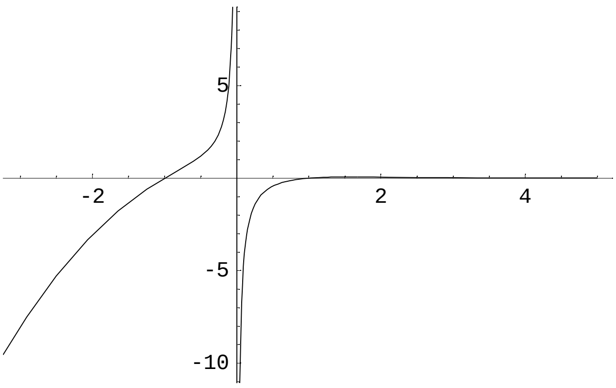
Lösungen

1

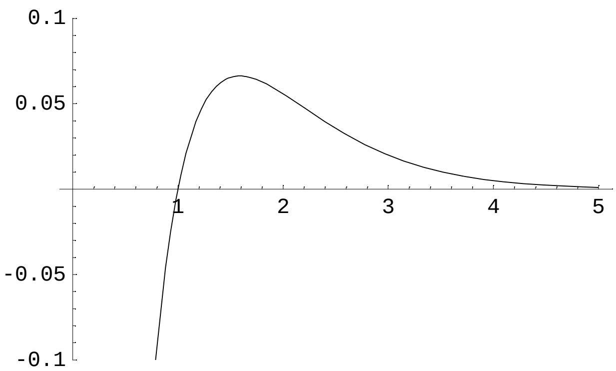
```
Remove["Global`*"]
```

a

```
f[x_] := (x^2 - 1) / (E^(2 x) - 1); Plot[f[x], {x, -5, 5}];
```




```
Plot[f[x], {x, 0.5, 5}, PlotRange → {-0.1, 0.1}];
```



b

```
(Evaluate[f'[y] // Simplify]) /. y → -1
```

$$-\frac{2}{-1+\frac{1}{e^2}}$$

```
N[%]
```

2.31304

```
ArcTan[%]
```

1.16273

```
% / Degree
```

66.6196

c

```
FindRoot[f[x] == 0, {x, 1}]
```

{x → 1.}

```
f[1]
```

0

```
FindRoot[f[x] == 0, {x, -1}]
```

{x → -1.}

```
f[-1]
```

0

```
FindRoot[1 / f[x] == 0, {x, 0}]
```

{x → 0.}

```
Limit[1 / f[x], x → 0]
```

0

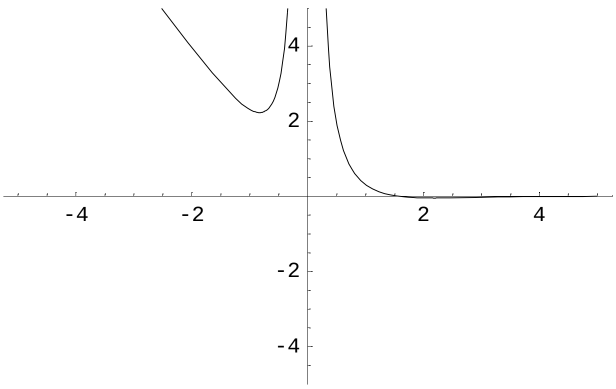
==> Pol

d

```
Evaluate[f'[x] // Simplify]
```

$$-\frac{2\left(x+e^{2x}\left(-1-x+x^2\right)\right)}{\left(-1+e^{2x}\right)^2}$$

```
Plot[Evaluate[f'[x] // Simplify], {x, -5, 5}, PlotRange → {-5, 5}];
```



```
FindRoot[(Evaluate[f'[y] // Simplify] == 0) /. y → x, {x, 2}]
```

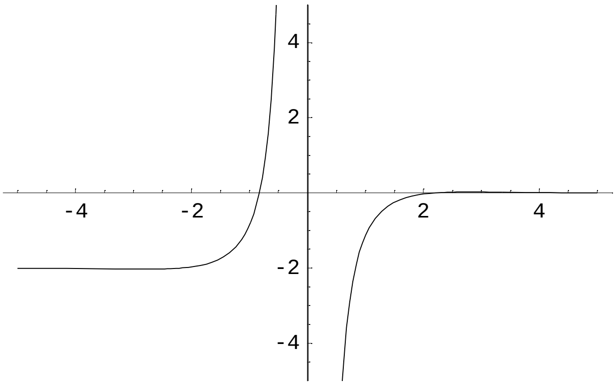
{x → 1.58798}

```
f[1.5879776025618713`]
```

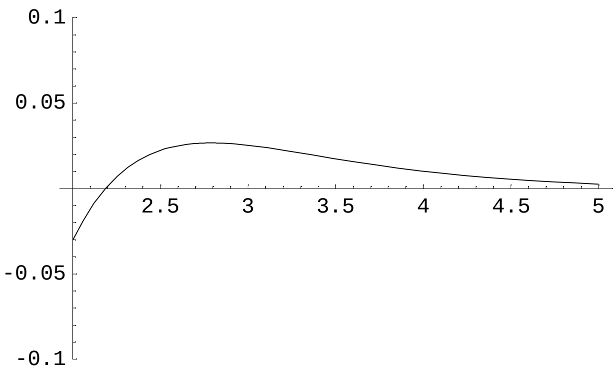
0.0663047

e

```
Plot[Evaluate[f''[x] // Simplify], {x, -5, 5}, PlotRange → {-5, 5}];
```




```
Plot[Evaluate[f''[x] // Simplify], {x, 2, 5}, PlotRange → {-0.1, 0.1}];
```




```
fr1 = FindRoot[(Evaluate[f''[y] // Simplify] == 0) /. y → x, {x, -1}]
```

{x → -0.827938}

```
x1 = x /. fr1
```

-0.827938

```
f[x1]
```

0.388738

```
fr2 = FindRoot[(Evaluate[f''[y] // Simplify] == 0) /. y → x, {x, 2}]
```

{x → 2.18734}

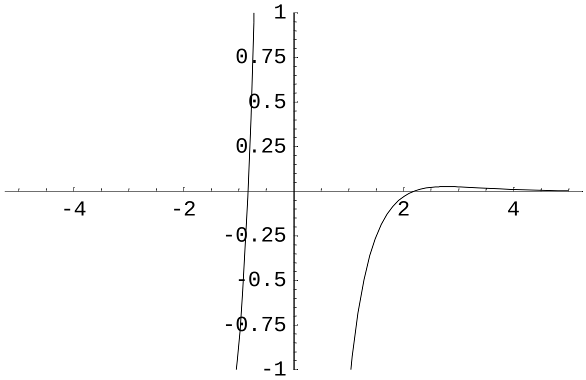
```
x2 = x /. fr2
```

2.18734

```
f[x2]
```

0.0482623

```
Plot[Evaluate[f''[x] // Simplify], {x, -5, 5}, PlotRange → {-1, 1}];
```



f

```
Limit[f[x], x → Infinity]
```

0

```
Limit[f[x], x → -Infinity]
```

$-\infty$

g

```
Normal[Series[f[x], {x, -3, 3}]] // N
```

$-8.01988 + 5.97505\,(3. + x) - 1.01265\,(3. + x)^2 - 0.00190931\,(3. + x)^3$

```
% // Expand
```

$0.739899 - 0.152383\,x - 1.02983\,x^2 - 0.00190931\,x^3$

h

```
A1 = Integrate[N[Normal[Series[f[x], {x, -3, 3}]]], {x, -4, -2}]
```

−16.7149

i

```
A2 = Integrate[N[f[x]], {x, -4, -2}] // Chop
```

−16.7141

```
Abs[A2 - A1]
```

0.000743575

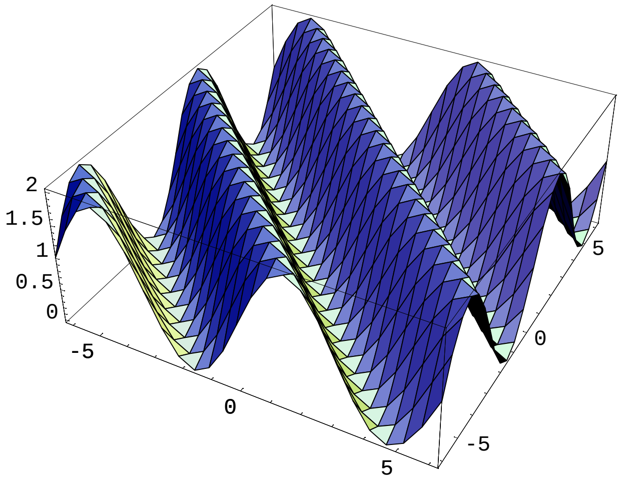
2

```
Remove["Global`*"]

f1[x_, y_] := 1 + Sin[x + y];
f2[x_, y_] := 1 + Sin[x y];
f3[x_, y_] := 1 + Sin[x] + Sin[y];
f4[x_, y_] := 1 + Sin[x] Sin[y];
```

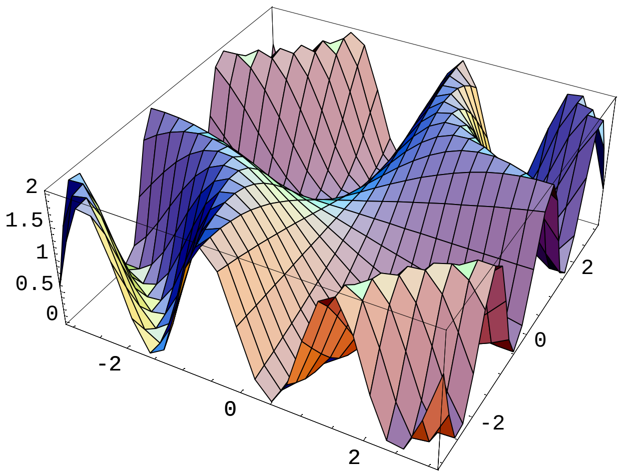
a

```
(* f1[x,y] *) Plot3D[1 + Sin[x + y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}];
```



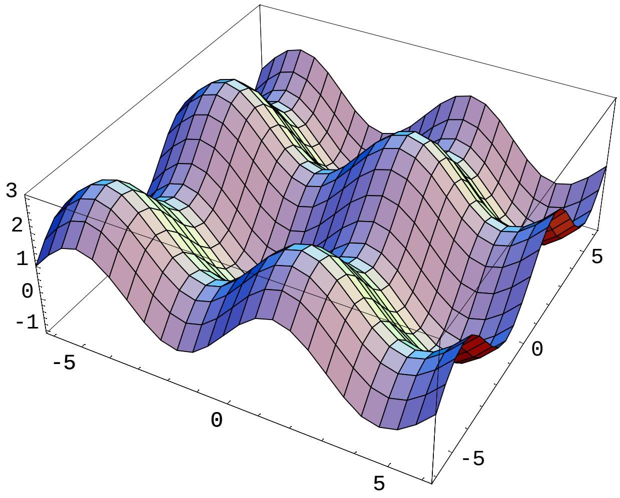
Wellblech

```
(* f2[x,y] *) Plot3D[1 + Sin[x y], {x, -Pi, Pi}, {y, -Pi, Pi}];
```



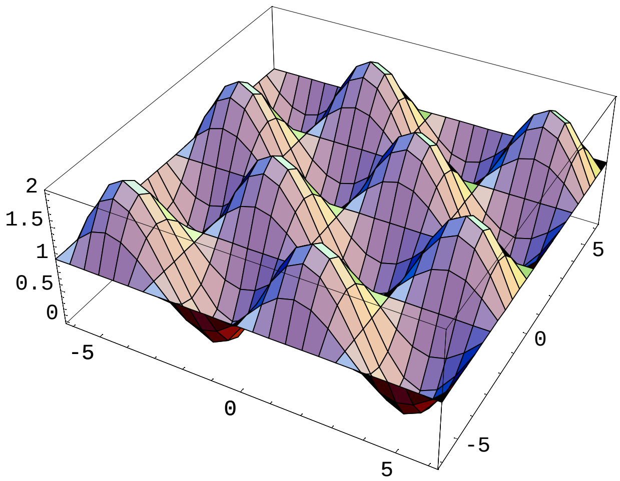
Keine der genannten Formen

```
(* f3[x,y] *) Plot3D[1 + Sin[x] + Sin[y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}];
```



Eierschachtel

```
(* f4[x,y] *) Plot3D[1 + Sin[x] Sin[y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}];
```



Eierschachtel

b

```
V1 = Integrate[f3[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

$16\pi^2$

```
N[%]
```

157.914

```
V2 = Integrate[f4[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

$16\pi^2$

```
N[%]
```

157.914

c

```
s1 = Integrate[(Evaluate[Sqrt[1 + D[f3[0, t], t]^2]] /. t → y), {y, -2 Pi, 2 Pi}]
```

$8\sqrt{2}\ \text{EllipticE}\left[\frac{1}{2}\right]$

```
N[%]
```

15.2808

```
s2 = Integrate[(Evaluate[Sqrt[1 + D[f4[0, t], t]^2]] /. t → y), {y, -2 Pi, 2 Pi}]
```

4π

```
N[%]
```

12.5664

```
s1 > s2
```

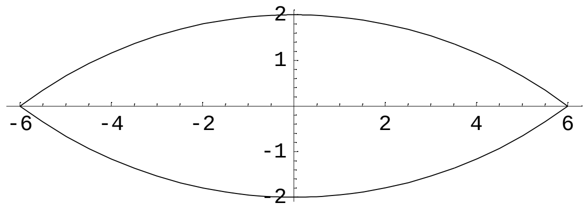
True

3

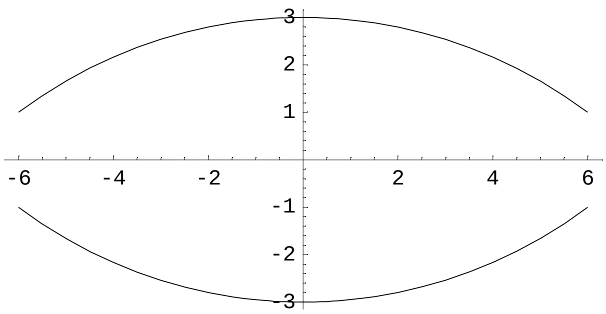
```
Remove["Global`*"]
```

a

```
s[x_, r_] := Sqrt[10^2 - x^2] - r;
Plot[{s[x, 8], -s[x, 8]}, {x, -6, 6}, AspectRatio → Automatic];
```




```
Plot[{s[x, 7], -s[x, 7]}, {x, -6, 6}, AspectRatio → Automatic];
```



b

```
A[r_] := s[6, r]^2 Pi; A[r]
```

$\pi\,(8-r)^2$

```
s[x, r]^2 Pi // Expand
```

$100\,\pi+\pi\,r^2-\pi\,x^2-2\,\pi\,r\,\sqrt{100-x^2}$

```
V[r_] := Integrate[s[x, r]^2 Pi, {x, -6, 6}]; V[r]
```

$4\,\pi\left(264+3\,r^2-2\,r\left(12+25\,\mathrm{ArcSin}\left[\frac{3}{5}\right]\right)\right)$

```
N[%]
```

$12.5664\,(264.-56.1751\,r+3.\,r^2)$

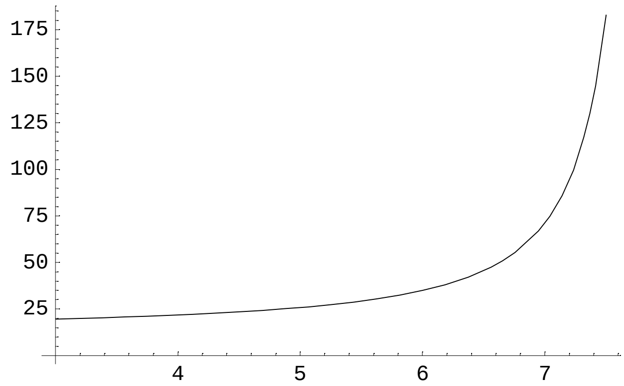
```
q[r_] := V[r] / A[r] // Simplify;  q[r]
```

$\frac{4\,(264+3\,r^2-2\,r\,(12+25\,\mathrm{ArcSin}[\frac{3}{5}]))}{(-8+r)^2}$

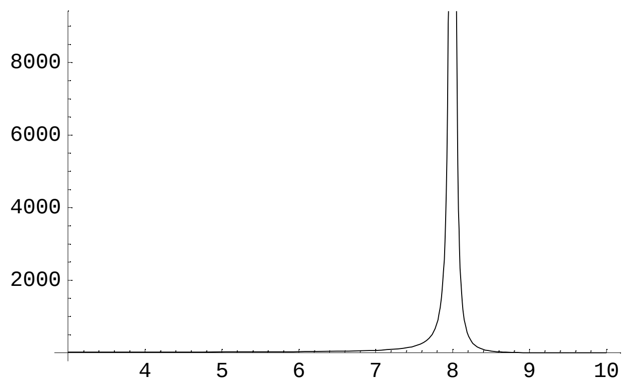
```
N[%]
```

$\frac{4.\,(264.-56.1751\,r+3.\,r^2)}{(-8.+r)^2}$

```
Plot[Evaluate[ q[r]], {r, 3, 7.5}];
```




```
Plot[Evaluate[ q[r]], {r, 3, 10}];
```



```
V[7]
```

$$4\pi\left(243-350\,\text{ArcSin}\left[\frac{3}{5}\right]\right)$$

```
N[%]
```

223.362

```
V[8]
```

$$-32\pi\left(-33+50\,\text{ArcSin}\left[\frac{3}{5}\right]\right)$$

```
N[%]
```

82.9325

```
q[7]
```

$$972-1400\,\text{ArcSin}\left[\frac{3}{5}\right]$$

```
N[%]
```

71.0984

```
q[8]
```

ComplexInfinity

c

```
solv = Solve[V[r] / A[r] == 25, {r}] // Flatten
```

$$\left\{r\to\frac{1}{26}\left(304-\sqrt{-28288+\left(304-200\,\text{ArcSin}\left[\frac{3}{5}\right]\right)^2}-200\,\text{ArcSin}\left[\frac{3}{5}\right]\right),\ r\to\frac{1}{26}\left(304+\sqrt{-28288+\left(304-200\,\text{ArcSin}\left[\frac{3}{5}\right]\right)^2}-200\,\text{ArcSin}\left[\frac{3}{5}\right]\right)\right\}$$

```
solv // N
```

{r → 4.84166, r → 8.64294}

Nur r = 4.84166 liegt im geometrisch möglichen Bereich

d

```
Integrate[(Evaluate[2 s[t, 7] Pi Sqrt[1 + D[s[t, 7], t]^2]] /. t → x), {x, -6, 6}]
```

$$40\pi\left(6-7\,\text{ArcSin}\left[\frac{3}{5}\right]\right)$$

```
N[%]
```

187.929

```
(* Vergleich ok *)
12 Pi 2 2 // N
```

150.796

4

```
Remove["Global`*"]
```

Daten

```
v1 = {2, -8, 0}; v2 = {-2, 6, 2};
OP = {-1, 2, 3};
v3 = Cross[v1, v2]
```

{-16, -4, -4}

a

```
mX = Transpose[{v1, v2, v3}]; mX // MatrixForm
```

$$\begin{pmatrix} 2 & -2 & -16 \\ -8 & 6 & -4 \\ 0 & 2 & -4 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
mS = mX.Dλ.Inverse[mX]; mS // MatrixForm
```

$$\begin{pmatrix} -\frac{7}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{8}{9} & -\frac{1}{9} \\ -\frac{4}{9} & -\frac{1}{9} & \frac{8}{9} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.777778 & -0.444444 & -0.444444 \\ -0.444444 & 0.888889 & -0.111111 \\ -0.444444 & -0.111111 & 0.888889 \end{pmatrix}$$

```
9 * mS // MatrixForm
```

$$\begin{pmatrix} -7 & -4 & -4 \\ -4 & 8 & -1 \\ -4 & -1 & 8 \end{pmatrix}$$

b

```
OP1 = mS.OP
```

$\left\{-\frac{13}{9}, \frac{17}{9}, \frac{26}{9}\right\}$

```
N[%]
```

{-1.44444, 1.88889, 2.88889}

c

```
Dreh[φ_] := {{Cos[φ], -Sin[φ], 0}, {Sin[φ], Cos[φ], 0}, {0, 0, 1}};
Dreh[Pi / 5] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4}\left(1+\sqrt{5}\right) & -\frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} & 0 \\ \frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} & \frac{1}{4}\left(1+\sqrt{5}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.809017 & -0.587785 & 0. \\ 0.587785 & 0.809017 & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

d

```
OP2 = Dreh[Pi / 5].OP1
```

$$\left\{-\frac{17}{18}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} - \frac{13}{36}\left(1+\sqrt{5}\right), -\frac{13}{18}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} + \frac{17}{36}\left(1+\sqrt{5}\right), \frac{26}{9}\right\}$$

```
N[%]
```

{-2.27884, 0.67912, 2.88889}

e

```
OP3 = mS.OP2 // N
```

{0.18665, 1.29549, 3.50526}

f

```
Dλ1 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}; Dλ1 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
mP = mX.Dλ1.Inverse[mX]; mP // MatrixForm
```

$$\begin{pmatrix} \frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{17}{18} & -\frac{1}{18} \\ -\frac{2}{9} & -\frac{1}{18} & \frac{17}{18} \end{pmatrix}$$

```
N[mP] // MatrixForm
```

$$\begin{pmatrix} 0.111111 & -0.222222 & -0.222222 \\ -0.222222 & 0.944444 & -0.0555556 \\ -0.222222 & -0.0555556 & 0.944444 \end{pmatrix}$$

```
18 * mP // MatrixForm
```

$$\begin{pmatrix} 2 & -4 & -4 \\ -4 & 17 & -1 \\ -4 & -1 & 17 \end{pmatrix}$$

g

```
mP.OP3 // N
```

{-1.0461, 0.987306, 3.19708}

5

```
Remove["Global`*"]
```

Vektornormierung

```
nor[m_] := Table[m[[k]] / Norm[m[[k]]], {k, 1, Length[m]}]
```

Daten

```
a1 = {2, 2, 2}; a2 = {2, -2, 2}; a3 = {2, 2, -2};
b1 = 2 a1;
OQ = {0, -2, 2};
```

```
A = Transpose[{a1, a2, a3}]; A // MatrixForm
```

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & -2 & 2 \\ 2 & 2 & -2 \end{pmatrix}$$

```
B = Transpose[{b1, a2, a3}]; B // MatrixForm
```

$$\begin{pmatrix} 4 & 2 & 2 \\ 4 & -2 & 2 \\ 4 & 2 & -2 \end{pmatrix}$$

a

```
ew1 = Eigenvalues[A]
```

$\{-4, 4, -2\}$

```
% // N
```

$\{-4., 4., -2.\}$

b

```
ew2 = Eigenvalues[B]
```

$\left\{2\left(1+\sqrt{5}\right), -4, 2\left(1-\sqrt{5}\right)\right\}$

```
% // N
```

$\{6.47214, -4., -2.47214\}$

c

Der Eigenwert -4 ist gemeinsam

d

```
ev1 = Eigenvectors[A]
```

$\{\{0, -1, 1\}, \{2, 1, 1\}, \{-1, 1, 1\}\}$

```
Transpose[ev1] // MatrixForm
```

$$\begin{pmatrix} 0 & 2 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

```
ev11 = nor[ev1] // N
```

$\{\{0., -0.707107, 0.707107\}, \{0.816497, 0.408248, 0.408248\}, \{-0.57735, 0.57735, 0.57735\}\}$

```
Transpose[ev11] // MatrixForm
```

$$\begin{pmatrix} 0. & 0.816497 & -0.57735 \\ -0.707107 & 0.408248 & 0.57735 \\ 0.707107 & 0.408248 & 0.57735 \end{pmatrix}$$

```
ev2 = Eigenvectors[B]
```

$\left\{\left\{\frac{1}{4}\left(2+2\sqrt{5}\right), 1, 1\right\}, \{0, -1, 1\}, \left\{\frac{1}{4}\left(2-2\sqrt{5}\right), 1, 1\right\}\right\}$

```
Transpose[ev2] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4}\left(2+2\sqrt{5}\right) & 0 & \frac{1}{4}\left(2-2\sqrt{5}\right) \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

```
ev21 = nor[ev2] // N
```

```
{{0.752938, 0.465341, 0.465341},
 {0., -0.707107, 0.707107}, {-0.400447, 0.647936, 0.647936}}
```

```
Transpose[ev21] // MatrixForm
```

$$\begin{pmatrix} 0.752938 & 0. & -0.400447 \\ 0.465341 & -0.707107 & 0.647936 \\ 0.465341 & 0.707107 & 0.647936 \end{pmatrix}$$

Der Eigenvektor {0,-1,1} zum Eigenwert -4 ist gemeinsam

e

```
ew3 = Eigenvalues[A.B]
```

$\left\{8\left(2+\sqrt{2}\right),\ 16,\ 8\left(2-\sqrt{2}\right)\right\}$

```
% // N
```

```
{27.3137, 16., 4.68629}
```

```
ew3 = Eigenvalues[B.A]
```

$\left\{8\left(2+\sqrt{2}\right),\ 16,\ 8\left(2-\sqrt{2}\right)\right\}$

```
% // N
```

```
{27.3137, 16., 4.68629}
```

Gleiche Eigenwerte

f

```
Apply[Plus, Eigenvalues[A]]
```

-2

```
Apply[Plus, Eigenvalues[B]]
```

$-4+2\left(1-\sqrt{5}\right)+2\left(1+\sqrt{5}\right)$

```
% // Simplify
```

0

```
Apply[Plus, Eigenvalues[A.B]] // Simplify
```

48

```
Apply[Plus, Eigenvalues[B.A]] // Simplify
```

```
48
```

Gleiche Eigenwerte von A.B und B.A ==> gleiche Summen

g

```
Apply[Times, Eigenvalues[A]]
```

```
32
```

```
Apply[Times, Eigenvalues[B]] // Simplify
```

```
64
```

```
Apply[Times, Eigenvalues[A.B]] // Simplify
```

```
2048
```

```
Apply[Times, Eigenvalues[B.A]] // Simplify
```

```
2048
```

Gleiche Eigenwerte von A mal Eigenwerte von B gleich Eigenwerte von B.A oder A.B

h

```
Det[A]
```

```
32
```

```
Det[B]
```

```
64
```

```
Det[A.B]
```

```
2048
```

```
Det[B.A]
```

```
2048
```

Die Determinante ist das Produkt der Eigenwerte: Resultate der vorangehenden Teilaufgabe

i

```
OP1 = A.OQ
```

```
{0, 8, -8}
```

```
OP2 = B.OQ
```

```
{0, 8, -8}
```

OQ ist Eigenvektor von A und von B !!!!

6

a

```
Remove["Global`*"]
```

$A.(A+X).A+A+A^{-1} = A.A^{T}+E$

==> $A^{-1}.A.(A+X).A.A^{-1}+A^{-1}.A.A^{-1}+A^{-1}.A^{-1}.A^{-1} = A^{-1}.A.A^{T}.A^{-1}+A^{-1}.E.A^{-1}$

==> $A+X+A^{-1}+A^{-3} = A^{T}.A^{-1}+A^{-2}$

==> $X = A^{T}.A^{-1}+A^{-2}-A-A^{-1}-A^{-3} = A^{T}.A^{-1} - A - A^{-1} + A^{-2} - A^{-3}$

$B.(B+X).B+B+B^{-1} = B.B^{T}+E$

== > $B^{-1}.B.(B+X).B.B^{-1}+B^{-1}.B.B^{-1}+B^{-1}.B^{-1}.B^{-1} = B^{-1}.B.B^{T}.B^{-1}+B^{-1}.E.B^{-1}$

== > $B+X+B^{-1}+B^{-3} = B^{T}.B^{-1}+B^{-2}$

== > $X = B^{T}.B^{-1}+B^{-2}-B-B^{-1}-B^{-3} = B^{T}.B^{-1}-B-B^{-1}+B^{-2}-B^{-3}$

b

i

```
U1 = {{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}}; U1 // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(U1.U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(U1.U1.U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

ii

```
(U1.U1).(U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(U1).(U1.U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(U1).(U1.U1.U1.U1) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

c

```
Remove["Global`*"]
```

```
U2 = {{1, 2, 3, 4, 5 } + {1, 1, 2, 1, 2},
{3, 2, 1, 5, 4},
{7, 2, -3, 7, 2 },
{1, 1, 2, 1, 2} - {1, 2, 3, 4, 5 }}; U2 // MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 5 & 5 & 7 \\ 3 & 2 & 1 & 5 & 4 \\ 7 & 2 & -3 & 7 & 2 \\ 0 & -1 & -1 & -3 & -3 \end{pmatrix}$$

```
b1 = {4 + 2, 3, 0, 2 - 4}
```

{6, 3, 0, -2}

```
b2 = {4 + 2, 3, 1, 2 - 4}
```

{6, 3, 1, -2}

i

```
x = {x1, x2, x3, x4, x5};
```

```
Solve[U2.x == b1, x]
```

{}

```
Solve[U2.x == b2, x]
```

$\{\{x1 \to -\frac{1}{4} + \frac{3\,x4}{4} + \frac{3\,x5}{4},\ x2 \to \frac{7}{4} - \frac{17\,x4}{4} - \frac{13\,x5}{4},\ x3 \to \frac{1}{4} + \frac{5\,x4}{4} + \frac{x5}{4}\}\}$

ii

Ordnung = 5

iii

Fall für b2: Dim Lösungsraum = 2

iiii

Rang = Ordnung - Dimension = 5 - 2 = 3

d

```
Remove["Global`*"]
```

```
v1[t1_] := {-3, 1, -1} + t1 {-1, 2, 1};
v2[t2_] := {1, 4, 2} + t2 {-2, 4, 2};
```

```
Solve[k {-1, 2, 1} == {-2, 4, 2}, {k}]
```

{{k → 2}}

==> Geraden parallel ==> Abstand:

```
a = Norm[Cross[(v1[0] - v2[0]), {-1, 2, 1}]] / Norm[{-1, 2, 1}]
```

$\sqrt{\frac{179}{6}}$

```
N[%]
```

5.46199

7

```
Remove["Global`*"]
```

a

```
OP1 = {2, 0, 4};
a = {3, 1, 4};
b = {1, 0, 0};
c = Cross[a, b]
```

{0, 4, -1}

```
d = Cross[a, c]
```

{-17, 3, 12}

```
ea = a / Norm[a]
```

$\left\{\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 2\sqrt{\frac{2}{13}}\right\}$

```
ea // N
```

{0.588348, 0.196116, 0.784465}

```
ec = c / Norm[c]
```

$\left\{0, \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right\}$

```
ec // N
```

{0., 0.970143, -0.242536}

```
ed = d / Norm[d]
```

$\left\{-\sqrt{\frac{17}{26}}, \frac{3}{\sqrt{442}}, 6\sqrt{\frac{2}{221}}\right\}$

```
ed // N
```

{-0.808608, 0.142695, 0.570782}

b

```
M = Transpose[{ea, ec, ed}]; M // MatrixForm
```

$$\begin{pmatrix} \frac{3}{\sqrt{26}} & 0 & -\sqrt{\frac{17}{26}} \\ \frac{1}{\sqrt{26}} & \frac{4}{\sqrt{17}} & \frac{3}{\sqrt{442}} \\ 2\sqrt{\frac{2}{13}} & -\frac{1}{\sqrt{17}} & 6\sqrt{\frac{2}{221}} \end{pmatrix}$$

```
M // N // MatrixForm
```

$$\begin{pmatrix} 0.588348 & 0. & -0.808608 \\ 0.196116 & 0.970143 & 0.142695 \\ 0.784465 & -0.242536 & 0.570782 \end{pmatrix}$$

c

Minv = Inverse[M]

$$\left\{\left\{\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 2\sqrt{\frac{2}{13}}\right\}, \left\{0, \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right\}, \left\{-\sqrt{\frac{17}{26}}, \frac{3}{\sqrt{442}}, 6\sqrt{\frac{2}{221}}\right\}\right\}$$

Minv // MatrixForm

$$\begin{pmatrix} \frac{3}{\sqrt{26}} & \frac{1}{\sqrt{26}} & 2\sqrt{\frac{2}{13}} \\ 0 & \frac{4}{\sqrt{17}} & -\frac{1}{\sqrt{17}} \\ -\sqrt{\frac{17}{26}} & \frac{3}{\sqrt{442}} & 6\sqrt{\frac{2}{221}} \end{pmatrix}$$

Minv // N // MatrixForm

$$\begin{pmatrix} 0.588348 & 0.196116 & 0.784465 \\ 0. & 0.970143 & -0.242536 \\ -0.808608 & 0.142695 & 0.570782 \end{pmatrix}$$

OP1s = Minv.OP1

$$\left\{11\sqrt{\frac{2}{13}}, -\frac{4}{\sqrt{17}}, 24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right\}$$

OP1s // N

{4.31455, -0.970143, 0.665912}

d

Dreh[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}}

Dreh[2 Pi / 3] // MatrixForm

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Dreh[2 Pi / 3] // N // MatrixForm

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & -0.5 & -0.866025 \\ 0. & 0.866025 & -0.5 \end{pmatrix}$$

Dreh[4 Pi / 3] // MatrixForm

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

```
Dreh[4 Pi / 3] // N // MatrixForm
```

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & -0.5 & 0.866025 \\ 0. & -0.866025 & -0.5 \end{pmatrix}$$

```
OP2s = Dreh[2 Pi / 3].OP1s
```

$$\left\{11\sqrt{\frac{2}{13}},\ \frac{2}{\sqrt{17}} - \frac{1}{2}\sqrt{3}\left(24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right),\ -2\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-24\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right\}$$

```
OP2s // N
```

{4.31455, −0.0916255, −1.17312}

```
OP3s = Dreh[4 Pi / 3].OP1s
```

$$\left\{11\sqrt{\frac{2}{13}},\ \frac{2}{\sqrt{17}} + \frac{1}{2}\sqrt{3}\left(24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right),\ 2\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-24\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right\}$$

```
OP3s // N
```

{4.31455, 1.06177, 0.507212}

```
OP3as = Dreh[2 Pi / 3].OP2s
```

$$\left\{11\sqrt{\frac{2}{13}},\right.$$
$$\frac{1}{2}\left(-\frac{2}{\sqrt{17}} + \frac{1}{2}\sqrt{3}\left(24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right) - \frac{1}{2}\sqrt{3}\left(-2\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-24\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right),$$
$$\left.\frac{1}{2}\left(2\sqrt{\frac{3}{17}} + \frac{1}{2}\left(24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right) + \frac{1}{2}\sqrt{3}\left(\frac{2}{\sqrt{17}} - \frac{1}{2}\sqrt{3}\left(24\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right)\right\}$$

```
OP3as // N
```

{4.31455, 1.06177, 0.507212}

e

```
OP2 = M.OP2s; OP2 // N
```

{3.48706, 0.589865, 2.73724}

```
OP3 = M.OP3s; OP3 // N
```

{2.12833, 1.9486, 3.41661}

```
OP3a = M.OP3as; OP3a // N
```

{2.12833, 1.9486, 3.41661}

f

```
V = Det[{OP1, OP2, OP3}] / 6
```

$$\frac{99 \sqrt{\frac{3}{26}}}{13}$$

```
V // N
```

2.58682

```
Va = Det[{OP1, OP2, OP3a}] / 6
```

$$\frac{99 \sqrt{\frac{3}{26}}}{13}$$

```
Va // N
```

2.58682