Lösungen

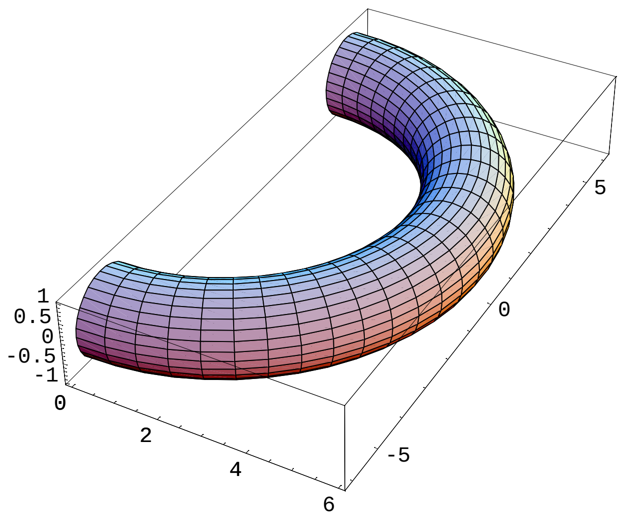
1

```
Remove["Global`*"]
```

a

```
x[r_,u_,t_]:= (r Cos[u]+5) Sin[t];
y[r_,u_,t_]:= (r Cos[u]+5) Cos[t];
z[r_,u_,t_]:= r Sin[u];
```

```
w[u_,t_]:={(Cos[u]+5) Sin[t], (Cos[u]+5) Cos[t],Sin[u]};
ParametricPlot3D[w[u,t],{u,0,2Pi},{t,0,Pi}];
```



Halber Ring

b

```
j[r_,u_,t_]:= Det[{
{D[x[r,u,t],{r}],D[x[r,u,t],{u}],D[x[r,u,t],{t}]},
{D[y[r,u,t],{r}],D[y[r,u,t],{u}],D[y[r,u,t],{t}]},
{D[z[r,u,t],{r}],D[z[r,u,t],{u}],D[z[r,u,t],{t}]}
}];
j[r,u,t]//Simplify
```

$r\,(5 + r\,\mathrm{Cos}[u])$

```
j[r_,u_,t_]:= r (5+r Cos[u]);
```

c

```
V1 = Integrate[j[r,u,t],{r,0,1},{u,0,2Pi},{t,0,Pi}]
```

$5\pi^2$

```
N[%]
```

49.348

d

```
V2 = Integrate[j[r,u,t],{r,0,2},{u,0,2Pi},{t,0,Pi}]
```

$20\pi^2$

```
N[%]
```

197.392

```
V2/V1
```

4

2 Rohr mit Schauglas

Programm

Programm laufen lassen

```
Remove["Global`*"];

volProgr[r_, α_, schritt_] := Module[{},
   h0 = 2 r / Cos[α];
   xh[h_, rR_, αα_] := rR - h Cos[αα];
   z1[x_, rR_, αα_] := -x Tan[αα] + rR Tan[αα];
   z2b[x_, rR_, αα_] := x Tan[Pi / 2 - αα] + rR Tan[αα] + b;
   solv = Solve[z1[xh[h, r, α], r, α] == z2b[xh[h, r, α], r, α], {b}] // Flatten;
   b = b /. solv;
   z2[x_, rR_, αα_, h_] = z2b[x, rR, αα] /. solv;
   IntZ[x_, h_] :=
    Integrate[1, {z, z1[x, r, α], z2[x, r, α, h]}
     , GenerateConditions → False];
   IntZY[x_, h_] :=
    Integrate[IntZ[x, h], {y, -Sqrt[r^2 - x^2], Sqrt[r^2 - x^2]}];
   IntZYX[h_] := (Integrate[IntZY[x, h], x] /. x -> r) -
     (Integrate[IntZY[x, h], x] /. x -> r - h Cos[α]);
   Print["Plot der Kurve 'Volumen als Funktion von h' "];
   Plot[Evaluate[IntZYX[h]], {h, 0, h0}];
   Print["Tabelle zur Kurve 'Volumen als Funktion von h' "];
   Prepend[Table[{h, IntZYX[h] // N}, {h, 0, h0, schritt}], {h, V}] // MatrixForm
  ];
```

Eingaben

```
r = 5;  schritt = 1; (* Tabelle am Schluss *)
α = 40 Degree;  α // N ;
```

```
volProgr[r, α, schritt]
```

Plot der Kurve 'Volumen als Funktion von h'

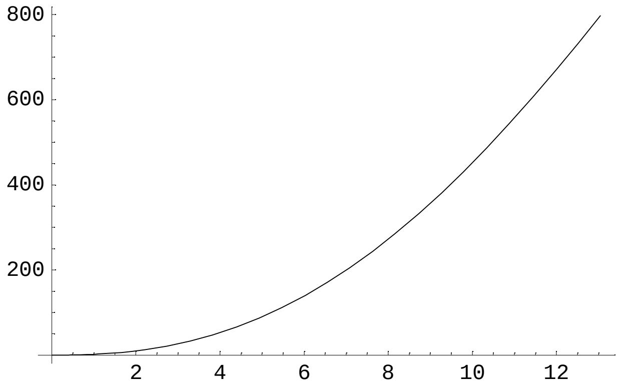


Tabelle zur Kurve 'Volumen als Funktion von h'

h	V
0	0.
1	1.73
2	9.61744
3	26.0259
4	52.4226
5	89.7775
6	138.694
7	199.465
8	272.099
9	356.318
10	451.536
11	556.806
12	670.693
13	790.908

```
Evaluate[IntZYX[8]]//N
```

272.099

3

```
Remove["Global`*"];
```

a

```
LaplaceTransform[Cosh[t]+Sinh[2 t],t,s]
```

$$\frac{2}{-4+s^2}+\frac{s}{-1+s^2}$$

```
%//Together
```

$$\frac{-2-4\,s+2\,s^2+s^3}{(-4+s^2)\,(-1+s^2)}$$

b

```
LaplaceTransform[ⅇ^t - 2 ⅇ^t Sin[t], t, s]
```

$$\frac{1}{-1+s}-\frac{2}{2-2s+s^2}$$

```
% // Together
```

$$\frac{4-4s+s^2}{(-1+s)\,(2-2s+s^2)}$$

c

```
LaplaceTransform[t/(1 + t^2), t, s] (* Achtung: Etwas schwierig *)
```

$$-\mathrm{Cos}[s]\ \mathrm{CosIntegral}[s]+\frac{1}{2}\mathrm{Sin}[s]\ (\pi-2\ \mathrm{SinIntegral}[s])$$

```
?CosIntegral
```

CosIntegral[z] gives the cosine integral function Ci(z). Mehr…

$$\mathrm{CosIntegral}[z]=-\int_z^\infty \cos(t)\,/\,t\,dt$$

d

```
?Dirac*
```

DiracDelta[x] represents the Dirac delta function δ(x). DiracDelta[x1, x2, ...] represents the multidimensional Dirac delta function δ(x1, x2, …). Mehr…

```
LaplaceTransform[DiracDelta[t] + 1 + t + t^2 + t^3, t, s]
```

$$1+\frac{6}{s^4}+\frac{2}{s^3}+\frac{1}{s^2}+\frac{1}{s}$$

```
%//Together
```

$$\frac{6+2s+s^2+s^3+s^4}{s^4}$$

e

```
InverseLaplaceTransform[s/(s^2 - 1), s, t]
```

$$\frac{1}{2}e^{-t}\,(1+e^{2t})$$

```
%//Expand
```

$$\frac{e^{-t}}{2}+\frac{e^t}{2}$$

f

```
InverseLaplaceTransform[10 s/((2 s)^2 - 4) + 5/(1 + s + s^2), s, t]
```

$$\frac{5}{12}\, e^{-t} \left(3 + 3\, e^{2 t} + 8 \sqrt{3}\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

```
%//Expand
```

$$\frac{5\, e^{-t}}{4} + \frac{5\, e^{t}}{4} + \frac{10\, e^{-t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$$

g

```
InverseLaplaceTransform[4 s/(8 + (2 s)^3), s, t]
```

$$\frac{1}{6}\, e^{-t} \left(-1 + e^{3 t/2} \left(\operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)\right)$$

```
%//TrigExpand
```

$$-\frac{e^{-t}}{6} + \frac{1}{6}\, e^{t/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \frac{e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{2 \sqrt{3}}$$

```
%//TrigFactor
```

$$\frac{1}{6}\, e^{-t} \left(-1 + e^{3 t/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, e^{3 t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

h

```
InverseLaplaceTransform[3/(1 + s^2 + s^4), s, t]
```

$$\frac{1}{2}\, e^{-t/2} \left(-3 \left(-1 + e^{t}\right) \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3} \left(1 + e^{t}\right) \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

```
%//TrigExpand
```

$$\frac{3}{2}\, e^{-t/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] - \frac{3}{2}\, e^{t/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \frac{1}{2} \sqrt{3}\, e^{-t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right] + \frac{1}{2} \sqrt{3}\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]$$

```
%//TrigFactor
```

$$\frac{1}{2}\, e^{-t/2} \left(3 \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] - 3\, e^{t} \operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, e^{t} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

4

```
Remove["Global`*"]
```

```
links = LaplaceTransform[4 y''[t] - 2 y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

$Y[s] - 2(-1 + s\,Y[s]) + 4(-1 - s + s^2\,Y[s])$

```
LaplaceTransform[DiracDelta[t-1],t,s]
```

e^{-s}

```
rechts = LaplaceTransform[DiracDelta[t] + DiracDelta[t - 1] + DiracDelta[t - 2] , t, s]
```

$1 + e^{-2 s} + e^{-s}$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{e^{-2 s}(1 + e^{s} + 3 e^{2 s} + 4 e^{2 s} s)}{1 - 2 s + 4 s^2}\right\}$$

```
%//ExpandAll
```

$$\left\{Y[s] \to \frac{1}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{e^{s}}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{3 e^{2 s}}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{4 e^{2 s} s}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2}\right\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{e^{-2 s}(1 + e^{s} + 3 e^{2 s} + 4 e^{2 s} s)}{1 - 2 s + 4 s^2}$$

```
%//ExpandAll
```

$$\frac{1}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{e^{s}}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{3 e^{2 s}}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2} + \frac{4 e^{2 s} s}{e^{2 s} - 2 e^{2 s} s + 4 e^{2 s} s^2}$$

```
U[s] // Apart
```

$$\frac{e^{-2 s}}{1 - 2 s + 4 s^2} + \frac{e^{-s}}{1 - 2 s + 4 s^2} + \frac{3 + 4 s}{1 - 2 s + 4 s^2}$$

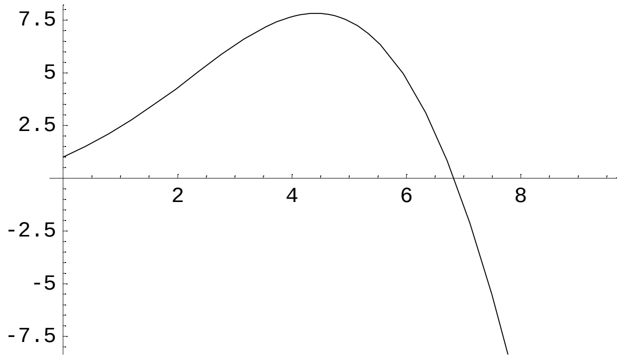
```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3} e^{\frac{1}{4}(-2+t)} \left(\sqrt{e}\left(3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{4}\right] + 4\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{4}\right]\right) + \sqrt{3}\,\mathrm{Sin}\left[\frac{1}{4}\sqrt{3}\,(-2 + t)\right] \mathrm{UnitStep}[-2 + t] + \sqrt{3}\,e^{1/4}\,\mathrm{Sin}\left[\frac{1}{4}\sqrt{3}\,(-1 + t)\right] \mathrm{UnitStep}[-1 + t]\right)$$

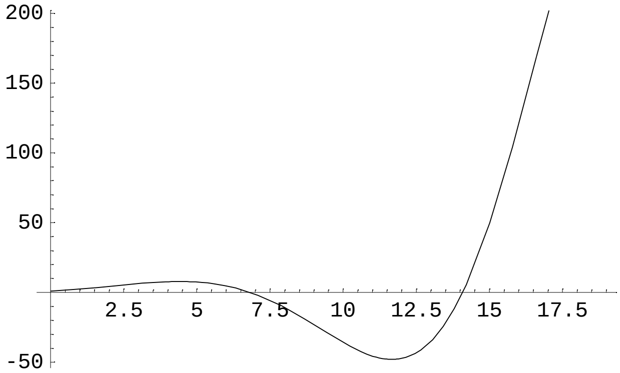
```
u0[t]//Expand
```

$$e^{t/4}\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{4}\right] + \frac{4 e^{t/4}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{4}\right]}{\sqrt{3}} + \frac{e^{-\frac{1}{2}+\frac{t}{4}}\,\mathrm{Sin}\left[\frac{1}{4}\sqrt{3}\,(-2 + t)\right] \mathrm{UnitStep}[-2 + t]}{\sqrt{3}} + \frac{e^{-\frac{1}{4}+\frac{t}{4}}\,\mathrm{Sin}\left[\frac{1}{4}\sqrt{3}\,(-1 + t)\right] \mathrm{UnitStep}[-1 + t]}{\sqrt{3}}$$

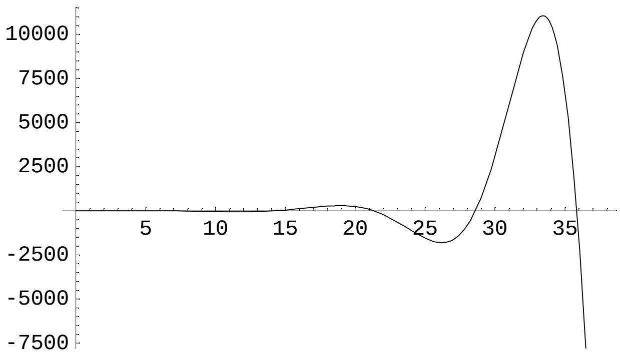
Plot[Evaluate[{u0[t]}],{t,0,3Pi}];



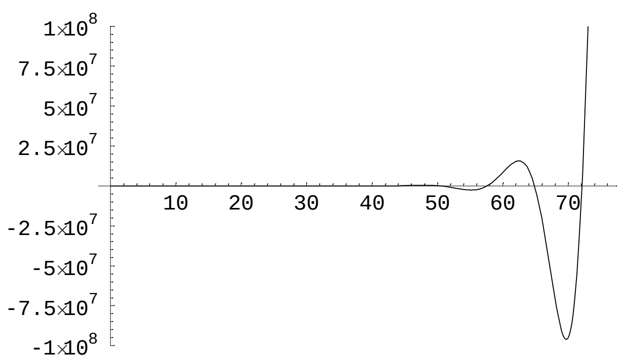
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];



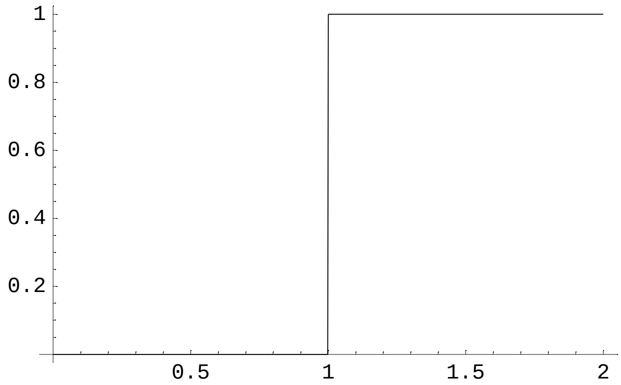
Plot[Evaluate[{u0[t]}],{t,0,12Pi}];



Plot[Evaluate[{u0[t]}],{t,0,24Pi},PlotRange->{-10^8,10^8}];



```
Plot[ UnitStep[-1+t],{t,0,2}];
```



5

a RBD y1[0]->0, y1'[0]->0, y2[0]->1, y2'[0]->0

```
Remove["Global`*"];

links1 = LaplaceTransform[y2''[t] + a y2[t] - b y2[t] + b y1[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->0, y1'[0]->0, y2[0]->1, y2'[0]->0}
```

$-s + b\,Y1[s] + a\,Y2[s] - b\,Y2[s] + s^2\,Y2[s]$

```
links2 = LaplaceTransform[y1''[t] + a y1[t] - b y1[t] + b y2[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->0, y1'[0]->0, y2[0]->1, y2'[0]->0}
```

$a\,Y1[s] - b\,Y1[s] + s^2\,Y1[s] + b\,Y2[s]$

```
solv=Solve[{links1 == 0, links2 == 0},{Y1[s],Y2[s]}] // Flatten
```

$$\left\{Y1[s] \to \frac{b\,s}{(-a+2\,b-s^2)\,(a+s^2)},\ Y2[s] \to -\frac{-a\,s+b\,s-s^3}{(a+s^2)\,(a-2\,b+s^2)}\right\}$$

```
%//ExpandAll
```

$$\left\{Y1[s] \to \frac{b\,s}{-a^2+2\,a\,b-2\,a\,s^2+2\,b\,s^2-s^4},\ Y2[s] \to \frac{a\,s}{a^2-2\,a\,b+2\,a\,s^2-2\,b\,s^2+s^4} - \frac{b\,s}{a^2-2\,a\,b+2\,a\,s^2-2\,b\,s^2+s^4} + \frac{s^3}{a^2-2\,a\,b+2\,a\,s^2-2\,b\,s^2+s^4}\right\}$$

```
U1[s]:=Y1[s]/. solv[[1]];
U2[s]:=Y2[s]/. solv[[2]];

u1[t_]:=InverseLaplaceTransform[U1[s],s,t]//Simplify; Print["u1(t) = ",u1[t]];
```

$$u1(t) = \frac{1}{4}\left(-e^{-\sqrt{-a+2\,b}\,t} - e^{\sqrt{-a+2\,b}\,t} + 2\,Cos[\sqrt{a}\,t]\right)$$

```
u2[t_]:=InverseLaplaceTransform[U2[s],s,t]//Simplify; Print["u2(t) = ",u2[t]];
```

$$u2(t) = \frac{1}{4}\left(e^{-\sqrt{-a+2\,b}\,t} + e^{\sqrt{-a+2\,b}\,t} + 2\,Cos[\sqrt{a}\,t]\right)$$

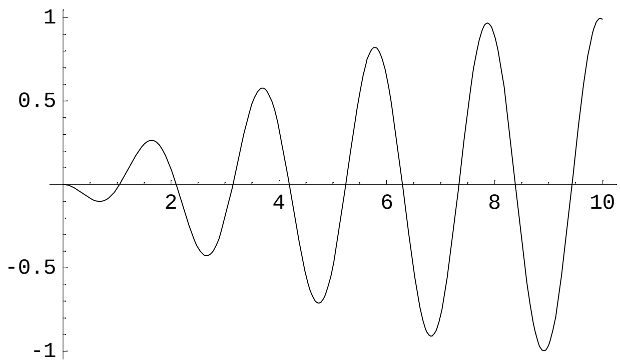
```
u1P[t]:=u1[t]/. {a -> 10, b->1};
u1P[t]
```

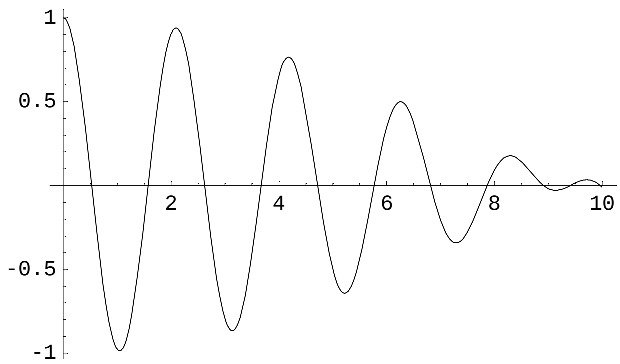
$$\frac{1}{4}\left(-e^{-2 i \sqrt{2} t}-e^{2 i \sqrt{2} t}+2 \operatorname{Cos}\left[\sqrt{10} t\right]\right)$$

```
u2P[t]:=u2[t]/. {a -> 10, b->1};
u2P[t]
```

$$\frac{1}{4}\left(e^{-2 i \sqrt{2} t}+e^{2 i \sqrt{2} t}+2 \operatorname{Cos}\left[\sqrt{10} t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,10}];
Plot[Evaluate[u2P[t]],{t,0,10}];
```





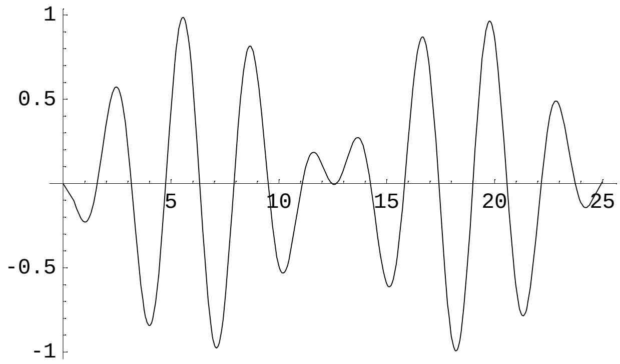

```
u1P[t]:=u1[t]/. {a -> 5, b->1};
u1P[t]
```

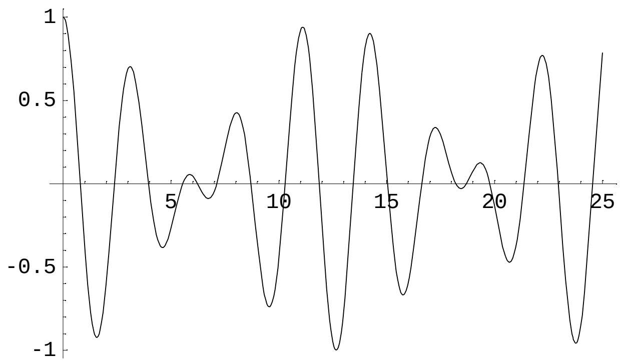
$$\frac{1}{4}\left(-e^{-i \sqrt{3} t}-e^{i \sqrt{3} t}+2 \operatorname{Cos}\left[\sqrt{5} t\right]\right)$$

```
u2P[t]:=u2[t]/. {a -> 5, b->1};
u2P[t]
```

$$\frac{1}{4}\left(e^{-i \sqrt{3} t}+e^{i \sqrt{3} t}+2 \operatorname{Cos}\left[\sqrt{5} t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```





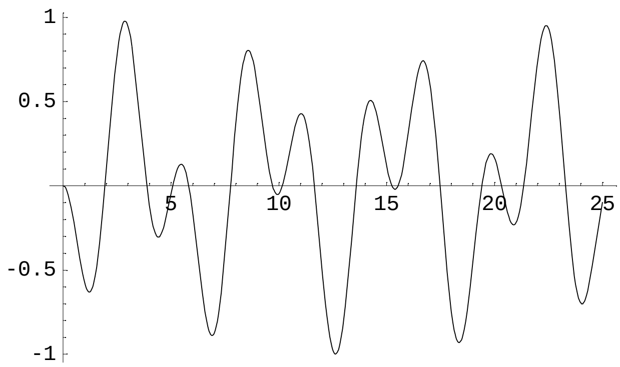

```
u1P[t]:=u1[t]/. {a -> 5, b->2};
u1P[t]
```

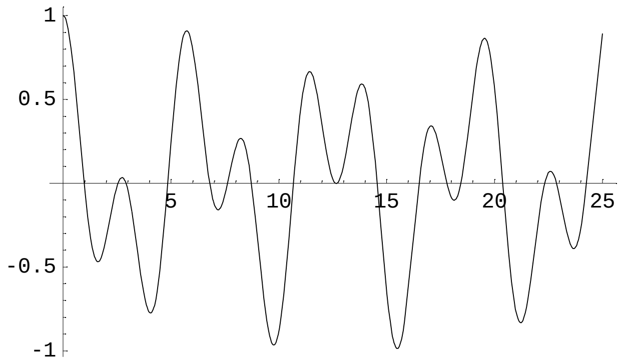
$$\frac{1}{4}\left(-e^{-i t}-e^{i t}+2 \operatorname{Cos}\left[\sqrt{5}\, t\right]\right)$$

```
u2P[t]:=u2[t]/. {a -> 5, b->2};
u2P[t]
```

$$\frac{1}{4}\left(e^{-i t}+e^{i t}+2 \operatorname{Cos}\left[\sqrt{5}\, t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```





b RBD y1[0]->0, y1'[0]->0, y2[0]->1, y2'[0]->0

```
Remove["Global`*"];
```

```
links1 = LaplaceTransform[y2''[t] + a y2[t] - b y2[t] + b y1[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->0, y1'[0]->1, y2[0]->1, y2'[0]->0}
```

$-s + b\,Y1[s] + a\,Y2[s] - b\,Y2[s] + s^2\,Y2[s]$

```
links2 = LaplaceTransform[y1''[t] + a y1[t] - b y1[t] + b y2[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->0, y1'[0]->1, y2[0]->1, y2'[0]->0}
```

$-1 + a\,Y1[s] - b\,Y1[s] + s^2\,Y1[s] + b\,Y2[s]$

```
solv=Solve[{links1 == 0, links2 == 0},{Y1[s],Y2[s]}] // Flatten
```

$$\left\{Y1[s] \to -\frac{a - b - b\,s + s^2}{(-a + 2\,b - s^2)\,(a + s^2)},\ Y2[s] \to -\frac{-b + a\,s - b\,s + s^3}{(-a + 2\,b - s^2)\,(a + s^2)}\right\}$$

```
%//ExpandAll
```

$$\left\{Y1[s] \to -\frac{a}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} + \frac{b}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} + \frac{b\,s}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} - \frac{s^2}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4},\right.$$
$$\left. Y2[s] \to \frac{b}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} - \frac{a\,s}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} + \frac{b\,s}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4} - \frac{s^3}{-a^2 + 2\,a\,b - 2\,a\,s^2 + 2\,b\,s^2 - s^4}\right\}$$

```
U1[s]:=Y1[s]/. solv[[1]];
U2[s]:=Y2[s]/. solv[[2]];
```

```
u1[t_]:=InverseLaplaceTransform[U1[s],s,t]//Simplify; Print["u1(t) = ",u1[t]];
```

$$u1(t) = \frac{1}{4}\left(-\frac{e^{-\sqrt{-a+2\,b}\,t}\left(1 + \sqrt{-a + 2\,b} + \left(-1 + \sqrt{-a + 2\,b}\right) e^{2\sqrt{-a+2\,b}\,t}\right)}{\sqrt{-a + 2\,b}} + 2\,\mathrm{Cos}[\sqrt{a}\,t] + \frac{2\,\mathrm{Sin}[\sqrt{a}\,t]}{\sqrt{a}}\right)$$

```
u2[t_]:=InverseLaplaceTransform[U2[s],s,t]//Simplify; Print["u2(t) = ",u2[t]];
```

$$u2(t) = \frac{1}{4}\left(\frac{e^{-\sqrt{-a+2\,b}\,t}\left(1 + \sqrt{-a + 2\,b} + \left(-1 + \sqrt{-a + 2\,b}\right) e^{2\sqrt{-a+2\,b}\,t}\right)}{\sqrt{-a + 2\,b}} + 2\,\mathrm{Cos}[\sqrt{a}\,t] + \frac{2\,\mathrm{Sin}[\sqrt{a}\,t]}{\sqrt{a}}\right)$$

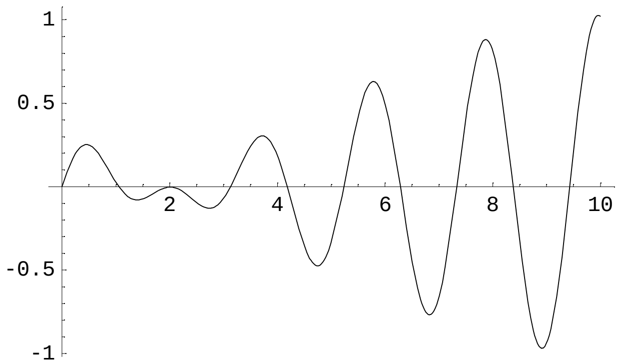
```
u1P[t]:=u1[t]/. {a -> 10, b->1};
u1P[t]
```

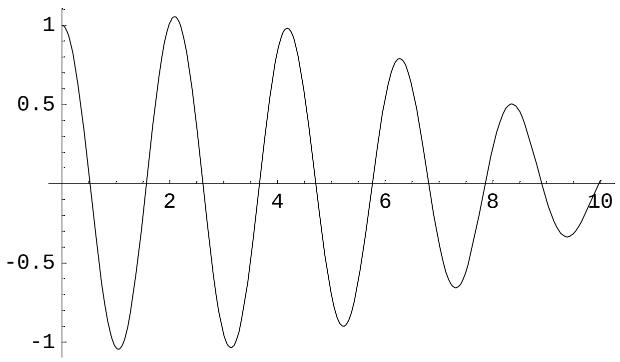
$$\frac{1}{4}\left(\frac{i\, e^{-2 i \sqrt{2} t}\left(1+2 i \sqrt{2}+\left(-1+2 i \sqrt{2}\right) e^{4 i \sqrt{2} t}\right)}{2 \sqrt{2}}+2 \mathrm{Cos}\left[\sqrt{10}\, t\right]+\sqrt{\frac{2}{5}}\, \mathrm{Sin}\left[\sqrt{10}\, t\right]\right)$$

```
u2P[t]:=u2[t]/. {a -> 10, b->1};
u2P[t]
```

$$\frac{1}{4}\left(-\frac{i\, e^{-2 i \sqrt{2} t}\left(1+2 i \sqrt{2}+\left(-1+2 i \sqrt{2}\right) e^{4 i \sqrt{2} t}\right)}{2 \sqrt{2}}+2 \mathrm{Cos}\left[\sqrt{10}\, t\right]+\sqrt{\frac{2}{5}}\, \mathrm{Sin}\left[\sqrt{10}\, t\right]\right)$$

```
pa1=Plot[Evaluate[u1P[t]],{t,0,10}];
pb1=Plot[Evaluate[u2P[t]],{t,0,10}];
```





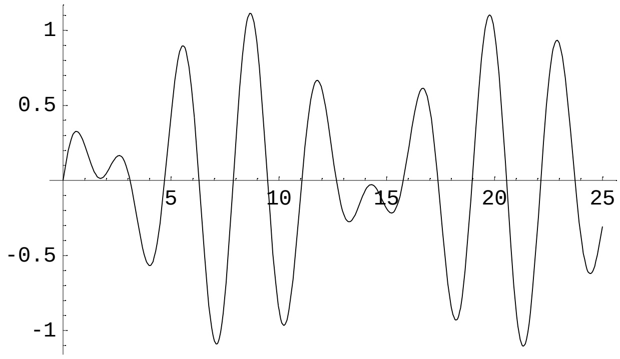

```
u1P[t]:=u1[t]/. {a -> 5, b->1};
u1P[t]
```

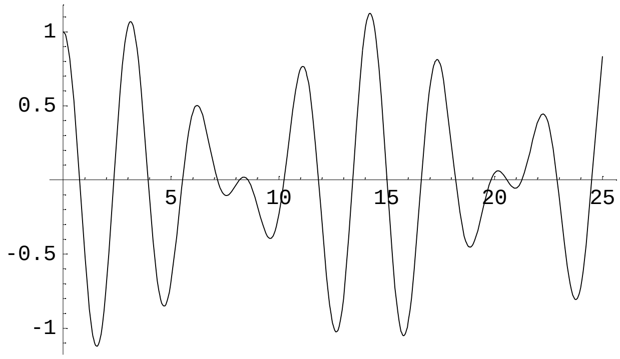
$$\frac{1}{4}\left(\frac{i\, e^{-i \sqrt{3} t}\left(1+i \sqrt{3}+\left(-1+i \sqrt{3}\right) e^{2 i \sqrt{3} t}\right)}{\sqrt{3}}+2 \mathrm{Cos}\left[\sqrt{5}\, t\right]+\frac{2 \mathrm{Sin}\left[\sqrt{5}\, t\right]}{\sqrt{5}}\right)$$

```
u2P[t]:=u2[t]/. {a -> 5, b->1};
u2P[t]
```

$$\frac{1}{4}\left(-\frac{i\, e^{-i \sqrt{3} t}\left(1+i \sqrt{3}+\left(-1+i \sqrt{3}\right) e^{2 i \sqrt{3} t}\right)}{\sqrt{3}}+2 \mathrm{Cos}\left[\sqrt{5}\, t\right]+\frac{2 \mathrm{Sin}\left[\sqrt{5}\, t\right]}{\sqrt{5}}\right)$$

```
pa2=Plot[Evaluate[u1P[t]],{t,0,25}];
pb2=Plot[Evaluate[u2P[t]],{t,0,25}];
```





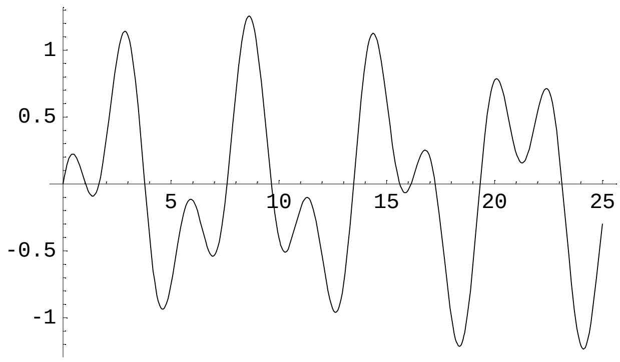

```
u1P[t]:=u1[t]/. {a -> 5, b->2};
u1P[t]
```

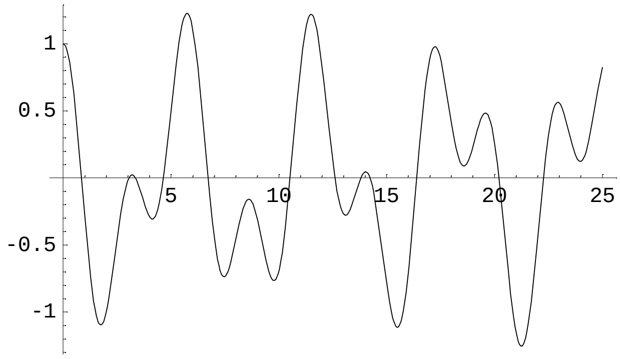
$$\frac{1}{4}\left(\mathbb{i}\,\mathbb{e}^{-\mathbb{i}\,t}\,((1+\mathbb{i})-(1-\mathbb{i})\,\mathbb{e}^{2\,\mathbb{i}\,t})+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]+\frac{2\,\mathrm{Sin}\left[\sqrt{5}\,t\right]}{\sqrt{5}}\right)$$

```
u2P[t]:=u2[t]/. {a -> 5, b->2};
u2P[t]
```

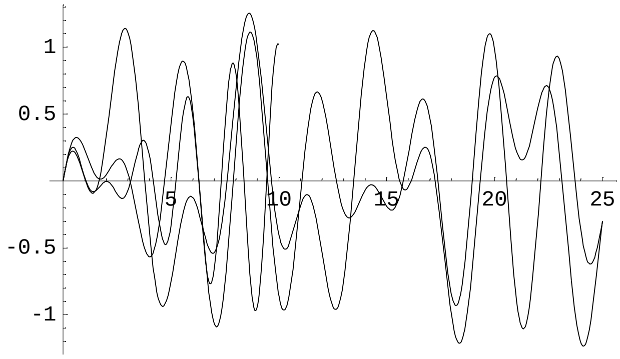
$$\frac{1}{4}\left(-\mathbb{i}\,\mathbb{e}^{-\mathbb{i}\,t}\,((1+\mathbb{i})-(1-\mathbb{i})\,\mathbb{e}^{2\,\mathbb{i}\,t})+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]+\frac{2\,\mathrm{Sin}\left[\sqrt{5}\,t\right]}{\sqrt{5}}\right)$$

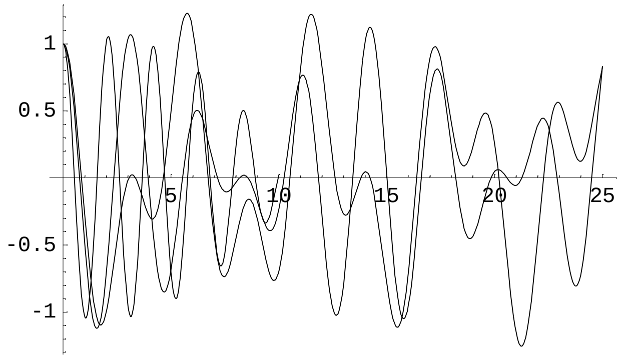
```
pa3=Plot[Evaluate[u1P[t]],{t,0,25}];
pb3=Plot[Evaluate[u2P[t]],{t,0,25}];
```






```
Show[pa1,pa2,pa3];
Show[pb1,pb2,pb3];
```





c RBD y1[0]->1, y1'[0]->0, y2[0]->0, y2'[0]->0

```
Remove["Global`*"];

links1 = LaplaceTransform[y2''[t] + g/L y2[t] - k/m y2[t] + k/m y1[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->a, y1'[0]->0, y2[0]->0, y2'[0]->0}
```

$$\frac{k\,Y1[s]}{m} + \frac{g\,Y2[s]}{L} - \frac{k\,Y2[s]}{m} + s^2\,Y2[s]$$

```
links2 = LaplaceTransform[y1''[t] + g/L y1[t] - k/m y1[t] + k/m y2[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->a, y1'[0]->0, y2[0]->0, y2'[0]->0}
```

$$-a\,s + \frac{g\,Y1[s]}{L} - \frac{k\,Y1[s]}{m} + s^2\,Y1[s] + \frac{k\,Y2[s]}{m}$$

```
solv=Solve[{links1 == 0, links2 == 0},{Y1[s],Y2[s]}] // Flatten
```

$$\left\{Y1[s] \to \frac{a\,L\,s\,(-k\,L + g\,m + L\,m\,s^2)}{(g + L\,s^2)\,(-2\,k\,L + g\,m + L\,m\,s^2)},\ Y2[s] \to \frac{a\,k\,L^2\,s}{(g + L\,s^2)\,(2\,k\,L - g\,m - L\,m\,s^2)}\right\}$$

```
U1[s]:=Y1[s]/. solv[[1]];
U2[s]:=Y2[s]/. solv[[2]];
```

```
u1[t_]:=InverseLaplaceTransform[U1[s],s,t]//Simplify; Print["u1(t) = ",u1[t]];
```

$$u1(t) = \frac{1}{4}\,a\left(e^{-\frac{\sqrt{2\,k\,L - g\,m}\;t}{\sqrt{L}\,\sqrt{m}}} + e^{\frac{\sqrt{2\,k\,L - g\,m}\;t}{\sqrt{L}\,\sqrt{m}}} + 2\,\mathrm{Cos}\left[\frac{\sqrt{g}\;t}{\sqrt{L}}\right]\right)$$

```
u2[t_]:=InverseLaplaceTransform[U2[s],s,t]//Simplify; Print["u2(t) = ",u2[t]];
```

$$u2(t) = \frac{1}{4}\,a\left(-e^{-\frac{\sqrt{2\,k\,L - g\,m}\;t}{\sqrt{L}\,\sqrt{m}}}\left(1 + e^{\frac{2\,\sqrt{2\,k\,L - g\,m}\;t}{\sqrt{L}\,\sqrt{m}}}\right) + 2\,\mathrm{Cos}\left[\frac{\sqrt{g}\;t}{\sqrt{L}}\right]\right)$$

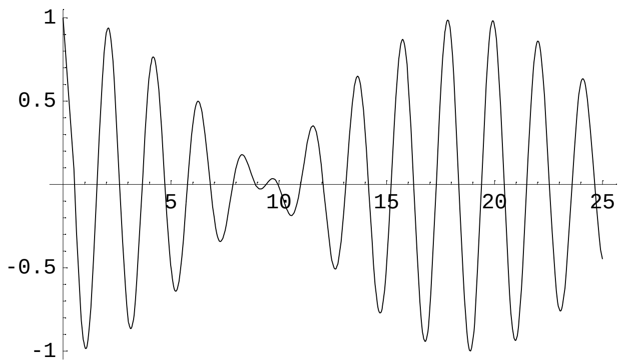
```
u1P[t]:=u1[t]/. {g -> 10, m->1, L->1, k->1, a->1};
u1P[t]
```

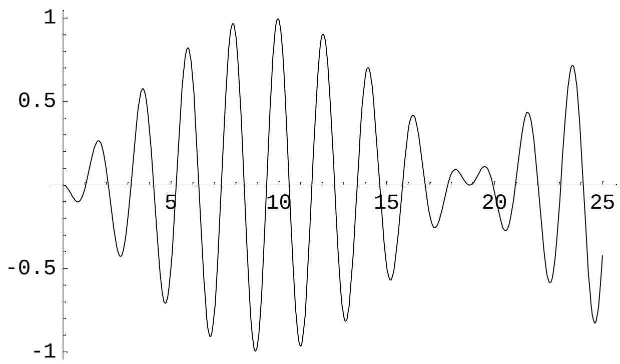
$$\frac{1}{4}\left(e^{-2\,i\,\sqrt{2}\,t} + e^{2\,i\,\sqrt{2}\,t} + 2\,\mathrm{Cos}\left[\sqrt{10}\;t\right]\right)$$

```
u2P[t]:=u2[t]/. {g -> 10, m->1, L->1, k->1, a->1};
u2P[t]
```

$$\frac{1}{4}\left(-e^{-2\,i\,\sqrt{2}\,t}\left(1 + e^{4\,i\,\sqrt{2}\,t}\right) + 2\,\mathrm{Cos}\left[\sqrt{10}\;t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```





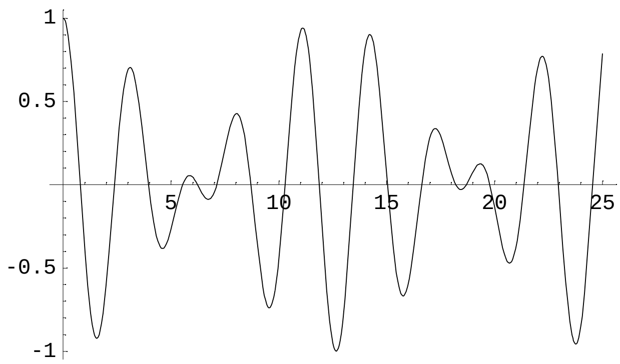
```
u1P[t]:=u1[t]/. {g -> 10, m->1, L->2, k->1, a->1};
u1P[t]
```

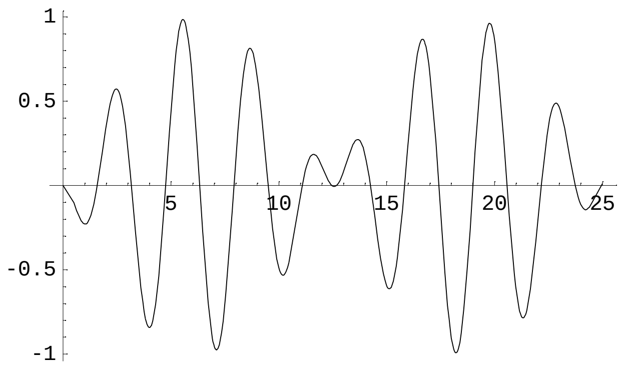
$$\frac{1}{4}\left(e^{-i\sqrt{3}\,t}+e^{i\sqrt{3}\,t}+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]\right)$$

```
u2P[t]:=u2[t]/. {g -> 10, m->1, L->2, k->1, a->1};
u2P[t]
```

$$\frac{1}{4}\left(-e^{-i\sqrt{3}\,t}\left(1+e^{2\,i\sqrt{3}\,t}\right)+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```





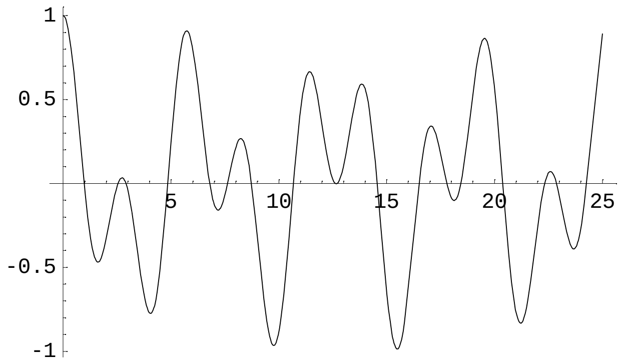

```
u1P[t]:=u1[t]/. {g -> 10, m->1, L->2, k->2, a->1};
u1P[t]
```

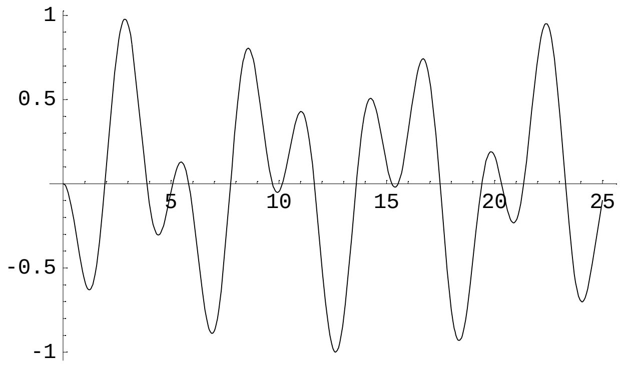
$$\frac{1}{4}\left(e^{-i\,t}+e^{i\,t}+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]\right)$$

```
u2P[t]:=u2[t]/. {g -> 10, m->1, L->2, k->2, a->1};
u2P[t]
```

$$\frac{1}{4}\left(-e^{-i\,t}\left(1+e^{2\,i\,t}\right)+2\,\mathrm{Cos}\left[\sqrt{5}\,t\right]\right)$$

```
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```





6

```
Remove["Global`*"];
```

a

i

```
f[x_,y_]:= E^(3 x+ 7 y)+c;
```

```
1/3 D[f[x,y],x]==1/7 D[f[x,y],y]
```

True

ii

```
E^(0)+c==2
```

$1 + c == 2$

```
f[1,1]
```

$c + e^{10}$

```
E^(3 1+ 7 1)+1
```

$1 + e^{10}$

```
E^(7 y)+1
```

$1 + e^{7 y}$

b

```
f[x_]:= a Cosh[x/a]
```

```
ϰ = f''[x]/(1+f'[x]^2)^(3/2)/.x->0
```

$\frac{1}{a}$

> Krümmungsradius ϱ = a