

Lösungen

1

```
Remove["Global`*"]
```

a

```
P={{1,1},{0,β}}; P//MatrixForm
```

$$\begin{pmatrix} 1 & 1 \\ 0 & \beta \end{pmatrix}$$

```
P//TeXForm
```

```
P.P//MatrixForm
```

$$\begin{pmatrix} 1 & 1 + \beta \\ 0 & \beta^2 \end{pmatrix}$$

```
P.P.P//ExpandAll//MatrixForm
```

$$\begin{pmatrix} 1 & 1 + \beta + \beta^2 \\ 0 & \beta^3 \end{pmatrix}$$

```
P.P.P.P//ExpandAll//MatrixForm
```

$$\begin{pmatrix} 1 & 1 + \beta + \beta^2 + \beta^3 \\ 0 & \beta^4 \end{pmatrix}$$

b

```
Q=Inverse[P]; Q//MatrixForm
```

$$\begin{pmatrix} 1 & -\frac{1}{\beta} \\ 0 & \frac{1}{\beta} \end{pmatrix}$$

```
Q.Q//MatrixForm
```

$$\begin{pmatrix} 1 & -\frac{1}{\beta^2} - \frac{1}{\beta} \\ 0 & \frac{1}{\beta^2} \end{pmatrix}$$

```
Q.Q.Q//ExpandAll//MatrixForm
```

$$\begin{pmatrix} 1 & -\frac{1}{\beta^3} - \frac{1}{\beta^2} - \frac{1}{\beta} \\ 0 & \frac{1}{\beta^3} \end{pmatrix}$$

Q.Q.Q.Q//ExpandAll//MatrixForm

$$\begin{pmatrix} 1 & -\frac{1}{\beta^4} - \frac{1}{\beta^3} - \frac{1}{\beta^2} - \frac{1}{\beta} \\ 0 & \frac{1}{\beta^4} \end{pmatrix}$$

C

H={{u,v},{0,w}}; H//MatrixForm

$$\begin{pmatrix} u & v \\ 0 & w \end{pmatrix}$$

hilf=H.Transpose[H]; hilf//MatrixForm

$$\begin{pmatrix} u^2 + v^2 & v w \\ v w & w^2 \end{pmatrix}$$

{{u,v},{0,w}}//TeXForm

Solve[hilf=={{2,4},{4,16}},{u,v,w}]

$$\{\{w \rightarrow -4, u \rightarrow -1, v \rightarrow -1\}, \{w \rightarrow -4, u \rightarrow 1, v \rightarrow -1\}, \{w \rightarrow 4, u \rightarrow -1, v \rightarrow 1\}, \{w \rightarrow 4, u \rightarrow 1, v \rightarrow 1\}\}$$

H={{1,1},{0,4}}; H//MatrixForm

$$\begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix}$$

hilf=H.Transpose[H]; hilf//MatrixForm

$$\begin{pmatrix} 2 & 4 \\ 4 & 16 \end{pmatrix}$$

P.Transpose[P]/.b->4//TeXForm

2

a

i

A={{5,-1,3},{3,5,1},{-1,3,\alpha}};A//MatrixForm

$$\begin{pmatrix} 5 & -1 & 3 \\ 3 & 5 & 1 \\ -1 & 3 & \alpha \end{pmatrix}$$

A//TeXForm

```
B=Transpose[{{1,1,1}}];B//MatrixForm
```

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

```
A.B//MatrixForm
```

$$\begin{pmatrix} 7 \\ 9 \\ 2 + \alpha \end{pmatrix}$$

```
Solve[A.B==Transpose[{{7,9,5}}],{α}]
```

```
{{α → 3}}
```

ii

```
(A.B/.α->5)//MatrixForm
```

$$\begin{pmatrix} 7 \\ 9 \\ 7 \end{pmatrix}$$

iii

```
Det[A]
```

```
28 + 28 α
```

```
Solve[Det[A]==0,{α}]
```

```
{{α → -1}}
```

b

i

```
B={{5,-1,3},{3,5,1},{-1,3,-1}};B//MatrixForm
```

$$\begin{pmatrix} 5 & -1 & 3 \\ 3 & 5 & 1 \\ -1 & 3 & -1 \end{pmatrix}$$

```
k[t_]:=Transpose[{{-4t,t,7t}}];
```

```
m[t_]:=Transpose[{{x,y,z}}];
```

```
hN[t_]:=B.k[t]; hN[t]//MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

ii

```
RowReduce[B]//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & \frac{4}{7} \\ 0 & 1 & -\frac{1}{7} \\ 0 & 0 & 0 \end{pmatrix}$$

3

```
I^(1/6)
```

$$(-1)^{1/12}$$

```
I^(1/6)//N
```

$$0.965926 + 0.258819 i$$

a

```
Remove["Global`*"]
```

```
solv3=Solve[z^6==I,{z]//Flatten
```

$$\{z \rightarrow -(-1)^{1/12}, z \rightarrow (-1)^{1/12}, z \rightarrow -(-1)^{5/12}, z \rightarrow (-1)^{5/12}, z \rightarrow -(-1)^{3/4}, z \rightarrow (-1)^{3/4}\}$$

```
%//N
```

$$\{z \rightarrow -0.965926 - 0.258819 i, z \rightarrow 0.965926 + 0.258819 i, z \rightarrow -0.258819 - 0.965926 i, z \rightarrow 0.258819 + 0.965926 i, z \rightarrow 0.707107 - 0.707107 i, z \rightarrow -0.707107 + 0.707107 i\}$$

```
tabZ = Table[zA[u]=z/.solv3[[u]],{u,1,6}]
```

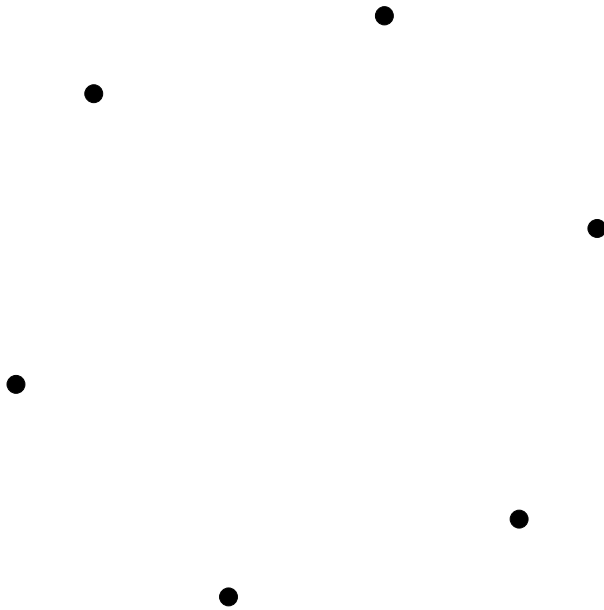
$$\{-(-1)^{1/12}, (-1)^{1/12}, -(-1)^{5/12}, (-1)^{5/12}, -(-1)^{3/4}, (-1)^{3/4}\}$$

b

```
tabPoints=Prepend[ Table[Point[{Re[zA[u]],Im[zA[u]]}],{u,1,6}],PointSize[0.03]]
```

$$\{\text{PointSize}[0.03], \text{Point}\left[\left\{-\frac{1+\sqrt{3}}{2\sqrt{2}}, -\frac{-1+\sqrt{3}}{2\sqrt{2}}\right\}\right], \text{Point}\left[\left\{\frac{1+\sqrt{3}}{2\sqrt{2}}, \frac{-1+\sqrt{3}}{2\sqrt{2}}\right\}\right], \text{Point}\left[\left\{-\frac{-1+\sqrt{3}}{2\sqrt{2}}, -\frac{1+\sqrt{3}}{2\sqrt{2}}\right\}\right], \text{Point}\left[\left\{\frac{-1+\sqrt{3}}{2\sqrt{2}}, \frac{1+\sqrt{3}}{2\sqrt{2}}\right\}\right], \text{Point}\left[\left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right], \text{Point}\left[\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right]\}$$

```
Show[Graphics[tabPoints],AspectRatio->Automatic];
```

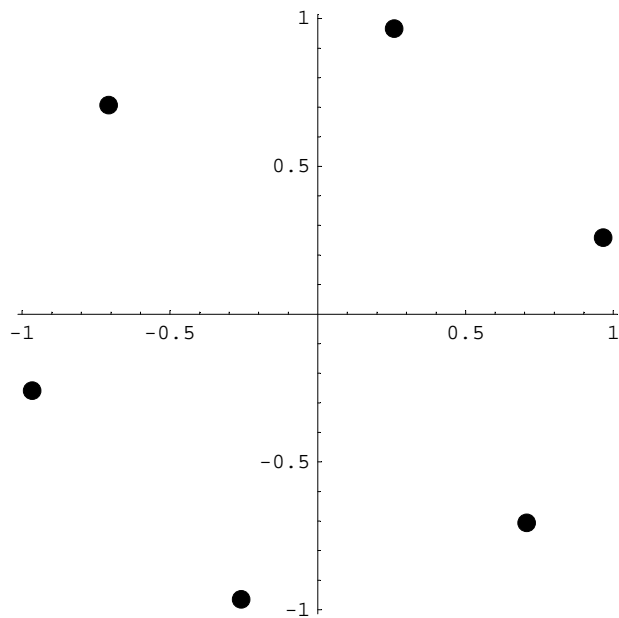


```
tabPoints=Table[{Re[zA[u]],Im[zA[u]]},{u,1,6}]
```

$$\left\{ \left\{ -\frac{1+\sqrt{3}}{2\sqrt{2}}, -\frac{-1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{1+\sqrt{3}}{2\sqrt{2}}, \frac{-1+\sqrt{3}}{2\sqrt{2}} \right\}, \right.$$

$$\left. \left\{ -\frac{-1+\sqrt{3}}{2\sqrt{2}}, -\frac{1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{-1+\sqrt{3}}{2\sqrt{2}}, \frac{1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\}, \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\} \right\}$$

```
ListPlot[tabPoints,PlotStyle->{PointSize[0.03]},AspectRatio->Automatic];
```



c**Plus@@tabPoints** $\{0, 0\}$ $\{0, 0\}$ **Apply[Plus, tabPoints]** $\{0, 0\}$ **Plus[tabPoints]**

$$\left\{ \left\{ -\frac{1+\sqrt{3}}{2\sqrt{2}}, -\frac{-1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{1+\sqrt{3}}{2\sqrt{2}}, \frac{-1+\sqrt{3}}{2\sqrt{2}} \right\}, \right.$$

$$\left. \left\{ -\frac{-1+\sqrt{3}}{2\sqrt{2}}, -\frac{1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{-1+\sqrt{3}}{2\sqrt{2}}, \frac{1+\sqrt{3}}{2\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\}, \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\} \right\}$$
d**z1=E^(I Pi/2/6)//N** $0.965926 + 0.258819 i$ **z1^5** $0.258819 + 0.965926 i$ $z1^5 = zA[[4]]$ **1/z1** $0.965926 - 0.258819 i$ Nicht in der Liste der $z[[u]]$, jedoch die Koordinaten sind in der Liste**Conjugate[z1]** $0.965926 - 0.258819 i$ **z1+1/z1** $1.93185 + 0. i$

Liegt auf der reellen Achse

4**Remove["Global`*"]**

a**i**

```
z[a_]:= a+I; w[a_]:=1/z[a]; Solve[z[a]+w[a]==0,{a}]
{{a -> 0}, {a -> -2 i}}
Solve[(a+I)^2+1==0,{a}]
{{a -> 0}, {a -> -2 i}}
```

$a = 0$ ist die einzige reelle Lösung. z ist dann gleich i .

ii

```
Solve[(a+I)^2+r==0,{a}]
{{a -> -i - sqrt(-r)}, {a -> -i + sqrt(-r)}}
```

b

```
Remove[z]
```

i

```
p1={4,1,-1};p2={3,-4,8};
q1={5,15,2};q2={8,-11,z};
{p1,p2,q1,q2}//Transpose//MatrixForm

$$\begin{pmatrix} 4 & 3 & 5 & 8 \\ 1 & -4 & 15 & -11 \\ -1 & 8 & 2 & z \end{pmatrix}$$

Det[{p2-p1,q1-p1,q2-p1}]
-717 - 9 z
slv=Solve[Det[{p2-p1,q1-p1,q2-p1}]==0,{z}]/Flatten;
z=z/.slv

$$-\frac{239}{3}$$

%/N
-79.6667
{p1,p2,q1,q2}//Transpose//MatrixForm

$$\begin{pmatrix} 4 & 3 & 5 & 8 \\ 1 & -4 & 15 & -11 \\ -1 & 8 & 2 & -\frac{239}{3} \end{pmatrix}$$

```

ii

```
g1[t1_]:=p1+t1 (p2-p1);
g2[t2_]:=q1+t2 (q2-q1);
slv1=Solve[g1[t1]==g2[t2],{t1,t2}]/Flatten
```

$$\left\{t1 \rightarrow -\frac{68}{41}, t2 \rightarrow \frac{9}{41}\right\}$$

```
{t1,t2}={t1,t2}/.slv1
```

$$\left\{-\frac{68}{41}, \frac{9}{41}\right\}$$

```
g1[t1]
```

$$\left\{\frac{232}{41}, \frac{381}{41}, -\frac{653}{41}\right\}$$

```
s1=g2[t2]
```

$$\left\{\frac{232}{41}, \frac{381}{41}, -\frac{653}{41}\right\}$$

```
g2[t2]/N
```

$$\{5.65854, 9.29268, -15.9268\}$$

iii

```
cr=Cross[p1-s1,q1-s1]
```

$$\left\{-\frac{9588}{41}, \frac{816}{41}, -\frac{612}{41}\right\}$$

```
Norm[cr]/2
```

$$\frac{102\sqrt{2234}}{41}$$

```
%/N
```

$$117.587$$