Lineare Algebra

1

```
Remove["Global`*"]
```

0

```
A = {{1, 1, -1}, {1, -1, 1}, {1, 1, 0}}; A // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

```
B = {{1, 2, 1}, {2, 1, 1}, {1, 1, 2}}; B // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

```
b1 = {1, 1, 1};
b2 = {1, -1, 1};
b3 = {-1, 1, 0};
```

```
z1 = -I;
z2 = 1 / 2 - 2 I ;
z3 = 2 - 4 I;
```

a

```
Det[A]
```

-2

b

```
Det[B]
```

-4

c

```
Det[B] / Det[A]
```

2

d

```
A.B // MatrixForm
```

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 3 & 3 & 2 \end{pmatrix}$$

e

```
B.A // MatrixForm
```

$$\begin{pmatrix} 4 & 0 & 1 \\ 4 & 2 & -1 \\ 4 & 2 & 0 \end{pmatrix}$$

f

```
Inverse[A] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

u = 2

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

g

```
Inverse[A].b1
```

{1, 0, 0}

h

```
Inverse[A].b2
```

{0, 1, 0}

i

```
Inverse[A].b3
```

{0, 0, 1}

j

```
Solve[A.Inverse[B].Transpose[Inverse[A]].{x1, x2, x3} == b1, {x1, x2, x3}]
```

{{x1 → 4, x2 → 0, x3 → 1}}

```
Inverse[A].b1
```

{1, 0, 0}

```
B.Inverse[A].b1
```

{1, 2, 1}

```
Transpose[A].B.Inverse[A].b1
```

{4, 0, 1}

k

```
Conjugate[z3] / z2
```

$-\frac{28}{17}+\frac{24\,i}{17}$

```
N[%]
```

$-1.64706+1.41176\,i$

l

```
(z2)^2 - z2 z3
```

$\frac{13}{4}+4\,i$

```
N[%]
```

$3.25+4.\,i$

m

```
Solve[z^4 == z1, {z}]
```

$\{\{z\to-(-1)^{3/8}\},\{z\to(-1)^{3/8}\},\{z\to-(-1)^{7/8}\},\{z\to(-1)^{7/8}\}\}$

```
solv = Solve[z^4 == z1, {z}] // Flatten // N
```

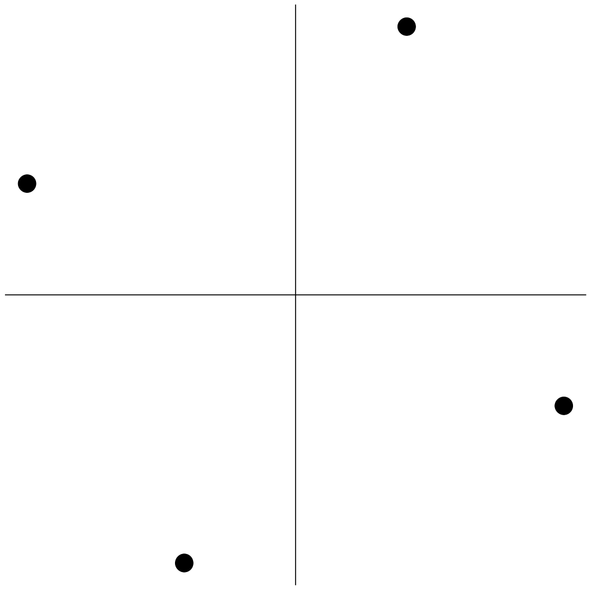
$\{z\to-0.382683-0.92388\,i,\ z\to0.382683+0.92388\,i,$
$z\to0.92388-0.382683\,i,\ z\to-0.92388+0.382683\,i\}$

```
solv[[1]]
```

$z\to-0.382683-0.92388\,i$

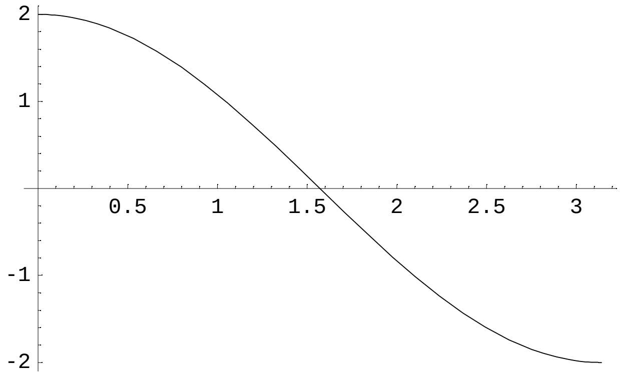
```
w1 = z /. solv[[1]];  p[1] = Point[{Re[w1], Im[w1]}];
w2 = z /. solv[[2]];  p[2] = Point[{Re[w2], Im[w2]}];
w3 = z /. solv[[3]];  p[3] = Point[{Re[w3], Im[w3]}];
w4 = z /. solv[[4]] ;  p[4] = Point[{Re[w4], Im[w4]}] ;
```

```
Show[Graphics[{PointSize[0.03], p[1], p[2], p[3], p[4],
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}]}], AspectRatio → Automatic];
```



n

```
Plot[2 Cos[x], {x, 0, Pi}];
```



o

12 22 32 42 52

p

3 2 1

```
v = Transpose[{{3, 2, 1}, -{3, 2, 1}, {3, 2, 1}}]; v // MatrixForm
```

$$\begin{pmatrix} 3 & -3 & 3 \\ 2 & -2 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

q

```
(v.Transpose[v]) // MatrixForm
```

$$\begin{pmatrix} 27 & 18 & 9 \\ 18 & 12 & 6 \\ 9 & 6 & 3 \end{pmatrix}$$

Kontrolle:

```
(Transpose[v].v) // MatrixForm
```

$$\begin{pmatrix} 14 & -14 & 14 \\ -14 & 14 & -14 \\ 14 & -14 & 14 \end{pmatrix}$$

2

```
Remove["Global`*"]
```

---

a

```
P1 = {1, 2, 1};
P2 = {-1, 3, 1};
P3 = {0, -1, 2};
Q = {-1, 0, 8};
```

a

```
Norm[Cross[P1, Q]] / 2
```

$$\frac{\sqrt{341}}{2}$$

```
Norm[Cross[P1, Q]] / 2 // N
```

9.23309

b

```
n = Cross[P2 - P1, P3 - P1]
```

{1, 2, 7}

```
n / Norm[n] // Simplify
```

$\left\{\frac{1}{3\sqrt{6}}, \frac{\sqrt{\frac{2}{3}}}{3}, \frac{7}{3\sqrt{6}}\right\}$

```
N[%]
```

{0.136083, 0.272166, 0.952579}

c

```
Solve[n.P1 + d == 0, {d}]
```

{{d → -12}}

```
HNFθ[x_, y_, z_] := (n.{x, y, z} - 12) / Norm[n] ;
HNFθ[{x_, y_, z_}] := HNFθ[x, y, z] ; HNFθ[x, y, z] ;
```

```
θVec[λ_, μ_] := P1 + λ (P2 - P1) + μ (P3 - P1); θVec[λ, μ]
```

$\{1 - 2\lambda - \mu, 2 + \lambda - 3\mu, 1 + \mu\}$

```
gLot[t_] := Q + t n;  gLot[t]
```

$\{-1 + t, 2t, 8 + 7t\}$

```
gLot[t] == θVec[λ, μ]
```

$\{-1 + t, 2t, 8 + 7t\} = \{1 - 2\lambda - \mu, 2 + \lambda - 3\mu, 1 + \mu\}$

```
solv = Solve[gLot[t] == θVec[λ, μ], {t, λ, μ}] // Flatten
```

$\left\{t \to -\frac{43}{54}, \lambda \to \frac{37}{54}, \mu \to \frac{77}{54}\right\}$

```
gLot[t] /. solv[[1]]
```

$\left\{-\frac{97}{54}, -\frac{43}{27}, \frac{131}{54}\right\}$

```
N[%]
```

{-1.7963, -1.59259, 2.42593}

d

```
HNFθ[Q]
```

$\frac{43}{3\sqrt{6}}$

```
N[%]
```

5.85156

e

```
dre[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}};
dre[φ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\varphi] & -\mathrm{Sin}[\varphi] \\ 0 & \mathrm{Sin}[\varphi] & \mathrm{Cos}[\varphi] \end{pmatrix}$$

```
U1 = dre[30 Degree].Q
```

$\{-1, -4, 4\sqrt{3}\}$

```
N[%]
```

{-1., -4., 6.9282}

```
U2 = dre[-30 Degree].Q
```

$\{-1, 4, 4\sqrt{3}\}$

```
N[%]
```

{-1., 4., 6.9282}

```
HNFe[U1]
```

$$\frac{-21 + 28\sqrt{3}}{3\sqrt{6}}$$

```
N[%]
```

3.74193

```
HNFe[U2]
```

$$\frac{-5 + 28\sqrt{3}}{3\sqrt{6}}$$

```
N[%]
```

5.91925

U1 liegt näher an der Ebene und U2 weiter entfernt von der Ebene als Q.

3

```
Remove["Global`*"]
```

```
A = {
  {1, 0, 0, 0, 1},
  {0, 1, 0, 1, 0},
  {0, 0, 1, 0, 0},
  {0, -1, 0, -1, 0},
  {1, 0, 0, 0, -1}
 }; A // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
B = Transpose[A]; B // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

a

```
A.A // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

b

```
B.B // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

c

```
A.B // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

d

```
B.A // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

e

```
A.A.B.B // MatrixForm
```

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

f

Im Falle, dass man den transponierten Vektor nimmt:

```
A.{1, 2, 3, 4, 5}
```

{6, 6, 3, -6, -4}

```
A.{2, 3, 4, 5, 6}
```

{8, 8, 4, -8, -4}

```
A.{3, 4, 5, 6, 7}
```

{10, 10, 5, -10, -4}

g

```
A1 = A + 2 B; Inverse[A1] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{3}{8} & 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{8} & 0 & -\frac{3}{8} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & -\frac{1}{6} \end{pmatrix}$$

```
N[Inverse[A1]] // MatrixForm
```

$$\begin{pmatrix} 0.166667 & 0. & 0. & 0. & 0.166667 \\ 0. & 0.375 & 0. & -0.125 & 0. \\ 0. & 0. & 0.333333 & 0. & 0. \\ 0. & 0.125 & 0. & -0.375 & 0. \\ 0.166667 & 0. & 0. & 0. & -0.166667 \end{pmatrix}$$

h

Im Falle, dass man statt einem Vektor nimmt eine Matrix X nimmt:

```
X = Inverse[A1].(A1 + Inverse[A1] - A1.A1) // MatrixForm
```

$$\begin{pmatrix} -\frac{35}{18} & 0 & 0 & 0 & -3 \\ 0 & -\frac{15}{8} & 0 & 1 & 0 \\ 0 & 0 & -\frac{17}{9} & 0 & 0 \\ 0 & -1 & 0 & \frac{33}{8} & 0 \\ -3 & 0 & 0 & 0 & \frac{73}{18} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -1.94444 & 0. & 0. & 0. & -3. \\ 0. & -1.875 & 0. & 1. & 0. \\ 0. & 0. & -1.88889 & 0. & 0. \\ 0. & -1. & 0. & 4.125 & 0. \\ -3. & 0. & 0. & 0. & 4.05556 \end{pmatrix}$$

4

```
Remove["Global`*"]
```

a

```
Solve[z^4 == -1 - I, {z}]
```

$\{\{z \to -(-1-i)^{1/4}\}, \{z \to -i\,(-1-i)^{1/4}\}, \{z \to i\,(-1-i)^{1/4}\}, \{z \to (-1-i)^{1/4}\}\}$

```
solv = Solve[z^4 == -1 - I, {z}] // Flatten // N
```

$\{z \to -0.906724 + 0.605854\, i,\ z \to -0.605854 - 0.906724\, i,$
$z \to 0.605854 + 0.906724\, i,\ z \to 0.906724 - 0.605854\, i\}$

```
solv[[1]]
```

$z \to -0.906724 + 0.605854\, i$

```
w1 = z /. solv[[1]];  p[1] = Point[{Re[w1], Im[w1]}];
w2 = z /. solv[[2]];  p[2] = Point[{Re[w2], Im[w2]}];
w3 = z /. solv[[3]];  p[3] = Point[{Re[w3], Im[w3]}];
w4 = z /. solv[[4]] ;  p[4] = Point[{Re[w4], Im[w4]}] ;
```

```
w1^4
```

−1. − 1. ⅈ

```
Norm[-1 - I] // Simplify
```

$\sqrt{2}$

```
Norm[-1 - I]^(1/4)
```

$2^{1/8}$

```
N[%]
```

1.09051

```
ArcTan[-1/(-1)]
```

$\frac{\pi}{4}$

```
ArcTan[-1/(-1)] + Pi
```

$\frac{5\pi}{4}$

```
(ArcTan[-1/(-1)] + Pi)/4
```

$\frac{5\pi}{16}$

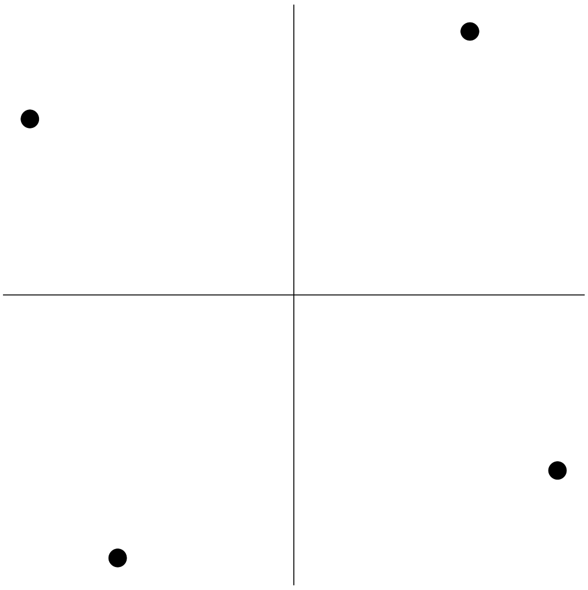
```
{(ArcTan[-1/(-1)] + Pi)/4, (ArcTan[-1/(-1)] + Pi)/4 + Pi/2,
 (ArcTan[-1/(-1)] + Pi)/4 + Pi, (ArcTan[-1/(-1)] + Pi)/4 + 3 Pi/2}
```

$\left\{\frac{5\pi}{16}, \frac{13\pi}{16}, \frac{21\pi}{16}, \frac{29\pi}{16}\right\}$

```
N[%]
```

{0.981748, 2.55254, 4.12334, 5.69414}

```
Show[Graphics[{PointSize[0.03], p[1], p[2], p[3], p[4],
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}]}], AspectRatio → Automatic];
```



b

```
(x - 1) (x + 2) (x^2 + 3) // Expand
```

$-6 + 3x + x^2 + x^3 + x^4$

```
-6 + 3 x + x^2 + x^3 + x^4 // Factor
```

$(-1 + x)(2 + x)(3 + x^2)$

```
solv = Solve[-6 + 3 x + x^2 + x^3 + x^4 == 0, {x}] // Flatten
```

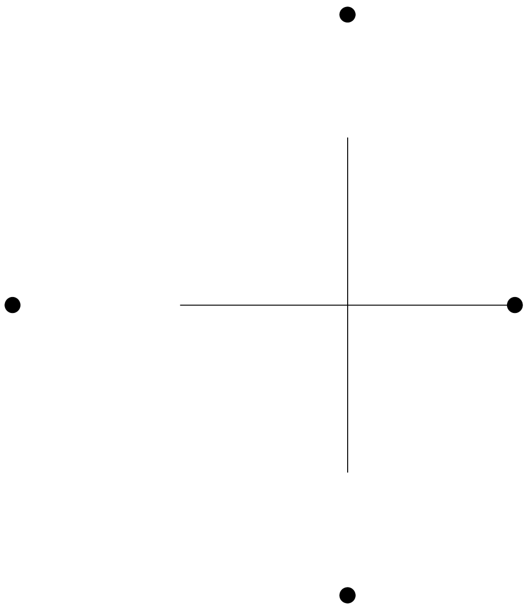
$\{x \to -2, x \to 1, x \to -i\sqrt{3}, x \to i\sqrt{3}\}$

```
N[%]
```

{x → -2., x → 1., x → 0. - 1.73205 i, x → 0. + 1.73205 i}

```
w1 = x /. solv[[1]];  p[1] = Point[{Re[w1], Im[w1]}];
w2 = x /. solv[[2]];  p[2] = Point[{Re[w2], Im[w2]}];
w3 = x /. solv[[3]];  p[3] = Point[{Re[w3], Im[w3]}];
w4 = x /. solv[[4]] ;  p[4] = Point[{Re[w4], Im[w4]}] ;
```

```
Show[Graphics[{PointSize[0.03], p[1], p[2], p[3], p[4],
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}]}], AspectRatio → Automatic];
```



```
Inhalt = 2 3 * Sqrt[3] / 2
```

$3\sqrt{3}$

```
N[%]
```

5.19615

```
q[x_] := 84 / (-6 + 3 x + x^2 + x^3 + x^4); q[x]
```

$$\frac{84}{-6+3x+x^2+x^3+x^4}$$

```
Apart[q[x]]
```

$$\frac{7}{-1+x}-\frac{4}{2+x}-\frac{3(5+x)}{3+x^2}$$

```
Apart[84 / (-6 + 3 x + x^2 + x^3 + x^4)] - 7 / (x - 1) + 3 (x + 5) / (x^2 + 3)
```

$$-\frac{4}{2+x}$$

5

```
Remove["Global`*"]
```

a

```
InhW[a_] := (2 a) ^3; InhW[a]
```

$8\,a^3$

```
InhOktW[a_] := 1 / 2 a^2 * 4 * a * 2 / 3; InhOktW[a]
```

$$\frac{4\,a^3}{3}$$

```
rK[a_] := Sqrt[3 a^2]; rK[a]
```

$$\sqrt{3}\,\sqrt{a^2}$$

```
InhOktK[a_] := InhOktW[rK[a]] ; InhOktK[a]
```

$$4\sqrt{3}\,(a^2)^{3/2}$$

```
InhOktW[rK[a]] / InhW[a] // Simplify
```

$$\frac{\sqrt{3}\,(a^2)^{3/2}}{2\,a^3}$$

```
InhOktW[rK[1]] / InhW[1] // Simplify
```

$$\frac{\sqrt{3}}{2}$$

```
N[%]
```

0.866025

b

```
p1[a_] := {a, 0, 0};
p2[a_] := {0, a, 0};
p3[a_] := {0, 0, a};
q1[a_] := (p1[a] + p2[a]) / 2;
s[a_] := q1[a] + 1 / 3 (p3[a] - q1[a])
```

```
hOkt[a_] := Norm[s[a]]; hOkt[a]
```

$$\frac{\text{Abs}[a]}{\sqrt{3}}$$

```
hOkt[1]
```

$$\frac{1}{\sqrt{3}}$$

Statt diesen Abstand steht im Falle von O2 der Radius von O1, d.h. rK[a]. Das Streckungsverhältnis für das Volumen ist damit (rK[1]/ hOkt[1])^3 :

```
InhOkt2K[a_] := InhOktW[rK[a]] (rK[1] / hOkt[1]) ^3; InhOkt2K[1]
```

$$108\sqrt{3}$$

```
InhOkt2K[1] / InhOktW[1]
```

$$81\sqrt{3}$$

```
N[%]
```

140.296