Teil 1

1

```
A = {{-b, b, b}, {a, c, c}, {c, a, c}}; A // MatrixForm
```

$$\begin{pmatrix} -b & b & b \\ a & c & c \\ c & a & c \end{pmatrix}$$

```
B = {{b, b, c}, {a, -a, a}, {c, c, b}};
B // MatrixForm
```

$$\begin{pmatrix} b & b & c \\ a & -a & a \\ c & c & b \end{pmatrix}$$

```
cC = {{b, b, a}, {a, -b, a}, {-c, c, b}};
cC // MatrixForm
```

$$\begin{pmatrix} b & b & a \\ a & -b & a \\ -c & c & b \end{pmatrix}$$

```
b1 = {1, -1, 1}
```

{1, -1, 1}

```
z1 = -1 / 2 I
```

$$-\frac{i}{2}$$

```
z2 = 1 / 4 + 1 / 2 I
```

$$\frac{1}{4} + \frac{i}{2}$$

```
z3 = 3 - 2 I
```

$3 - 2 i$

a

```
Det[A]
```

$a^2 b - b c^2$

```
Det[A] // Factor
```

$b (a - c) (a + c)$

b

```
Det[B]
```

$-2\,a\,b^2 + 2\,a\,c^2$

```
Det[B] // Factor
```

$-2\,a\,(b-c)\,(b+c)$

c

```
Det[A] /. {a → 1, b → 0, c → -1}
```

0

```
Det[B] /. {a → 1, b → 0, c → -1}
```

2

```
Det[A.B] /. {a → 1, b → 0, c → -1}
```

0

```
Det[B.A] /. {a → 1, b → 0, c → -1}
```

0

d

```
A.B
```

$\{\{a\,b - b^2 + b\,c,\ -a\,b - b^2 + b\,c,\ a\,b + b^2 - b\,c\},$
$\{a\,b + a\,c + c^2,\ a\,b - a\,c + c^2,\ 2\,a\,c + b\,c\},\ \{a^2 + b\,c + c^2,\ -a^2 + b\,c + c^2,\ a^2 + b\,c + c^2\}\}$

```
(A.B /. {a → 1, b → 0, c → -1}) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 2 & 0 & 2 \end{pmatrix}$$

```
(B.A /. {a → 1, b → 0, c → -1}) // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 1 \\ -2 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

Faktoren nicht vertauschbar

e

```
Det[cC] /. {a → 1, b → 0, c → -1}
```

-1

```
Det[A.cC] /. {a → 1, b → 0, c → -1}
```

0

```
Det[B.cC] /. {a → 1, b → 0, c → -1}
```

-2

f

```
(Det[B.cC] /. {b → 0, c → -1}) == 16
```

$-2\,a^3 == 16$

```
Solve[(Det[B.cC] /. {b → 0, c → -1}) == 16, {a}]
```

$\{\{a \to -2\}, \{a \to 2\,(-1)^{1/3}\}, \{a \to -2\,(-1)^{2/3}\}\}$

```
N[%]
```

{{a → -2.}, {a → 1. + 1.73205 ⅈ}, {a → 1. - 1.73205 ⅈ}}

g

```
(Inverse[cC] /. {a → 1, b → 0, c → -1}) // MatrixForm
```

$$\begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

```
(Inverse[cC] /. {a → 1, b → 0, c → -1}).b1
```

{-2, -3, 1}

h

```
G = {{-1, 1}, {2, 1}}; G // MatrixForm
```

$$\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$$

```
Inverse[G] // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.333333 & 0.333333 \\ 0.666667 & 0.333333 \end{pmatrix}$$

i

```
Inverse[G].(IdentityMatrix[2] - X^T + G).G == G.G;
```

```
IdentityMatrix[2] - X^T + G == G.G
```

False

```
-X^T == G.G - IdentityMatrix[2] - G
```

False

```
X = Transpose[-(G.G - IdentityMatrix[2] - G)]; X // MatrixForm
```

$$\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$$

j

```
Conjugate[z1] 1 / z2
```

$$\frac{4}{5} + \frac{2 i}{5}$$

```
N[%]
```

0.8 + 0.4 i

k

```
solv = Solve[z^6 == z1, {z}]
```

$$\left\{\left\{z \to -\frac{(-1)^{1/4}}{2^{1/6}}\right\}, \left\{z \to \frac{(-1)^{1/4}}{2^{1/6}}\right\}, \left\{z \to -\frac{(-1)^{7/12}}{2^{1/6}}\right\}, \left\{z \to \frac{(-1)^{7/12}}{2^{1/6}}\right\}, \left\{z \to -\frac{(-1)^{11/12}}{2^{1/6}}\right\}, \left\{z \to \frac{(-1)^{11/12}}{2^{1/6}}\right\}\right\}$$

```
z[k_] := z /. solv[[k]]
```

```
tab0 = Table[z[k], {k, 1, 6}]
```

$$\left\{-\frac{(-1)^{1/4}}{2^{1/6}}, \frac{(-1)^{1/4}}{2^{1/6}}, -\frac{(-1)^{7/12}}{2^{1/6}}, \frac{(-1)^{7/12}}{2^{1/6}}, -\frac{(-1)^{11/12}}{2^{1/6}}, \frac{(-1)^{11/12}}{2^{1/6}}\right\}$$

```
tab0N = tab0 // N
```

{-0.629961 - 0.629961 i, 0.629961 + 0.629961 i, 0.230582 - 0.860542 i,
-0.230582 + 0.860542 i, 0.860542 - 0.230582 i, -0.860542 + 0.230582 i}

```
tab0N[[1]] + tab0N[[2]]
```

0. + 0. i

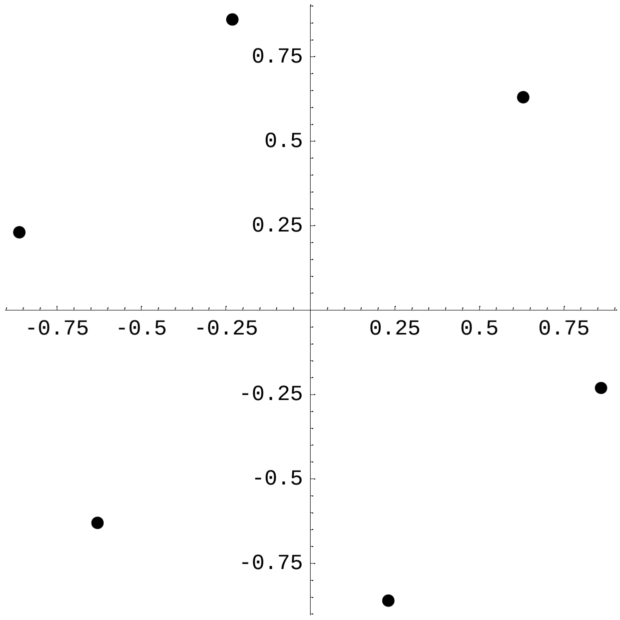
```
tab0N[[3]] + tab0N[[4]]
```

0. + 0. i

```
tab0N[[5]] + tab0N[[6]]
```

0. + 0. i

```
ListPlot[Table[{Re[tab0N[[k]]], Im[tab0N[[k]]]}, {k, 1, 6}],
  PlotStyle → PointSize[0.02], AspectRatio → Automatic];
```



l

```
(z2^2 + z2) / z3
```

$-\frac{21}{208} + \frac{19\,i}{104}$

```
N[%]
```

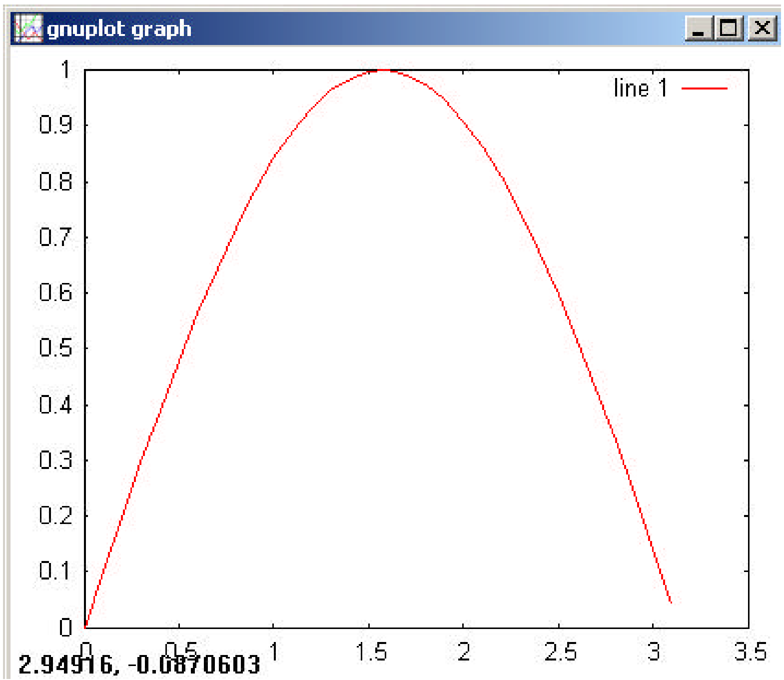
$-0.100962 + 0.182692\,i$

2 Matlab - Octave - Output

a

```
>> x = 0 : 0.1 : pi; y = sin (x); plot (x, y)
```

(Eingefügt mit Copy-Paste aus einem Window)



b

```
>> x = 3; y = 2; clear; y = 3; z = (x * y) ^ (1 / 2)

error : `x' undefined near line 2 column 26
error : evaluating binary operator ` * ' near line 2, column 27
error : evaluating binary operator ` ^ ' near line 2, column 30
error : evaluating assignment expression near line 2, column 24
```

c

```
>> exp (1) - log10 (10)

ans = 1.7183
```

d

```
>> ((0 : 5) - 5) * 5

ans = -25 - 20 - 15 - 10 - 5 0
```

e

```
>> u = [2 * 3, 4, sqrt (25)]; v = [u' (4 + u)' 2 * u']

v = 6 10 12
    4  8  8
    5  9 10
```

f

```
>> v * v

ans = 136 248 272
       96 176 192
      116 212 232
```

Teil 2

3

a

```
P1 = {2, -1, 0};
P2 = {3, 4, 0};
P3 = {1, 6, 0};
Q = {1, 5, 7};
```

```
solv = Solve[{Norm[{x, y, 0} - P1] == Norm[{x, y, 0} - P2],
    Norm[{x, y, 0} - P1] == Norm[{x, y, 0} - P3]}, {x, y}] // Flatten
```

$\{x \to -\frac{5}{6}, y \to \frac{13}{6}\}$

```
N[%]
```

{x → -0.833333, y → 2.16667}

```
M = {x, y, 0} /. solv
```

$\{-\frac{5}{6}, \frac{13}{6}, 0\}$

```
N[%]
```

{-0.833333, 2.16667, 0.}

b

```
v1 = (P2 - P1)
```

{1, 5, 0}

```
g[t_] := P1 + t v1; g[t]
```

{2 + t, -1 + 5 t, 0}

```
v1s = {-v1[[2]], v1[[1]], 0}
```

$\{-5, 1, 0\}$

```
gM[t_] := M + t v1s; gM[t]
```

$\left\{-\frac{5}{6} - 5\,t,\ \frac{13}{6} + t,\ 0\right\}$

```
solv1 = Solve[gM[t1] == g[t2], {t1, t2}] // Flatten
```

$\left\{t1 \to -\frac{2}{3},\ t2 \to \frac{1}{2}\right\}$

```
F = g[t2] /. solv1
```

$\left\{\frac{5}{2},\ \frac{3}{2},\ 0\right\}$

```
Ms = M + 2 (F - M)
```

$\left\{\frac{35}{6},\ \frac{5}{6},\ 0\right\}$

```
N[%]
```

$\{5.83333, 0.833333, 0.\}$

c

```
winkel = ArcCos[(M - P3).(Ms - P3) / (Norm[M - P3] Norm[Ms - P3])]
```

$\text{ArcCos}\left[\frac{197}{5\sqrt{11713}}\right]$

```
N[%]
```

1.19818

```
% / Degree
```

68.6508

d

```
V = Abs[Det[{(P1 - Q), (P2 - Q), (P3 - Q)}] / 6]
```

14

e

```
abst[A_, v_] := Norm[Cross[A, v] / Norm[v]];
abst[M - P1, v1]
```

$\frac{2\sqrt{26}}{3}$

```
abst[M - P2, v1]
```

$$\frac{2\sqrt{26}}{3}$$

```
abst[Ms - P2, v1]
```

$$\frac{2\sqrt{26}}{3}$$

```
N[%]
```

3.39935

```
abst[P3 - P1, v1]
```

$$6\sqrt{\frac{2}{13}}$$

```
abst[P3 - P2, v1]
```

$$6\sqrt{\frac{2}{13}}$$

```
N[%]
```

2.35339

4

a

$$p[x\_] := \frac{-1 + x - 5x^2 - 9x^3 - 8x^4 - 4x^5}{x^2\,(1 + x)};\ p[x]\ //\ Apart$$

$$-5 - \frac{1}{x^2} + \frac{2}{x} - 4x - 4x^2 - \frac{2}{1 + x}$$

b

$$q[x\_] := \frac{4 - x - x^2 - 4x^3 - 4x^4}{(-1 + x)\,(1 + x)};$$

```
q[x] // Apart
```

$$-5 - \frac{3}{-1 + x} - 4x - 4x^2 - \frac{2}{1 + x}$$

c

```
(p[x] - q[x]) // Apart
```

$$\frac{3}{-1 + x} - \frac{1}{x^2} + \frac{2}{x}$$

Kein ganzer Anteil

5

a

```
solv1 = Solve[z^8 == 1, {z}]
```

{{z → -1}, {z → -i}, {z → i}, {z → 1},
{z → -(-1)^{1/4}}, {z → (-1)^{1/4}}, {z → -(-1)^{3/4}}, {z → (-1)^{3/4}}}

```
solv2 = Solve[z^8 == -1, {z}]
```

{{z → -(-1)^{1/8}}, {z → (-1)^{1/8}}, {z → -(-1)^{3/8}}, {z → (-1)^{3/8}},
{z → -(-1)^{5/8}}, {z → (-1)^{5/8}}, {z → -(-1)^{7/8}}, {z → (-1)^{7/8}}}

```
z[k_] := z /. solv1[[k]]
```

```
tab1 = Table[z[k], {k, 1, 8}]
```

{-1, -i, i, 1, -(-1)^{1/4}, (-1)^{1/4}, -(-1)^{3/4}, (-1)^{3/4}}

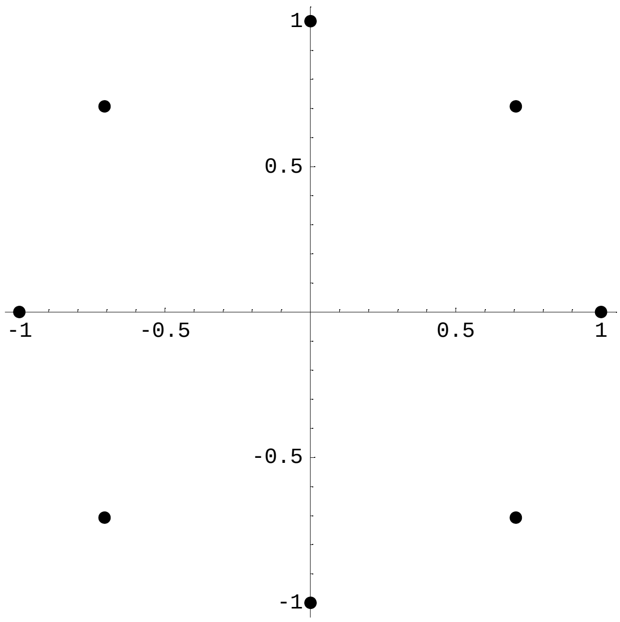
```
tab1a = Table[(E^(I 2 Pi / 8))^k, {k, 1, 8}]
```

$\left\{e^{\frac{i\pi}{4}}, i, e^{\frac{3i\pi}{4}}, -1, e^{-\frac{3i\pi}{4}}, -i, e^{-\frac{i\pi}{4}}, 1\right\}$

```
tab1aN = tab1a // N
```

{0.707107 + 0.707107 i, 0. + 1. i, -0.707107 + 0.707107 i,
-1., -0.707107 - 0.707107 i, 0. - 1. i, 0.707107 - 0.707107 i, 1.}

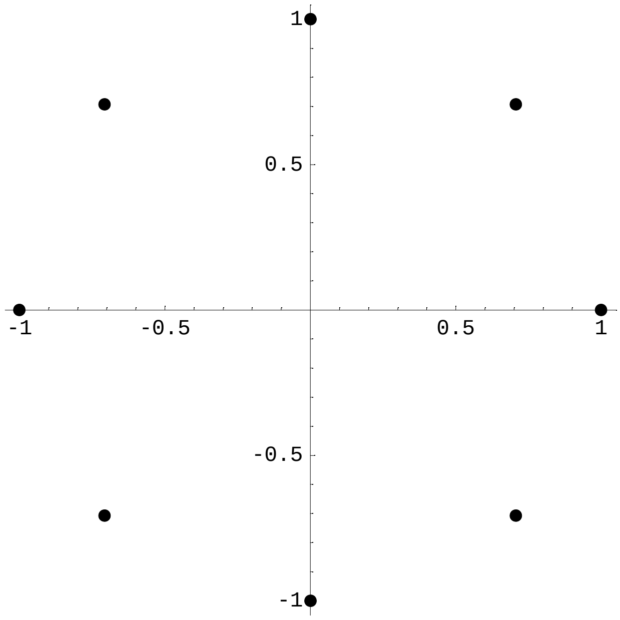
```
ListPlot[Table[{Re[tab1aN[[k]]], Im[tab1aN[[k]]]}, {k, 1, 8}],
  PlotStyle → PointSize[0.02], AspectRatio → Automatic];
```



```
tab1uN = -tab1aN
```

{-0.707107 - 0.707107 ⅈ, 0. - 1. ⅈ, 0.707107 - 0.707107 ⅈ,
1., 0.707107 + 0.707107 ⅈ, 0. + 1. ⅈ, -0.707107 + 0.707107 ⅈ, -1.}

```
ListPlot[Table[{Re[tab1uN[[k]]], Im[tab1uN[[k]]]}, {k, 1, 8}],
  PlotStyle → PointSize[0.02], AspectRatio → Automatic];
```



Dieselbe Gleichung

b

```
sum[j_] := Sum[tab1a[[k]], {k, 1, j}]
```

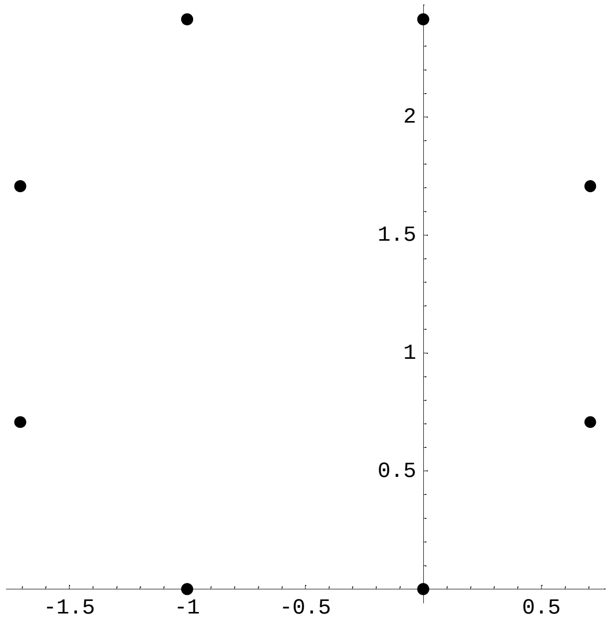
```
tab1b = Table[sum[j], {j, 1, 8}]
```

$$\left\{e^{\frac{i\pi}{4}},\ i+e^{\frac{i\pi}{4}},\ i+e^{\frac{i\pi}{4}}+e^{\frac{3i\pi}{4}},\ (-1+i)+e^{\frac{i\pi}{4}}+e^{\frac{3i\pi}{4}},\ (-1+i)+e^{\frac{i\pi}{4}}+e^{-\frac{3i\pi}{4}}+e^{\frac{3i\pi}{4}},\right.$$
$$\left.-1+e^{\frac{i\pi}{4}}+e^{-\frac{3i\pi}{4}}+e^{\frac{3i\pi}{4}},\ -1+e^{-\frac{i\pi}{4}}+e^{\frac{i\pi}{4}}+e^{-\frac{3i\pi}{4}}+e^{\frac{3i\pi}{4}},\ e^{-\frac{i\pi}{4}}+e^{\frac{i\pi}{4}}+e^{-\frac{3i\pi}{4}}+e^{\frac{3i\pi}{4}}\right\}$$

```
tab1bN = tab1b // N // Chop
```

{0.707107 + 0.707107 ⅈ, 0.707107 + 1.70711 ⅈ, 2.41421 ⅈ,
-1. + 2.41421 ⅈ, -1.70711 + 1.70711 ⅈ, -1.70711 + 0.707107 ⅈ, -1., 0}

```
ListPlot[Table[{Re[tab1bN[[k]]], Im[tab1bN[[k]]]}, {k, 1, 8}],
  PlotStyle → PointSize[0.02], AspectRatio → Automatic];
```




```
a = 1;
((Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /. k → a) +
    (Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /.
      k → a + 4)) / 2 // N
```

{0.5, -1.20711}

```
a = 2;
((Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /. k → a) +
    (Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /.
      k → a + 4)) / 2 // N
```

{0.5, -1.20711}

```
a = 3;
((Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /. k → a) +
    (Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /.
      k → a + 4)) / 2 // N
```

{0.5, -1.20711}

```
a = 4;
((Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /. k → a) +
    (Sum[{Re[E^(I ( j 2 Pi / 8 + Pi))], Im[E^(I ( j 2 Pi / 8 + Pi))]}, {j, 1, k}] /.
      k → a + 4)) / 2 // N
```

{0.5, -1.20711}

Vermutung: Kreis.(Einfach beweisbar mit Hilfe der endlichen geometrischen Reihen)

6

```
A = {{1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {0, 1, 1, 1, 0}, {0, 0, 1, 1, 1}, {0, 0, 0, 1, 1}}
```

{{1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {0, 1, 1, 1, 0}, {0, 0, 1, 1, 1}, {0, 0, 0, 1, 1}}

```
A // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

```
B = {{1, -1, 0, 0, 0}, {1, 1, -1, 0, 0}, {0, 1, 1, -1, 0}, {0, 0, 1, 1, -1}, {0, 0, 0, 1, 1}}
```

{{1, -1, 0, 0, 0}, {1, 1, -1, 0, 0}, {0, 1, 1, -1, 0}, {0, 0, 1, 1, -1}, {0, 0, 0, 1, 1}}

```
B // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

a

```
A.A // MatrixForm
```

$$\begin{pmatrix} 2 & 2 & 1 & 0 & 0 \\ 2 & 3 & 2 & 1 & 0 \\ 1 & 2 & 3 & 2 & 1 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 0 & 1 & 2 & 2 \end{pmatrix}$$

```
A.A.A // MatrixForm
```

$$\begin{pmatrix} 4 & 5 & 3 & 1 & 0 \\ 5 & 7 & 6 & 3 & 1 \\ 3 & 6 & 7 & 6 & 3 \\ 1 & 3 & 6 & 7 & 5 \\ 0 & 1 & 3 & 5 & 4 \end{pmatrix}$$

Matrix wird nach aussen aufgefüllt, Nullen werden eliminiert. Symmetrisch da A symmetrisch.

b

```
B.B // MatrixForm
```

$$\begin{pmatrix} 0 & -2 & 1 & 0 & 0 \\ 2 & -1 & -2 & 1 & 0 \\ 1 & 2 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 & -2 \\ 0 & 0 & 1 & 2 & 0 \end{pmatrix}$$

```
B.B.B // MatrixForm
```

$$\begin{pmatrix} -2 & -1 & 3 & -1 & 0 \\ 1 & -5 & 0 & 3 & -1 \\ 3 & 0 & -5 & 0 & 3 \\ 1 & 3 & 0 & -5 & -1 \\ 0 & 1 & 3 & 1 & -2 \end{pmatrix}$$

Matrix wird nach aussen aufgefüllt, Nullen werden eliminiert. Symmetrisch da B symmetrisch.

c

```
A.B // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & -1 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 \\ 1 & 2 & 1 & 0 & -1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{pmatrix}$$

```
B.A // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 \\ 1 & 2 & 1 & 0 & -1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 \end{pmatrix}$$

An der x=y-Achse gespiegelt, identisch bis auf Endelemente der Hauptdiagonalen.

```
A.B == B.A
```

```
False
```

d

```
Inverse[B]
```

$$\left\{\left\{\frac{5}{8}, \frac{3}{8}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}, \left\{-\frac{3}{8}, \frac{3}{8}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}, \left\{\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right\}, \left\{-\frac{1}{8}, \frac{1}{8}, -\frac{1}{4}, \frac{3}{8}, \frac{3}{8}\right\}, \left\{\frac{1}{8}, -\frac{1}{8}, \frac{1}{4}, -\frac{3}{8}, \frac{5}{8}\right\}\right\}$$

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} \frac{5}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ -\frac{3}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{8} & -\frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{8} & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{8} & \frac{5}{8} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.625 & 0.375 & 0.25 & 0.125 & 0.125 \\ -0.375 & 0.375 & 0.25 & 0.125 & 0.125 \\ 0.25 & -0.25 & 0.5 & 0.25 & 0.25 \\ -0.125 & 0.125 & -0.25 & 0.375 & 0.375 \\ 0.125 & -0.125 & 0.25 & -0.375 & 0.625 \end{pmatrix}$$

```
A.B.A + A == X.B.B + B;
X == (A.B.A + A - B).Inverse[B].Inverse[B]
```

$$X == \left\{\left\{-\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{5}{8}, \frac{5}{8}, \frac{7}{4}, \frac{11}{8}, \frac{13}{8}\right\}, \left\{\frac{1}{8}, \frac{5}{8}, \frac{7}{4}, \frac{11}{8}, \frac{25}{8}\right\}, \left\{\frac{5}{4}, -\frac{3}{4}, \frac{5}{2}, -\frac{1}{4}, \frac{17}{4}\right\}, \left\{\frac{7}{8}, -\frac{5}{8}, \frac{5}{4}, -\frac{3}{8}, \frac{23}{8}\right\}\right\}$$

```
(A.B.A + A - B).Inverse[B].Inverse[B] // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{5}{8} & \frac{5}{8} & \frac{7}{4} & \frac{11}{8} & \frac{13}{8} \\ \frac{1}{8} & \frac{5}{8} & \frac{7}{4} & \frac{11}{8} & \frac{25}{8} \\ \frac{5}{4} & -\frac{3}{4} & \frac{5}{2} & -\frac{1}{4} & \frac{17}{4} \\ \frac{7}{8} & -\frac{5}{8} & \frac{5}{4} & -\frac{3}{8} & \frac{23}{8} \end{pmatrix}$$

e

```
x1 = {0, 0, 0, 0, 0};
x2 = {1, -1, 0, 1, -1}
```

```
{1, -1, 0, 1, -1}
```

```
A.x1
```

```
{0, 0, 0, 0, 0}
```

```
A.x2
```

```
{0, 0, 0, 0, 0}
```

```
A.x1 == A.x2
```

```
True
```

```
Det[A]
```

```
0
```

Zwei verschiedene Urbilder haben dieselben Bilder. A ist nicht regulär.

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```
Remove["Global`*"]
```

Für die Ebene braucht es 3 Punkte. Beispiel:

```
r1 = 5;  k1[x_, y_, z_] := {x, y, z}.{x, y, z} - r1^2;
r2 = 6; M2 = {5, 5, 5};
k2[x_, y_, z_] := ({x, y, z} - M2).({x, y, z} - M2) - r2^2;
```

```
Solve[{k1[x, y, z] == 0, k2[x, y, z] == 0}, {x, y, z}]
```

$$\left\{\left\{x \to \frac{1}{10}\left(32 - 5 z - \sqrt{226 + 320 z - 75 z^2}\right), y \to \frac{1}{10}\left(32 - 5 z + \sqrt{226 + 320 z - 75 z^2}\right)\right\}, \left\{x \to \frac{1}{10}\left(32 - 5 z + \sqrt{226 + 320 z - 75 z^2}\right), y \to \frac{1}{10}\left(32 - 5 z - \sqrt{226 + 320 z - 75 z^2}\right)\right\}\right\}$$

```
solv1 = Solve[({k1[x, y, z] == 0, k2[x, y, z] == 0} /. z → 1), {x, y, z}] // N
```

{{x → 0.529747, y → 4.87025}, {x → 4.87025, y → 0.529747}}

```
p1 = {x, y} /. solv1[[1]]; p1 = Join[p1, {1}]
```

{0.529747, 4.87025, 1}

```
solv2 = Solve[
   ({k1[x, y, z] == 0, k2[x, y, z] == 0, k1[x, y, z] == k2[x, y, z]} /. z → 2), {x, y, z}] // N
```

{{x → -0.179075, y → 4.57908}, {x → 4.57908, y → -0.179075}}

```
p2 = {x, y} /. solv2[[1]]; p2 = Join[p2, {2}]
```

{-0.179075, 4.57908, 2}

```
solv3 = Solve[
   ({k1[x, y, z] == 0, k2[x, y, z] == 0, k1[x, y, z] == k2[x, y, z]} /. z → 3), {x, y, z}] // N
```

{{x → -0.560531, y → 3.96053}, {x → 3.96053, y → -0.560531}}

```
p3 = {x, y} /. solv3[[1]]; p3 = Join[p3, {3}]
```

{-0.560531, 3.96053, 3}

```
Φ[λ_, μ_] := p1 + λ (p2 - p1) + μ (p3 - p1);
Φ[λ, μ]
```

{0.529747 - 0.708822 λ - 1.09028 μ, 4.87025 - 0.291178 λ - 0.909723 μ, 1 + λ + 2 μ}

```
solve = Solve[Φ[λ, μ] == {t, 0, 0}, {t, λ, μ}] // Flatten
```

{t → 6.4, λ → -32.533, μ → 15.7665}

```
x = t /. solve
```

6.4

8

```
Remove["Global`*"]

P1 = {1, 0, 0}; P2 = {0, 2, 0}; P3 = {0, 0, 3};
Q1 = {4, 0, 0}; Q2 = {1, 5, 1};
Φ[λ_, μ_] := P1 + λ (P2 - P1) + μ (P3 - P1);
gSQ2Φ[t_] := Q2 + t Cross[ (P2 - P1), (P3 - P1)];
solvQ2F = Solve[Φ[λ, μ] == gSQ2Φ[t], {t, λ, μ}] // Flatten
```

$\left\{t \to -\frac{17}{49}, \lambda \to \frac{97}{49}, \mu \to \frac{5}{49}\right\}$

```
FusspunktQ2 = gSQ2Φ[t] /. solvQ2F
```

$\left\{-\frac{53}{49}, \frac{194}{49}, \frac{15}{49}\right\}$

```
N[%]
```

{-1.08163, 3.95918, 0.306122}

```
Q2Gespiegelt = Q2 + 2 (FusspunktQ2 - Q2)
```

$\left\{-\frac{155}{49}, \frac{143}{49}, -\frac{19}{49}\right\}$

```
N[%]
```

{-1.08163, 3.95918, 0.306122}

```
gQ2GespQ1[t_] := Q1 = t (Q2Gespiegelt - Q1);
solvgQ2GespQ1 = Solve[Φ[λ, μ] == gQ2GespQ1[t], {t, λ, μ}] // Flatten
```

$\left\{t \to -\frac{6}{35}, \lambda \to -\frac{429}{1715}, \mu \to \frac{38}{1715}\right\}$

```
L = gQ2GespQ1[t] /. solvgQ2GespQ1
```

$\left\{\frac{45186}{60025}, -\frac{35178}{60025}, \frac{4674}{60025}\right\}$

```
N[%]
```

{0.752786, -0.586056, 0.0778676}