}$$


M == Inverse[M]

True

Eigenvalues[Inverse[M]]

{-1, -1, 1}
```

■ d Eigenvektoren zu doppelten Eigenwerten nicht eindeutig

```
Eigenvectors[Inverse[M]] // Transpose // MatrixForm


$$\begin{pmatrix} -1 & 5 & 1 \\ 0 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

```

```
EVNorm[Eigenvectors[Inverse[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.447214 & 0.780869 & 0.57735 \\ 0. & 0.624695 & 0.57735 \\ 0.894427 & 0. & 0.57735 \end{pmatrix}$$

■ e

```
Eigenvalues[Transpose[M]]
```

```
{-1, -1, 1}
```

■ f

```
Eigenvectors[Transpose[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} -1 & -1 & 4 \\ 0 & 1 & -5 \\ 1 & 0 & 2 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Transpose[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.707107 & -0.707107 & 0.596285 \\ 0. & 0.707107 & -0.745356 \\ 0.707107 & 0. & 0.298142 \end{pmatrix}$$

■ g

```
w = Cross[v1, v2]
```

```
{-35, -35, 7}
```

```
InhPar = Norm[Cross[v1, v2]]
```

```
7  $\sqrt{51}$ 
```

```
N[%]
```

```
49.99
```

■ h

```
InhMPar = Norm[Cross[M.v1, M.v2]]
```

```
77  $\sqrt{123}$ 
```

```
N[%]
```

```
853.971
```

```
InhMPar / InhPar
```

$$11 \sqrt{\frac{41}{17}}$$

```
N[%]
```

```
17.0828
```

```
Det[M]
```

```
1
```

Kein direkter Einfluss sichtbar.

2

■ a

```
X = {v1, v2, w} // Transpose; X // MatrixForm
```

$$\begin{pmatrix} 4 & -3 & -35 \\ -3 & 4 & -35 \\ 5 & 5 & 7 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, -2, 0}, {0, 0, 3}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} \frac{509}{357} & \frac{611}{357} & \frac{35}{51} \\ \frac{614}{357} & \frac{359}{357} & -\frac{70}{51} \\ \frac{260}{357} & -\frac{505}{357} & -\frac{22}{51} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 1.42577 & 1.71148 & 0.686275 \\ 1.71989 & 1.0056 & -1.37255 \\ 0.728291 & -1.41457 & -0.431373 \end{pmatrix}$$

```
357 A // MatrixForm
```

$$\begin{pmatrix} 509 & 611 & 245 \\ 614 & 359 & -490 \\ 260 & -505 & -154 \end{pmatrix}$$

■ b

```
OQStrich = A.(2 v1 - 3 v2)
```

```
{-10, 18, 40}
```

■ c

```
OQStrichStrich = A.OQStrich
```

```
{44, -54, -50}
```

3

```
Remove["Global`*"]
```

■ a

```
a = {1, -1, 2}; b = {-2, 1, 4}; w = Cross[a, b]
```

```
{-6, -8, -1}
```

```
X = Transpose[{a, b, w}]; X // MatrixForm
```

$$\begin{pmatrix} 1 & -2 & -6 \\ -1 & 1 & -8 \\ 2 & 4 & -1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
B = X.Dλ.Inverse[X]; B // MatrixForm
```

$$\begin{pmatrix} \frac{29}{101} & -\frac{96}{101} & -\frac{12}{101} \\ -\frac{96}{101} & -\frac{27}{101} & -\frac{16}{101} \\ -\frac{12}{101} & -\frac{16}{101} & \frac{99}{101} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.287129 & -0.950495 & -0.118812 \\ -0.950495 & -0.267327 & -0.158416 \\ -0.118812 & -0.158416 & 0.980198 \end{pmatrix}$$

```
101 B // MatrixForm
```

$$\begin{pmatrix} 29 & -96 & -12 \\ -96 & -27 & -16 \\ -12 & -16 & 99 \end{pmatrix}$$

■ b

```
B.{5, 4, 2}
```

$$\left\{ -\frac{263}{101}, -\frac{620}{101}, \frac{74}{101} \right\}$$

```
N[%]
```

```
{-2.60396, -6.13861, 0.732673}
```

■ c

```
Dλ^100
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
Bhoch100 = X.(Dλ^100).Inverse[X]; Bhoch100 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Bhoch100 == IdentityMatrix[3]
```

```
True
```

4

$$\begin{aligned} (U^{-1}W)^{-1} (U^{-1}W) X W^T - E &= \left(\left((U^{-1})^T W^T \right)^{-1} \right)^T = > (U^{-1}W)^{-1} (U^{-1}W) = E \text{ oder} \\ &= > W^{-1} U^{-1} W X W^T - E = \left(\left((W U^{-1})^T \right)^{-1} \right)^T = \left(\left((W U^{-1})^T \right)^T \right)^{-1} = (W U^{-1})^{-1} = U W^{-1} \\ &= > X W^T = U W^{-1} + E \\ &= > X = U W^{-1} (W^T)^{-1} + (W^T)^{-1} = U (W^T W)^{-1} + (W^T)^{-1} \end{aligned}$$

5

```
Remove["Global`*"]
```

■ a

```
OP1 = {5, 0, 1}; OP2 = {4, 1, -1}; OP3 = {7 / 2, 2, 10};
```

```
OP4 = {2, 6, 1}; OP5 = {-1, 5, 8}; OP6 = {-2, 12, 0};
```

G1 = {OP1, OP2, OP3} // Transpose;

G2 = {OP4, OP5, OP6} // Transpose;

Det[G1]

$$\frac{129}{2}$$

Det[G2]

$$-290$$

G.G1 == G2

$$G \cdot \left\{ \left\{ 5, 4, \frac{7}{2} \right\}, \{0, 1, 2\}, \{1, -1, 10\} \right\} = \left\{ \{2, -1, -2\}, \{6, 5, 12\}, \{1, 8, 0\} \right\}$$

G = G2.Inverse[G1]; G // MatrixForm

$$\begin{pmatrix} \frac{16}{43} & -\frac{101}{43} & \frac{6}{43} \\ \frac{140}{129} & \frac{53}{43} & \frac{74}{129} \\ \frac{56}{129} & \frac{219}{43} & -\frac{151}{129} \end{pmatrix}$$

129 G // MatrixForm

$$\begin{pmatrix} 48 & -303 & 18 \\ 140 & 159 & 74 \\ 56 & 657 & -151 \end{pmatrix}$$

G // N // MatrixForm

$$\begin{pmatrix} 0.372093 & -2.34884 & 0.139535 \\ 1.08527 & 1.23256 & 0.573643 \\ 0.434109 & 5.09302 & -1.17054 \end{pmatrix}$$

■ b

Dreh[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}}; Dreh[φ] // MatrixForm

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix}$$

Dreh[12 Degree] // N // MatrixForm

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 0.978148 & -0.207912 \\ 0. & 0.207912 & 0.978148 \end{pmatrix}$$

■ c

OP7 = Dreh[12 Degree].OP1

$$\{5, -\sin[12^\circ], \cos[12^\circ]\}$$

Dreh[12 Degree].OP1 // N

$$\{5., -0.207912, 0.978148\}$$

■ d

G.OP7 // MatrixForm

$$\begin{pmatrix} \frac{80}{43} + \frac{6}{43} \cos[12^\circ] + \frac{101}{43} \sin[12^\circ] \\ \frac{700}{129} + \frac{74}{129} \cos[12^\circ] - \frac{53}{43} \sin[12^\circ] \\ \frac{280}{129} - \frac{151}{129} \cos[12^\circ] - \frac{219}{43} \sin[12^\circ] \end{pmatrix}$$

G.OP7 // N // MatrixForm

$$\begin{pmatrix} 2.4853 \\ 5.7312 \\ -0.0333199 \end{pmatrix}$$

6

Remove["Global`*"]

Hmatrix = {{1, -2, 3}, {1, 4, -3}, {2, 2, 4}}; Det[Hmatrix]

24

Eigenvalues[Hmatrix]

{4, 3, 2}

Inverse[Hmatrix]

$$\left\{ \left\{ \frac{11}{12}, \frac{7}{12}, -\frac{1}{4} \right\}, \left\{ -\frac{5}{12}, -\frac{1}{12}, \frac{1}{4} \right\}, \left\{ -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4} \right\} \right\}$$

N[%]

{{0.916667, 0.583333, -0.25}, {-0.416667, -0.0833333, 0.25}, {-0.25, -0.25, 0.25}}

Smatrix = Hmatrix + Transpose[Hmatrix]

{{2, -1, 5}, {-1, 8, -1}, {5, -1, 8}}

Smatrix // MatrixForm

$$\begin{pmatrix} 2 & -1 & 5 \\ -1 & 8 & -1 \\ 5 & -1 & 8 \end{pmatrix}$$

Det[Smatrix]

-72

Eigenvalues[Smatrix] // Simplify

{Root[72 + 69 #1 - 18 #1² + #1³ &, 3],
Root[72 + 69 #1 - 18 #1² + #1³ &, 2], Root[72 + 69 #1 - 18 #1² + #1³ &, 1]}

Eigenvalues[Smatrix] // N

{11.3821, 7.46528, -0.847354}

Det[Smatrix - x IdentityMatrix[3]]

-72 - 69 x + 18 x² - x³

Solve[Det[Smatrix - x IdentityMatrix[3]] == 0, {x}] // Simplify

$$\left\{ \left\{ x \rightarrow 6 + \frac{13}{\left(-27 + 2 i \sqrt{367} \right)^{1/3}} + \left(-27 + 2 i \sqrt{367} \right)^{1/3} \right\}, \right. \\ \left\{ x \rightarrow 6 - \frac{13 \left(1 + i \sqrt{3} \right)}{2 \left(-27 + 2 i \sqrt{367} \right)^{1/3}} + \frac{1}{2} i \left(i + \sqrt{3} \right) \left(-27 + 2 i \sqrt{367} \right)^{1/3} \right\}, \\ \left. \left\{ x \rightarrow 6 + \frac{13 i \left(i + \sqrt{3} \right)}{2 \left(-27 + 2 i \sqrt{367} \right)^{1/3}} - \frac{1}{2} \left(1 + i \sqrt{3} \right) \left(-27 + 2 i \sqrt{367} \right)^{1/3} \right\} \right\}$$

```

N[%]

{{x → 11.3821 - 4.44089 × 10-16 i}, {x → -0.847354 + 4.44089 × 10-16 i}, {x → 7.46528 + 0. i}}

N[%] // Chop

{{x → 11.3821}, {x → -0.847354}, {x → 7.46528}}

a1 = {1, 1, 1};
Hmatrix.a1

{2, 2, 8}

Transpose[Hmatrix].a1

{4, 4, 4}

Smatrix.a1

{6, 6, 12}

(Hmatrix + Inverse[Hmatrix]).a1

{13/4, 7/4, 31/4}

N[%]

{3.25, 1.75, 7.75}

```

7

```

Remove["Global`*"]

a = {2, 1, 3}; OQ = {-1, 4, 2}; φ = Pi / 8;

```

■ Matrixkonstruktion : Lokale Basis

```

(* Normiert einen Vektor *)
NVec[a_] := a / Norm[a];

(* Quadriert Komponenten eines Vektors *)
QVec[a_] := Table[a[[k]]^2, {k, 1, Length[a]}];

(* Numeriert Komponenten eines Vektors *)
QVecNr[a_] := Table[{k, a[[k]]^2}, {k, 1, Length[a]}];

(* Sucht die Nummer einer absolut maximal grossen Komponente *)
NrMaxQVec[a_] := Max[Table[If[a[[k]]^2 == Max[QVec[a]], k, 0], {k, 1, Length[a]}]];

(* Sucht die Nummer einer absolut minimal grossen Komponente *)
NrMinQVec[a_] :=
  Min[Table[If[a[[k]]^2 == Min[QVec[a]], k, Length[a] + 1], {k, 1, Length[a]}]];
b[a_, x_] := Table[If[k == NrMaxQVec[a], 1, If[k == NrMinQVec[a], 0, x]],
  {k, 1, Length[a]}];
solv = Solve[b[a, x].a == 0, {x}] // Flatten;
b[a_] := b[a, x] /. solv
e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};
If[Element[NVec[a], Union[{e1, e2, e3}, -{e1, e2, e3}]],
  b[a_] := Cross[e1 + e2 + e3, NVec[a]], b[a] = b[a]];
basis[a_] := {NVec[a], NVec[b[a]], Cross[NVec[a], NVec[b[a]]]};
TrBasis[a_] := basis[a] // Transpose;
aVec1 = NVec[a]; aVec2 = NVec[b[a]];
aVec3 = Cross[NVec[a], NVec[b[a]]];

```

■ Kontrolle

```

Cross[a, basis[a][[1]]]
{0, 0, 0}

basis[a][[1]].basis[a][[2]]
0

basis[a][[1]].basis[a][[3]]
0

basis[a][[2]].basis[a][[3]]
0

basis[a][[1]] // Norm
1

basis[a][[2]] // Norm
1

basis[a][[3]] // Norm
1

TrBasis[a].e1 == aVec1
True

TrBasis[a].e2 == aVec2
True

TrBasis[a].e3 == aVec3
True

```

■ Matrixzusammensetzung

```

Print[Inverse[TrBasis[a]] // MatrixForm];
mDrehung[phi_] := {{1, 0, 0}, {0, Cos[phi], -Sin[phi]}, {0, Sin[phi], Cos[phi]}};
mDrehung[Pi / 8];
Print[mDrehung[Pi / 8] // MatrixForm];
matrix[phi_] := TrBasis[a].mDrehung[phi].Inverse[TrBasis[a]];
Print[matrix[Pi / 8] // MatrixForm];
Print[matrix[Pi / 8] // N // MatrixForm];

```


$$\begin{pmatrix} \sqrt{\frac{2}{7}} & \frac{1}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ \sqrt{\frac{2}{91}} & -\sqrt{\frac{13}{14}} & \frac{3}{\sqrt{182}} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\left[\frac{\pi}{8}\right] & -\sin\left[\frac{\pi}{8}\right] \\ 0 & \sin\left[\frac{\pi}{8}\right] & \cos\left[\frac{\pi}{8}\right] \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{7} - \frac{3\left(-\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \sqrt{\frac{2}{91}}\sin\left[\frac{\pi}{8}\right]\right)}{\sqrt{13}} + \sqrt{\frac{2}{91}}\left(\sqrt{\frac{2}{91}}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) & \frac{1}{7} - \sqrt{\frac{13}{14}}\left(\sqrt{\frac{2}{91}}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) & \frac{3}{7} + \frac{2}{14} \\ \frac{1}{7} - \frac{1}{7}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{14}} & \frac{1}{14} + \frac{13}{14}\cos\left[\frac{\pi}{8}\right] & \\ \frac{3}{7} + \sqrt{\frac{2}{91}}\left(\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{182}} - \frac{2\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) - \frac{3\left(\frac{2\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{182}}\right)}{\sqrt{13}} & \frac{3}{14} - \sqrt{\frac{13}{14}}\left(\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{182}} - \frac{2\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) & -\frac{1}{14} \end{pmatrix}$$

$$\begin{pmatrix} 0.945628 & -0.295955 & 0.1349 \\ 0.317704 & 0.929317 & -0.188241 \\ -0.0696534 & 0.220864 & 0.972814 \end{pmatrix}$$

■ Drehung

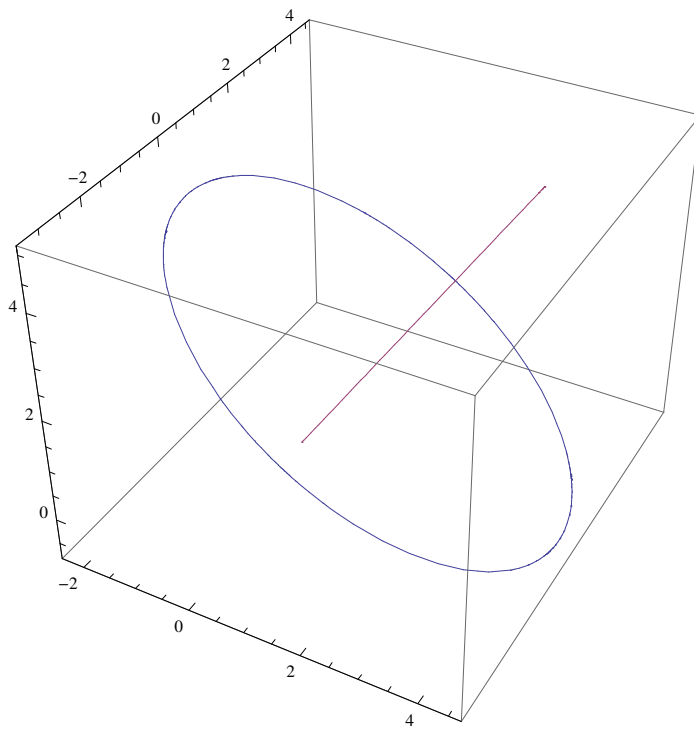
OQStrich = matrix[Pi / 8].OQ

$$\left\{ -\frac{2}{7} + \frac{3\left(-\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \sqrt{\frac{2}{91}}\sin\left[\frac{\pi}{8}\right]\right)}{\sqrt{13}} - \sqrt{\frac{2}{91}}\left(\sqrt{\frac{2}{91}}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) + 4\left(\frac{1}{7} - \sqrt{\frac{13}{14}}\left(\sqrt{\frac{2}{91}}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right)\right) + 2\left(\frac{3}{7} + \frac{2\left(-\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \sqrt{\frac{2}{91}}\sin\left[\frac{\pi}{8}\right]\right)}{\sqrt{13}} + \frac{3\left(\sqrt{\frac{2}{91}}\cos\left[\frac{\pi}{8}\right] + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right)}{\sqrt{182}}\right), -\frac{1}{7} + 4\left(\frac{1}{14} + \frac{13}{14}\cos\left[\frac{\pi}{8}\right]\right) + \frac{1}{7}\cos\left[\frac{\pi}{8}\right] - \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{14}} + 2\left(\frac{3}{14} - \frac{3}{14}\cos\left[\frac{\pi}{8}\right] - \sqrt{\frac{2}{7}}\sin\left[\frac{\pi}{8}\right]\right), -\frac{3}{7} - \sqrt{\frac{2}{91}}\left(\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{182}} - \frac{2\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right) + \frac{3\left(\frac{2\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{182}}\right)}{\sqrt{13}} + 4\left(\frac{3}{14} - \sqrt{\frac{13}{14}}\left(\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{182}} - \frac{2\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right)\right) + 2\left(\frac{9}{14} + \frac{3\left(\frac{3\cos\left[\frac{\pi}{8}\right]}{\sqrt{182}} - \frac{2\sin\left[\frac{\pi}{8}\right]}{\sqrt{13}}\right)}{\sqrt{182}} + \frac{2\left(\frac{2\cos\left[\frac{\pi}{8}\right]}{\sqrt{13}} + \frac{3\sin\left[\frac{\pi}{8}\right]}{\sqrt{182}}\right)}{\sqrt{13}}\right) \right\}$$

OQStrich // N

{-1.85965, 3.02308, 2.89874}

```
p1 = ParametricPlot3D[{matrix[phi].OQ, phi aVec1}, {phi, 0, 2 Pi}]
```



```
p2 = Graphics3D[{Sphere[OQ, 0.2], Sphere[OQStrich, 0.2]}];  
Show[p1, p2]
```

