Lösungen

1

```
rem; Remove["Global`*"]
```

```
M = {{2, -2, 3}, {-3, 1, 3}, {-2, -1, 2}};
MatrixForm[M]
```

$$\begin{pmatrix} 2 & -2 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 2 \end{pmatrix}$$

```
MM = M + M;
MatrixForm[MM]
```

$$\begin{pmatrix} 4 & -4 & 6 \\ -6 & 2 & 6 \\ -4 & -2 & 4 \end{pmatrix}$$

a

```
Det[M]
```

25

```
Det[MM]
```

200

b

Det[M] ungleich 0, Inverse existiert

c

```
{Inverse[M] // MatrixForm, Inverse[M] // N // MatrixForm}
```

$$\left\{\begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0.04 & -0.36 \\ 0. & 0.4 & -0.6 \\ 0.2 & 0.24 & -0.16 \end{pmatrix}\right\}$$

d

```
{Inverse[Transpose[M]] // MatrixForm, Inverse[Transpose[M]] // N // MatrixForm}
```

$$\left\{\begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0. & 0.2 \\ 0.04 & 0.4 & 0.24 \\ -0.36 & -0.6 & -0.16 \end{pmatrix}\right\}$$

```
{Transpose[Inverse[M]] // MatrixForm, Transpose[Inverse[M]] // N // MatrixForm}
```

$$\left\{\begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0. & 0.2 \\ 0.04 & 0.4 & 0.24 \\ -0.36 & -0.6 & -0.16 \end{pmatrix}\right\}$$

```
{Inverse[M] // MatrixForm, Inverse[Transpose[M]] // MatrixForm}
```

$$\left\{\begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}\right\}$$

Die Transponierte der Inversen ist die Inverse der Transponierten.

e

```
Inverse[Transpose[M]] == Transpose[Inverse[M]]
```

True

f

```
IM = Inverse[M];
IM7 = IM.IM.IM.IM.IM.IM.IM; IM7 // MatrixForm (* exakt *)
```

$$\begin{pmatrix} -\frac{823454}{1220703125} & -\frac{3516614}{6103515625} & \frac{974151}{6103515625} \\ -\frac{245403}{244140625} & -\frac{659023}{1220703125} & -\frac{493293}{1220703125} \\ -\frac{244574}{1220703125} & \frac{1395591}{6103515625} & -\frac{9278194}{6103515625} \end{pmatrix}$$

```
IM7 // N // MatrixForm (* nicht exakt *)
```

$$\begin{pmatrix} -0.000674574 & -0.000576162 & 0.000159605 \\ -0.00100517 & -0.000539872 & -0.000404106 \\ -0.000200355 & 0.000228654 & -0.00152014 \end{pmatrix}$$

```
Det[IM]
```

$$\frac{1}{25}$$

```
Det[IM]^7
```

$$\frac{1}{6103515625}$$

```
Det[IM7]
```

$$\frac{1}{6103515625}$$

g

```
OP0 = {{2}, {5}, {8}}; OP0 // MatrixForm
```

$$\begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$$

```
OP1 = M.OP0; OP1 // MatrixForm
```

$$\begin{pmatrix} 18 \\ 23 \\ 7 \end{pmatrix}$$

```
{OP2 = Inverse[M].OP0; OP2 // MatrixForm, OP2 = Inverse[M].OP0; OP2 // N // MatrixForm}
```

$$\left\{ \begin{pmatrix} -\frac{57}{25} \\ -\frac{14}{5} \\ \frac{8}{25} \end{pmatrix}, \begin{pmatrix} -2.28 \\ -2.8 \\ 0.32 \end{pmatrix} \right\}$$

```
{Norm[OP2 - OP1], Norm[OP2 - OP1] // N}
```

$$\left\{ \frac{\sqrt{700963}}{25}, 33.4894 \right\}$$

h

```
M2 = M.M; M2 // MatrixForm
```

$$\begin{pmatrix} 4 & -9 & 6 \\ -15 & 4 & 0 \\ -5 & 1 & -5 \end{pmatrix}$$

```
M2.OP2 == OP1
```

True

i

```
Eigenvalues[M] // N
```

$\{3.83195, 0.584025 + 2.48657\, i, 0.584025 - 2.48657\, i\}$

```
Det[M - λ IdentityMatrix[3]] == 0
```

$25 - 11\lambda + 5\lambda^2 - \lambda^3 == 0$

```
Solve[Det[M - λ IdentityMatrix[3]] == 0, {λ}]
```

$$\left\{\left\{\lambda \to \frac{1}{3}\left(5 - \frac{8}{\left(215 + 9\sqrt{577}\right)^{1/3}} + \left(215 + 9\sqrt{577}\right)^{1/3}\right)\right\},\right.$$

$$\left\{\lambda \to \frac{5}{3} + \frac{4\left(1 + i\sqrt{3}\right)}{3\left(215 + 9\sqrt{577}\right)^{1/3}} - \frac{1}{6}\left(1 - i\sqrt{3}\right)\left(215 + 9\sqrt{577}\right)^{1/3}\right\},$$

$$\left.\left\{\lambda \to \frac{5}{3} + \frac{4\left(1 - i\sqrt{3}\right)}{3\left(215 + 9\sqrt{577}\right)^{1/3}} - \frac{1}{6}\left(1 + i\sqrt{3}\right)\left(215 + 9\sqrt{577}\right)^{1/3}\right\}\right\}$$

```
Eigenvectors[M] // N
```

{{-3.07369, 4.31542, 1.}, {0.322557 - 0.770142 i, 0.770862 - 0.946281 i, 1.},
{0.322557 + 0.770142 i, 0.770862 + 0.946281 i, 1.}}

Aufgabe kann auch anders interpretiert werden:

```
Eigenvalues[M2] // N
```

{14.6838, -5.84192 + 2.90443 i, -5.84192 - 2.90443 i}

```
Eigenvectors[M2] // N
```

{{-3.07369, 4.31542, 1.}, {0.322557 - 0.770142 i, 0.770862 - 0.946281 i, 1.},
{0.322557 + 0.770142 i, 0.770862 + 0.946281 i, 1.}}

2

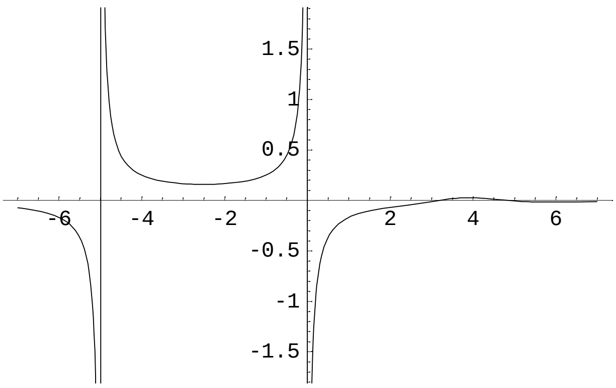
```
rem; Remove["Global`*"]
```

```
f[x_] := (2 E^(-(x - 4)^2) - 1) / (x (x + 5)); f[x]
```

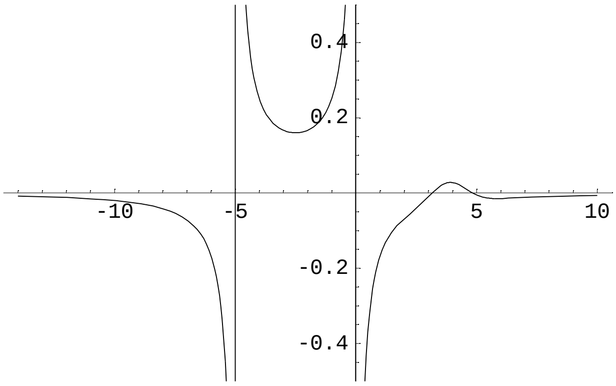
$$\frac{-1 + 2\,e^{-(-4+x)^2}}{x\,(5 + x)}$$

a

```
Plot[f[x], {x, -7, 7}];
```



```
Plot[f[x], {x, -14, 10}, PlotRange → {-0.5, 0.5}];
```



b

```
f'[x]
```

$$-\frac{-1+2\,e^{-(-4+x)^2}}{x\,(5+x)^2}-\frac{-1+2\,e^{-(-4+x)^2}}{x^2\,(5+x)}-\frac{4\,e^{-(-4+x)^2}\,(-4+x)}{x\,(5+x)}$$

```
f'[x] /. x → -1
```

$$\frac{1}{4}\left(1-\frac{2}{e^{25}}\right)+\frac{1}{16}\left(-1+\frac{2}{e^{25}}\right)-\frac{5}{e^{25}}$$

```
steig = (f'[x] /. x → -1) // N
```

0.1875

```
ArcTan[steig]
```

0.185348

```
ArcTan[steig] / Degree // N
```

10.6197

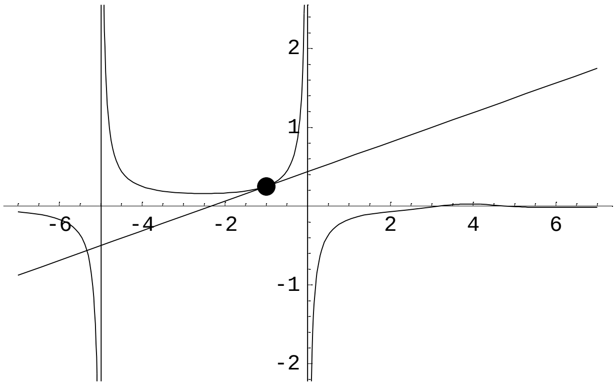
```
g[x_] := f[-1] + steig (x - (-1)); g[x]
```

$$\frac{1}{4}\left(1-\frac{2}{e^{25}}\right)+0.1875\,(1+x)$$

```
g[x] // N
```

$0.25+0.1875\,(1.+x)$

```
Plot[{f[x], g[x]}, {x, -7, 7}, Epilog → {PointSize[0.03], Point[{-1, f[-1]}]}];
```



c

c1

```
Solve[f[x] == 0, {x}]
```

$\left\{\left\{x \to 4 - \sqrt{\text{Log}[2]}\right\}, \left\{x \to 4 + \sqrt{\text{Log}[2]}\right\}\right\}$

```
solv1 = NSolve[f[x] == 0, {x}]
```

{{x → 3.16745}, {x → 4.83255}}

```
{x1 = x /. solv1[[1]], x2 = x /. solv1[[2]]}
```

{3.16745, 4.83255}

c2

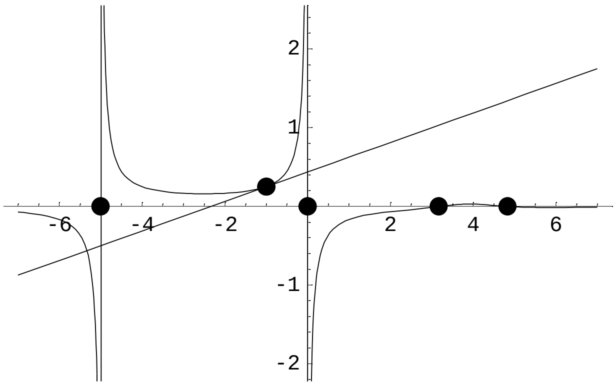
```
solv2 = Solve[1 / f[x] == 0, {x}]
```

{{x → -5}, {x → 0}}

```
{x3 = x /. solv2[[1]], x4 = x /. solv2[[2]]}
```

{-5, 0}

```
Plot[{f[x], g[x]}, {x, -7, 7}, Epilog → {PointSize[0.03], Point[{-1, f[-1]}],
    Point[{x1, f[x1]}], Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}]}];
```



d

```
f'[x]
```

$$-\frac{-1+2\,e^{-(-4+x)^2}}{x\,(5+x)^2}-\frac{-1+2\,e^{-(-4+x)^2}}{x^2\,(5+x)}-\frac{4\,e^{-(-4+x)^2}\,(-4+x)}{x\,(5+x)}$$

```
solv3 = FindRoot[f'[x] == 0, {x, -3}]
```

{x → -2.5}

```
x5 = x /. solv3
```

-2.5

```
solv4 = FindRoot[f'[x] == 0, {x, 4}]
```

{x → 3.90875}

```
x6 = x /. solv4
```

3.90875

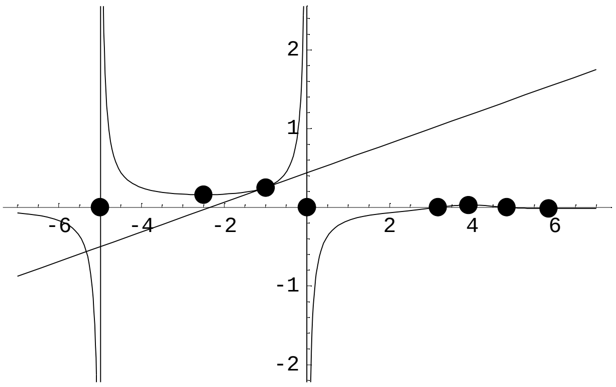
```
solv5 = FindRoot[f'[x] == 0, {x, 5}]
```

{x → 5.84439}

```
x7 = x /. solv5
```

5.84439

```
Plot[{f[x], g[x]}, {x, -7, 7},
  Epilog → {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],
    Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],
    Point[{x6, f[x6]}], Point[{x7, f[x7]}]}];
```



e

```
solv6 = FindRoot[Evaluate[f''[x] == 0], {x, 2}]
```

{x → 2.20542}

```
solv7 = FindRoot[Evaluate[f''[x] == 0], {x, 3}]
```

{x → 3.00981}

```
solv8 = FindRoot[Evaluate[f''[x] == 0], {x, 4}]
```

{x → 4.55421}

```
solv9 = FindRoot[Evaluate[f''[x] == 0], {x, 7}]
```

{x → 6.52488}

```
x8 = x /. solv6
```

2.20542

```
x9 = x /. solv7
```

3.00981

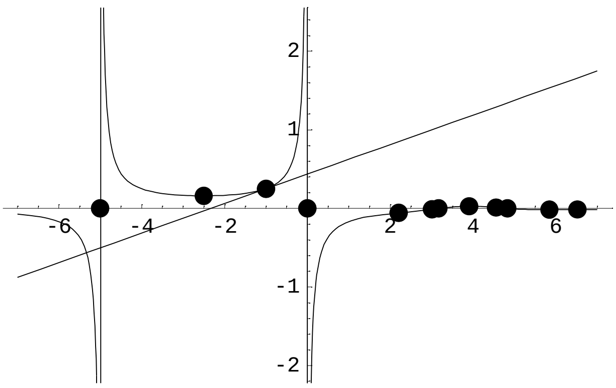
```
x10 = x /. solv8
```

4.55421

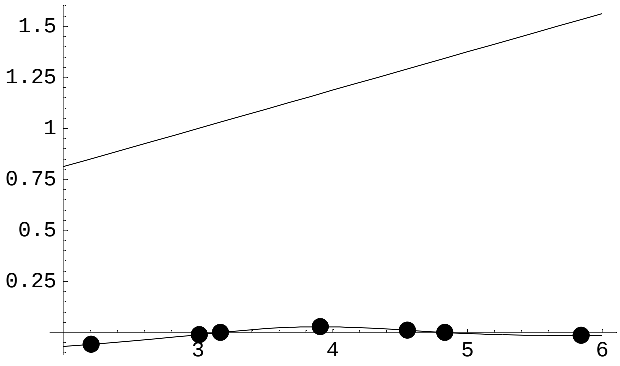
```
x11 = x /. solv9
```

6.52488

```
Plot[{f[x], g[x]}, {x, -7, 7},
  Epilog → {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],
    Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],
    Point[{x6, f[x6]}], Point[{x7, f[x7]}], Point[{x8, f[x8]}],
    Point[{x9, f[x9]}], Point[{x10, f[x10]}], Point[{x11, f[x11]}]}];
```



```
Plot[{f[x], g[x]}, {x, 2, 6},
  Epilog → {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],
    Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],
    Point[{x6, f[x6]}], Point[{x7, f[x7]}], Point[{x8, f[x8]}],
    Point[{x9, f[x9]}], Point[{x10, f[x10]}], Point[{x11, f[x11]}]}];
```



f

```
Limit[f[x], x -> Infinity]
```

0

```
Limit[f[x], x → -Infinity]
```

0

g

```
ser = Series[f[x], {x, 4, 3}]
```

$$\frac{1}{36}-\frac{13\,(x-4)}{1296}-\frac{2459\,(x-4)^2}{46656}+\frac{32435\,(x-4)^3}{1679616}+O[x-4]^4$$

```
N[%]
```

$$0.0277778-0.0100309\,(x-4.)-0.0527049\,(x-4.)^2+0.019311\,(x-4.)^3+O[x-4.]^4$$

```
serN = Series[f[x], {x, 4, 3}] // Normal
```

$$\frac{1}{36}-\frac{13\,(-4+x)}{1296}-\frac{2459\,(-4+x)^2}{46656}+\frac{32435\,(-4+x)^3}{1679616}$$

```
N[%]
```

$$0.0277778-0.0100309\,(-4.+x)-0.0527049\,(-4.+x)^2+0.019311\,(-4.+x)^3$$

```
ExpandAll[Evaluate[serN]]
```

$$-\frac{13196}{6561}+\frac{23419\,x}{17496}-\frac{9953\,x^2}{34992}+\frac{32435\,x^3}{1679616}$$

```
N[%]
```

$$-2.01128+1.33853\,x-0.284436\,x^2+0.019311\,x^3$$

h

```
Integrate[-13196/6561 + 23419 x/17496 - 9953 x^2/34992 + 32435 x^3/1679616, {x, 3, 5}]
```

$\frac{1429}{69984}$

```
int1 = Integrate[-2.0112787684804148` + 1.3385345221764975` x -
   0.284436442615455` x^2 + 0.019310961553116904` x^3, {x, 3, 5}]
```

0.020419

i

```
int2 = NIntegrate[f[x], {x, 3, 5}]
```

0.0276849

```
Abs[int1 - int2]
```

0.00726595

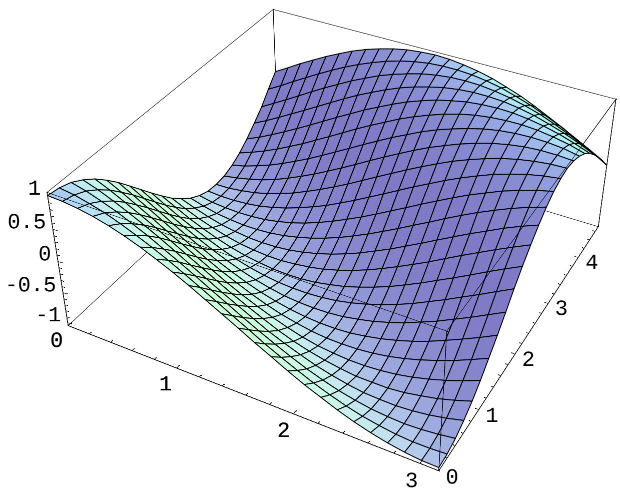
3

```
rem; Remove["Global`*"]
```

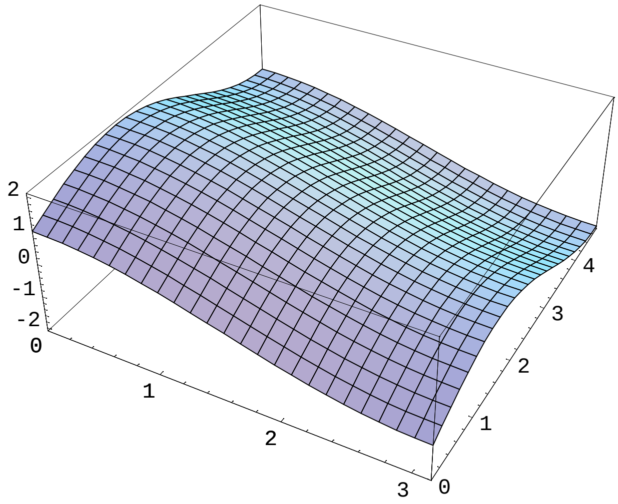
```
x1 = 0; x2 = Pi; y1 = 0; y2 = 3 Pi / 2;
f1[x_, y_] := Cos[x + y];
f2[x_, y_] := Cos[x] + Sin[y];
f3[x_, y_] := Cos[x] Sin[y];
```

a

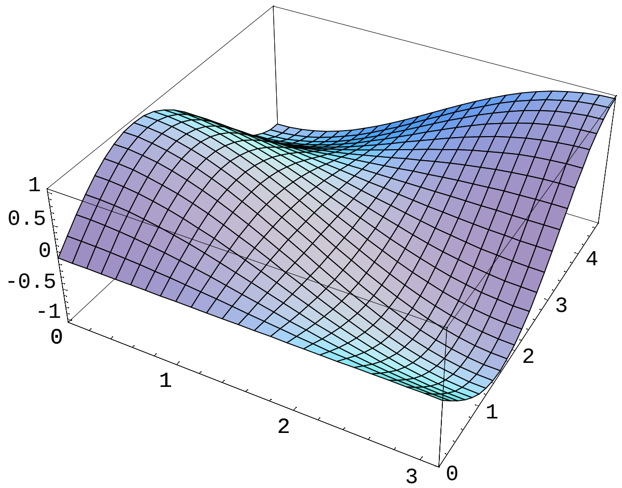
```
Plot3D[f1[x, y], {x, x1, x2}, {y, y1, y2}];
```



```
Plot3D[f2[x, y], {x, x1, x2}, {y, y1, y2}];
```




```
Plot3D[f3[x, y], {x, x1, x2}, {y, y1, y2}];
```



b

```
A1[y_] := Integrate[f1[x, y], {x, x1, x2}];  A1[y]
```

```
-2 Sin[y]
```

```
A2[y_] := Integrate[f2[x, y], {x, x1, x2}];  A2[y]
```

```
π Sin[y]
```

```
A3[y_] := Integrate[f3[x, y], {x, x1, x2}];  A3[y]
```

```
0
```

c

```
V1 = Integrate[A1[y], {y, y1, y2}]
```

```
-2
```

```
V2 = Integrate[A2[y], {y, y1, y2}]
```

π

```
V3 = Integrate[A3[y], {y, y1, y2}]
```

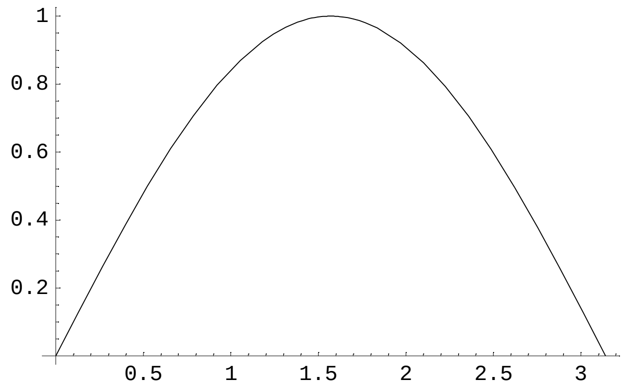
0

d

```
D[f2[x, y], x] == 0
```

-Sin[x] == 0

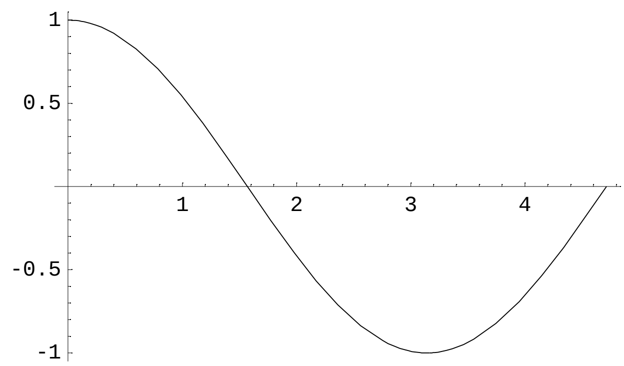
```
Plot[Sin[x], {x, x1, x2}];
```




```
D[f2[x, y], y] == 0
```

Cos[y] == 0

```
Plot[Cos[y], {y, y1, y2}];
```




```
Cos[Pi / 2]
```

0

```
Cos[3 Pi / 2]
```

0

```
f2[0, Pi / 2]
```

2

```
f2[0, 3 Pi / 2]
```

0

```
f2[Pi, Pi / 2]
```

0

```
f2[Pi, 3 Pi / 2]
```

−2

Minimalstelle bei {Pi, 3Pi/2}

4

```
rem; Remove["Global`*"]
```

```
{d / 2 == x4 Sqrt[2] / 2,  h^2 == 4^2 - (d / 2) ^2}
```

$$\left\{\frac{d}{2} == \frac{x4}{\sqrt{2}},\ h^2 == 16 - \frac{d^2}{4}\right\}$$

```
solv1 = Solve[d / 2 == x4 Sqrt[2] / 2, {d}] // Flatten
```

$$\left\{d \to \sqrt{2}\ x4\right\}$$

```
d[x4_] = d /. solv1;  d[k]
```

$$\sqrt{2}\ k$$

```
d[x4]
```

$$\sqrt{2}\ x4$$

$$h^2 == 16 - \frac{d[x4]^2}{4}$$

$$h^2 == 16 - \frac{x4^2}{2}$$

$$solv2 = Solve\left[h^2 == 16 - \frac{x4^2}{2}, \{h\}\right] // Flatten$$

$$\left\{h \to -\frac{\sqrt{32 - x4^2}}{\sqrt{2}},\ h \to \frac{\sqrt{32 - x4^2}}{\sqrt{2}}\right\}$$

```
h[x4_] = h /. solv2[[2]]
```

$$\frac{\sqrt{32 - x4^2}}{\sqrt{2}}$$

```
h[k]
```

$$\frac{\sqrt{32 - k^2}}{\sqrt{2}}$$

a

```
V4[x4_] := x4^2 h[x4] / 3;  V4[x4]
```

$$\frac{x4^2 \sqrt{32 - x4^2}}{3 \sqrt{2}}$$

```
V4'[x4]
```

$$-\frac{x4^3}{3 \sqrt{2} \sqrt{32 - x4^2}} + \frac{1}{3} \sqrt{2}\, x4 \sqrt{32 - x4^2}$$

```
solv3 = Solve[Evaluate[V4'[x4] == 0], {x4}] // Flatten
```

$$\left\{x4 \to 0,\ x4 \to -\frac{8}{\sqrt{3}},\ x4 \to \frac{8}{\sqrt{3}}\right\}$$

```
N[%]
```

$\{x4 \to 0.,\ x4 \to -4.6188,\ x4 \to 4.6188\}$

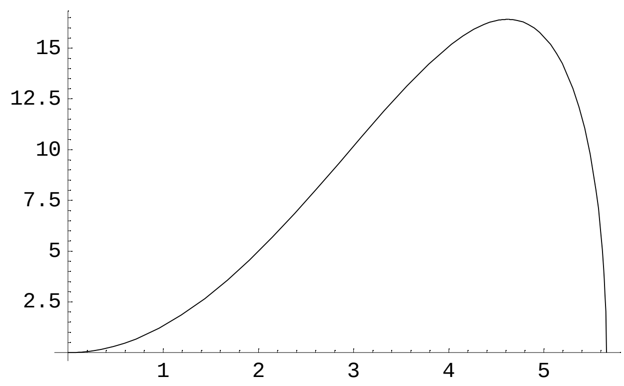
```
xMax = x4 /. solv3[[3]]
```

$$\frac{8}{\sqrt{3}}$$

```
V4[k]
```

$$\frac{k^2 \sqrt{32 - k^2}}{3 \sqrt{2}}$$

```
Plot[V4[x4], {x4, 0, 2 4 / Sqrt[2]}];
```



x0 4.6188

b

```
V4[x4]
```

$$\frac{x4^2 \sqrt{32 - x4^2}}{3 \sqrt{2}}$$

```
V4[x4] /. x4 -> xMax
```

$$\frac{256}{9\sqrt{3}}$$

```
N[%]
```

16.4224

c

```
rem; Remove["Global`*"]
```

```
{d^2 == x3^2 - (x3/2)^2, s^2 + (x3/2)^2 == 4^2}
```

$$\left\{d^2 == \frac{3\,x3^2}{4},\ s^2 + \frac{x3^2}{4} == 16\right\}$$

```
solv1 = Solve[d^2 == x3^2 - (x3/2)^2, {d}] // Flatten
```

$$\left\{d \to -\frac{\sqrt{3}\,x3}{2},\ d \to \frac{\sqrt{3}\,x3}{2}\right\}$$

```
d[x3_] = d /. solv1[[2]]; d[k]
```

$$\frac{\sqrt{3}\,k}{2}$$

```
d[x3]
```

$$\frac{\sqrt{3}\,x3}{2}$$

```
solv2 = Solve[s^2 + (x3/2)^2 == 4^2, {s}] // Flatten
```

$$\left\{s \to -\frac{1}{2}\sqrt{64 - x3^2},\ s \to \frac{\sqrt{64 - x3^2}}{2}\right\}$$

```
s[x3_] = s /. solv2[[2]]; s[k]
```

$$\frac{\sqrt{64 - k^2}}{2}$$

```
s[x3]
```

$$\frac{\sqrt{64 - x3^2}}{2}$$

```
h[x3_] := Sqrt[s[x3]^2 - (d[x3]/3)^2]; h[k]
```

$$\sqrt{-\frac{k^2}{12} + \frac{1}{4}\left(64 - k^2\right)}$$

```
h[x3]
```

$$\sqrt{-\frac{x3^2}{12} + \frac{1}{4}\left(64 - x3^2\right)}$$

```
V3[x3_] := x3 d[x3] / 2 h[x3] / 3; V3[k]
```

$$\frac{k^2 \sqrt{-\frac{k^2}{12} + \frac{1}{4}\,(64 - k^2)}}{4\sqrt{3}}$$

```
V3[x3]
```

$$\frac{x3^2 \sqrt{-\frac{x3^2}{12} + \frac{1}{4}\,(64 - x3^2)}}{4\sqrt{3}}$$

```
V3'[x3]
```

$$-\frac{x3^3}{12\sqrt{3}\sqrt{-\frac{x3^2}{12} + \frac{1}{4}\,(64 - x3^2)}} + \frac{x3\sqrt{-\frac{x3^2}{12} + \frac{1}{4}\,(64 - x3^2)}}{2\sqrt{3}}$$

```
solv3 = Solve[Evaluate[V3'[x3] == 0], {x3}] // Flatten
```

$\{x3 \to 0,\ x3 \to -4\sqrt{2},\ x3 \to 4\sqrt{2}\}$

```
N[%]
```

$\{x3 \to 0.,\ x3 \to -5.65685,\ x3 \to 5.65685\}$

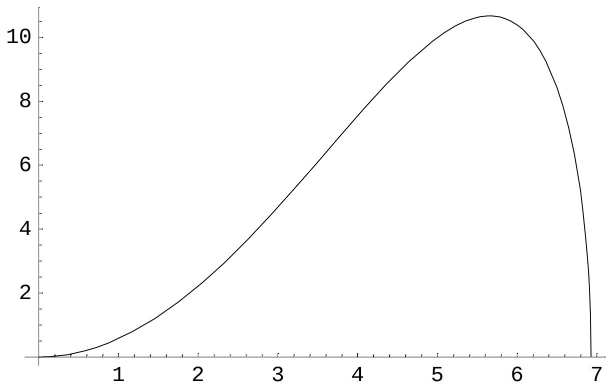
```
xMax = x3 /. solv3[[3]]
```

$4\sqrt{2}$

```
V3[k]
```

$$\frac{k^2 \sqrt{-\frac{k^2}{12} + \frac{1}{4}\,(64 - k^2)}}{4\sqrt{3}}$$

```
Plot[V3[x3], {x3, 0, 2 Sqrt[4^2 - 2^2]}];
```



d

```
V3[xMax]
```

$\frac{32}{3}$

```
N[%]
```

10.6667

5

```
rem; Remove["Global`*"]
```

```
f[x_] := 1 / 2 (x - 2) (x + 3) E^x;  f[y]
```

$$\frac{1}{2} e^{y} (-2+y) (3+y)$$

```
h[x_] := f[x] E^x; h[x]
```

$$\frac{1}{2} e^{2x} (-2+x) (3+x)$$

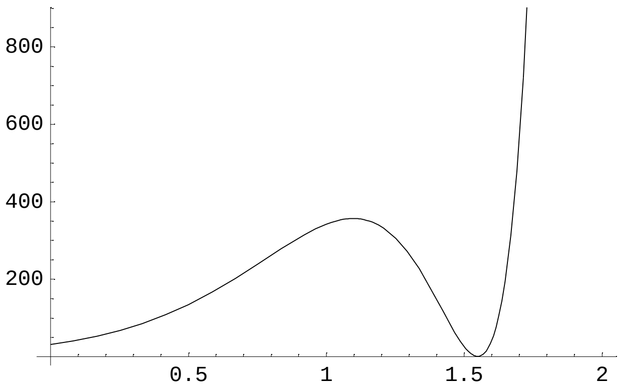
```
1 + Evaluate[h'[y]]^2 // Simplify
```

$$1+\frac{1}{4} e^{4y} (-11+4y+2y^2)^2$$

```
(1 + Evaluate[h'[u]]^2) /. u → 2
```

$$1+\frac{25 e^{8}}{4}$$

```
Plot[(1 + Evaluate[h'[u]]^2) /. u → t, {t, 0, 2}];
```

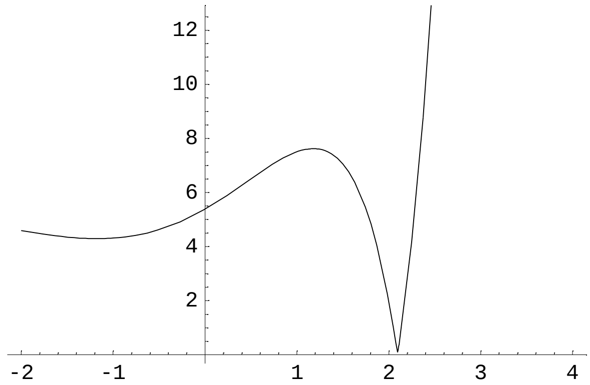


a

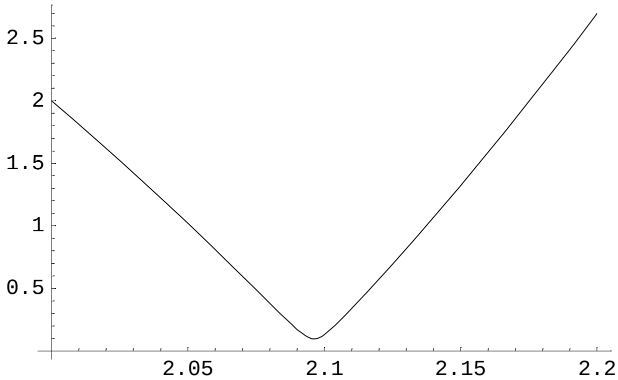
```
dist[x_] := Sqrt[(x - 2)^2 + (f[x] - 2)^2]; dist[x]
```

$$\sqrt{(-2+x)^2+\left(-2+\frac{1}{2} e^{x} (-2+x) (3+x)\right)^2}$$

```
Plot[dist[x], {x, -2, 4}];
```




```
Plot[dist[x], {x, 2, 2.2}];
```




```
dist'[x] // Simplify
```

$$\frac{4\,(-2+x)-4\,e^{x}\,(-5+3\,x+x^{2})+e^{2\,x}\,(30-23\,x-8\,x^{2}+4\,x^{3}+x^{4})}{4\,\sqrt{(-2+x)^{2}+\left(2-\frac{1}{2}\,e^{x}\,(-6+x+x^{2})\right)^{2}}}$$

```
solv = FindRoot[Evaluate[D[dist[k], k] /. k → x], {x, 2.1}]
```

{x → 2.09627}

```
x1 = x /. solv
```

2.09627

```
dist[x1]
```

0.0963626

b

```
Sqrt[1 + Evaluate[h'[x]]^2]
```

$$\sqrt{1+\left(\frac{1}{2}\,e^{2\,x}\,(-2+x)+\frac{1}{2}\,e^{2\,x}\,(3+x)+e^{2\,x}\,(-2+x)\,(3+x)\right)^{2}}$$

```
laenge1 = NIntegrate[Sqrt[1 + Evaluate[h'[x]]^2], {x, 0, 2}]
```

42.5487

c

```
s[t_] := {t, t^2, Sqrt[1 + t]}; s[t]
```

$\{t, t^2, \sqrt{1+t}\}$

```
norm[v_] := Sqrt[v.v]; D[s[t], t].D[s[t], t]
```

$1 + 4 t^2 + \frac{1}{4 (1 + t)}$

```
len = NIntegrate[Evaluate[D[s[t], t].D[s[t], t]] /. t → u, {u, 0, 2}]
```

12.9413

d

```
v1 = {-1, 2, -3}; a1 = {0, 1, -2};
v2 = {1, -2, 2}; a2 = {1, 0, 1};
```

```
V = Det[{v2 - v1, a1, a2}] / Norm[Cross[a1, a2]]
```

$\frac{5}{\sqrt{6}}$

```
N[%]
```

2.04124

6

```
rem; Remove["Global`*"]
```

```
a = {-1, 2, -3}; b = {0, 1, -2}; c = Cross[a, b]
```

{-1, -2, -1}

a

```
X = Transpose[{a, b, c}]; X // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & -2 \\ -3 & -2 & -1 \end{pmatrix}$$

```
Eλ = {{2, 0, 0}, {0, -1, 0}, {0, 0, 3}}; Eλ // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
M = X.Eλ.Inverse[X];  M // MatrixForm
```

$$\begin{pmatrix} \frac{13}{6} & \frac{1}{3} & \frac{1}{6} \\ -\frac{11}{3} & \frac{11}{3} & \frac{7}{3} \\ \frac{49}{6} & -\frac{5}{3} & -\frac{11}{6} \end{pmatrix}$$

```
M // N // MatrixForm
```

$$\begin{pmatrix} 2.16667 & 0.333333 & 0.166667 \\ -3.66667 & 3.66667 & 2.33333 \\ 8.16667 & -1.66667 & -1.83333 \end{pmatrix}$$

b

```
M.M - Transpose[M].Y.M + M == IdentityMatrix[3];
M.M - IdentityMatrix[3] + M == Transpose[M].Y.M;
Y == (Inverse[Transpose[M]].(M.M - IdentityMatrix[3] + M).Inverse[M] // MatrixForm)
```

$$Y == \begin{pmatrix} \frac{4589}{216} & -\frac{1081}{108} & -\frac{1}{216} \\ -\frac{649}{108} & \frac{215}{54} & \frac{53}{108} \\ -\frac{1729}{216} & \frac{485}{108} & \frac{53}{216} \end{pmatrix}$$

```
Inverse[Transpose[M]].(M.M - IdentityMatrix[3] + M).Inverse[M] // N // MatrixForm
```

$$\begin{pmatrix} 21.2454 & -10.0093 & -0.00462963 \\ -6.00926 & 3.98148 & 0.490741 \\ -8.00463 & 4.49074 & 0.24537 \end{pmatrix}$$

c

```
detQ = (1 / Det[M])^50
```

$$\frac{1}{808281277464764060643139600456536293376}$$

```
detQ // N
```

1.23719×10^{-39}

```
detQ 10^39
```

$$\frac{1818989403545856475830078125}{1470255078792914101801469952}$$

```
detQ 10^39  // N
```

1.23719

d

```
M5 = M.M.M.M.M;
M5 // MatrixForm
```

$$\begin{pmatrix} \frac{403}{6} & \frac{211}{3} & \frac{211}{6} \\ \frac{79}{3} & \frac{551}{3} & \frac{277}{3} \\ \frac{739}{6} & \frac{145}{3} & \frac{139}{6} \end{pmatrix}$$

```
M5 // N // MatrixForm
```

$$\begin{pmatrix} 67.1667 & 70.3333 & 35.1667 \\ 26.3333 & 183.667 & 92.3333 \\ 123.167 & 48.3333 & 23.1667 \end{pmatrix}$$

```
v = Transpose[{b + x b - x^2 b - x^3 b}] // MatrixForm
```

$$\begin{pmatrix} 0 \\ 1 + x - x^2 - x^3 \\ -2 - 2x + 2x^2 + 2x^3 \end{pmatrix}$$

```
M5.v // Simplify // MatrixForm
```

$$\begin{pmatrix} 0 \\ (-1 + x)(1 + x)^2 \\ -2(-1 + x)(1 + x)^2 \end{pmatrix}$$

```
M5.v // Expand // MatrixForm
```

$$\begin{pmatrix} 0 \\ -1 - x + x^2 + x^3 \\ 2 + 2x - 2x^2 - 2x^3 \end{pmatrix}$$

Kommetar: Eigenvektoren ändern nur die Länge!

e

```
Dφ[φ_] := {{Cos[φ], -Sin[φ], 0}, {Sin[φ], Cos[φ], 0}, {0, 0, 1}};
MatrixForm[Dφ[φ]]
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] & 0 \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

f

```
U = Inverse[M].Dφ[φ].M // Simplify; MatrixForm[U]
```

$$\begin{pmatrix} \frac{1}{216}(-49 + 265\,\text{Cos}[\varphi] + 348\,\text{Sin}[\varphi]) & \frac{1}{108}(5 - 5\,\text{Cos}[\varphi] - 189\,\text{Sin}[\varphi]) & \frac{1}{216}(11 - 11 \\ \frac{1}{108}(833 - 833\,\text{Cos}[\varphi] - 606\,\text{Sin}[\varphi]) & \frac{1}{54}(-85 + 139\,\text{Cos}[\varphi] + 423\,\text{Sin}[\varphi]) & \frac{1}{108}(-187 + 187 \\ \frac{1}{216}(-2695 + 2695\,\text{Cos}[\varphi] + 2652\,\text{Sin}[\varphi]) & \frac{1}{108}(275 - 275\,\text{Cos}[\varphi] - 1611\,\text{Sin}[\varphi]) & \frac{1}{216}(605 - 389 \end{pmatrix}$$

```
Eigenvalues[U]
```

{1, Cos[φ] − ⅈ Sin[φ], Cos[φ] + ⅈ Sin[φ]}

Reell für Sin[ ]=0, also für = n , n = ganze Zahl.

g

```
X = Transpose[{a, b, c}]; X // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & -2 \\ -3 & -2 & -1 \end{pmatrix}$$

```
EX = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; ; MatrixForm[EX]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
S = X.EX.Inverse[X];  S // MatrixForm
```

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

```
S // N // MatrixForm
```

$$\begin{pmatrix} 0.666667 & -0.666667 & -0.333333 \\ -0.666667 & -0.333333 & -0.666667 \\ -0.333333 & -0.666667 & 0.666667 \end{pmatrix}$$

```
e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};
OA = Transpose [{e1 + e2 + e3}];  OA // MatrixForm
```

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

```
S.OA // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{3} \\ -\frac{5}{3} \\ -\frac{1}{3} \end{pmatrix}$$

```
S.OA // N // MatrixForm
```

$$\begin{pmatrix} -0.333333 \\ -1.66667 \\ -0.333333 \end{pmatrix}$$

7

```
rem; Remove["Global`*"]
```

```
f[x_] := x Cos[(x + x^2) Pi] - x x^x;  f[x]
```

$-x^{1+x} + x \, Cos[\pi \, (x + x^2)]$

```
h[x_] := 2 x Sin[x^2 + 1] + x E^(2 x^2) + Cos[x] - x Sin[x]; h[x]
```

$e^{2 x^2} x + Cos[x] - x \, Sin[x] + 2 x \, Sin[1 + x^2]$

a

```
f'[x]
```

$Cos[\pi \, (x + x^2)] - x^{1+x} \left(\frac{1 + x}{x} + Log[x]\right) - \pi \, x \, (1 + 2 x) \, Sin[\pi \, (x + x^2)]$

```
f'[x] // Simplify
```

$Cos[\pi \, x \, (1 + x)] - x^x \, (1 + x + x \, Log[x]) - \pi \, x \, (1 + 2 x) \, Sin[\pi \, x \, (1 + x)]$

b

```
steig = (f'[x] /. x → 1) // Simplify
```

-1

```
ArcTan[steig]
```

$-\frac{\pi}{4}$

```
ArcTan[steig] // N
```

-0.785398

```
ArcTan[steig] / Degree // N
```

$-45.$

c

```
Integrate[h[x], x]
```

$\frac{e^{2 x^2}}{4} + x \, Cos[x] - Cos[1 + x^2]$