

Lösungen

1

```
Remove["Global`*"]
```

a

```
f[x_] := ((a4 x + a3) x + a2) x + a1) x + a0 // Expand;  
Print[f[x]];  
f'''[x]  
  
a0 + a1 x + a2 x2 + a3 x3 + a4 x4  
  
6 a3 + 24 a4 x
```

b

```
Solve[6 a3 + 24 a4 1 == 0, {a3}]  
  
{{a3 → -4 a4}}
```

```
Solve[6 a3 + 24 a4 1 == 0, {a4}]  
  
{{a4 → - $\frac{a3}{4}$ }}
```

c

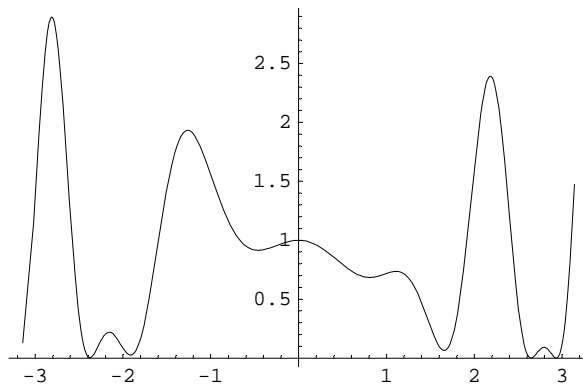
```
Limit[(1 + 1/n)^(n x), n → Infinity]  
  
ex  
  
Limit[(1 + 1/n)^(n x), n → Infinity] /. x → Pi  
  
eπ
```

2

```
Remove["Global`*"]  
  
f[x_] := (x/4 - Sin[x^2])^2 + E^(-x^2);  
h[x_] := x E^(-x^2) + 1/2 Sin[x] Cos[x];
```

a

```
Plot[f[x], {x, -Pi, Pi}];
```

**b**

```
f'[x]
```

$$-2 e^{-x^2} x + 2 \left(\frac{1}{4} - 2 x \cos[x^2] \right) \left(\frac{x}{4} - \sin[x^2] \right)$$

```
f'[x] // Simplify
```

$$-2 e^{-x^2} x - \frac{1}{8} (-1 + 8 x \cos[x^2]) (x - 4 \sin[x^2])$$

```
% // ExpandAll
```

$$\frac{x}{8} - 2 e^{-x^2} x - x^2 \cos[x^2] - \frac{\sin[x^2]}{2} + 4 x \cos[x^2] \sin[x^2]$$

c

```
f'[x] /. x -> 0
```

```
0
```

d

```
FindRoot[Evaluate[f'[x] == 0], {x, -0.8}]
```

```
{x -> -0.459065}
```

```
f'[x] /. x -> -0.45906450412214833`
```

```
0.
```

e

```
Limit[1 / (1 + x) h[x], {x -> -Infinity}]
{0}
```

f

```
Integrate[h[x], x] + C[1]
-  $\frac{e^{-x^2}}{2}$  + C[1] -  $\frac{\text{Cos}[x]^2}{4}$ 
```

g

```
Integrate[h[x], {x, 0, 1}]
-  $\frac{4 + e^{-5 + \text{Cos}[2]}}{8 e}$ 
Integrate[h[x], {x, 0, 1}] // N
0.493079
```

h

```
Series[f[x], {x, 0, 3}]
1 -  $\frac{15 x^2}{16}$  -  $\frac{x^3}{2}$  + O[x]^4
Series[f[x], {x, 0, 3}] // N
1. - 0.9375 (x + 0.)^2 - 0.5 (x + 0.)^3 + O[x + 0.]^4
```

i

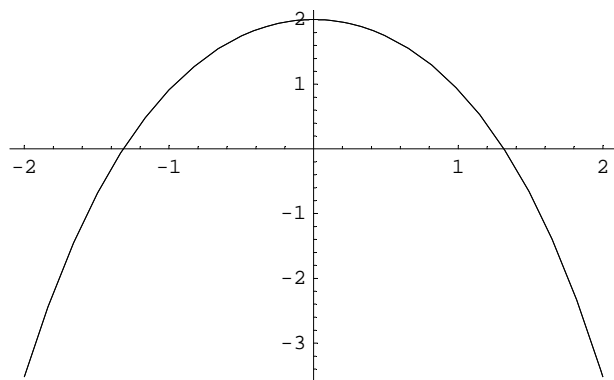
```
FindRoot[Evaluate[f''[x] == 0], {x, -0.8}]
{x -> -0.984813}
FindRoot[Evaluate[f''[x] == 0], {x, -0.1}]
{x -> -0.252331}
```

3

```
Remove["Global`*"]
```

a

```
f[x_] := 4 - 2 ((E^x + E^(-x)) / 2);
f1[x_] := 4 - 2 Cosh[x];
Plot[{f[x], f1[x]}, {x, -2, 2}];
```

**b**

```
Solve[f[x] == 0, {x}]
{{x -> Log[2 - Sqrt[3]]}, {x -> Log[2 + Sqrt[3]]}}
```

```
Solve[f[x] == 0, {x}] // N
{{x -> -1.31696}, {x -> 1.31696}}
```

```
x1 = -1.3169578969248164`;
```

```
x2 = 1.3169578969248164`;
```

c

```
gew = Integrate[f[x], {x, x1, x2}]
3.60746
```

```
S = 1 / gew Integrate[f[x]^2 / 2, {x, x1, x2}]
0.809609
```

d

```
rotV = Integrate[f[x]^2 Pi, {x, x1, x2}]
18.3509
```

e

```
kurvL = NIntegrate[Evaluate[Sqrt[1 + (f'[y])^2] /. y -> x], {x, x1, x2}]
```

```
5.03943
```

f

```
rotFl = NIntegrate[Evaluate[f[y] 2 Pi Sqrt[1 + (f'[y])^2] /. y -> x], {x, x1, x2}]
```

```
35.889
```

4

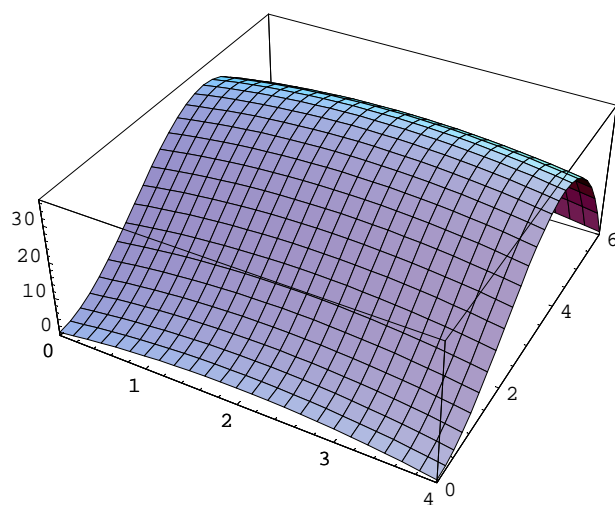
```
Remove["Global`*"]
```

```
f[x_, y_] := (4 - x) x + (6 - y) y^2;
```

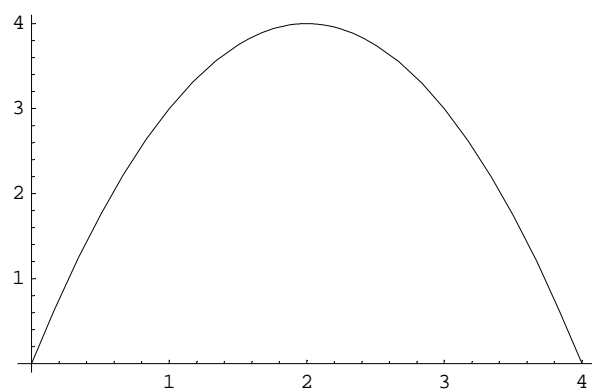
```
h[x_] := f[x, 6];
```

```
g[x_, a_] := a x;
```

```
Plot3D[f[x, y], {x, 0, 4}, {y, 0, 6}];
```



```
Plot[h[x], {x, 0, 4}];
```



a

```
Solve[h[x] == g[x, a], {x}]
```

```
{{x -> 0}, {x -> 4 - a}}
```

```
z[a_] := 4 - a
```

```
A1[a_] := Integrate[h[x] - g[x, a], {x, 0, z[a]}];
```

```
A1[a]
```

$$-\frac{1}{6}(-4 + a)^3$$

```
A2[a_] := z[a] g[z[a], a] / 2 + Integrate[h[x], {x, z[a], 4}];
```

```
A2[a]
```

$$\frac{1}{2}(4 - a)^2 a + 2 a^2 - \frac{a^3}{3}$$

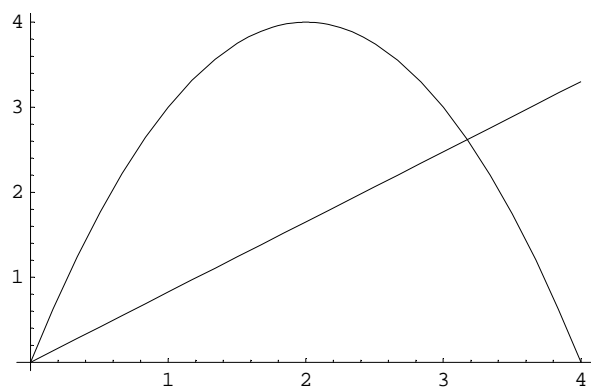
```
Solve[A1[a] == A2[a], {a}] // N // Chop
```

```
{{a -> 0.825198}, {a -> 5.5874 - 2.74946 i}, {a -> 5.5874 + 2.74946 i}}
```

```
a1 = 0.8251978960636008`
```

```
0.825198
```

```
Plot[{h[x], g[x, 0.8251978960636008`]}, {x, 0, 4}];
```

**b**

```
inh[a_] := z[a] g[z[a], a] / 2;
```

```
inh[a]
```

$$\frac{1}{2}(4 - a)^2 a$$

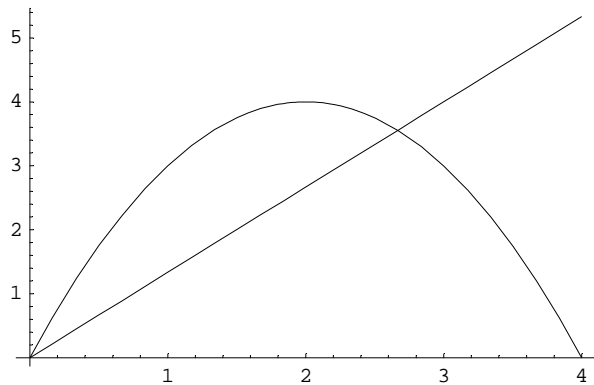
```
solv = Solve[Evaluate[inh'[u] == 0] /. u -> a, {a}] // Flatten
```

```
{a -> 4/3, a -> 4}
```

```
a1 = a /. solv[[1]]
```

$$\frac{4}{3}$$

```
Plot[{h[x], g[x, a1]}, {x, 0, 4}];
```



```
u[x_] := f[x, 0]; u[x]
```

$$(4 - x) x$$

```
u[x] == g[x, a1]
```

$$(4 - x) x == \frac{4 x}{3}$$

```
solv1 = Solve[u[x] == g[x, a1], {x}] // Flatten
```

$$\left\{x \rightarrow 0, x \rightarrow \frac{8}{3}\right\}$$

```
x1 = x /. solv1[[2]]
```

$$\frac{8}{3}$$

```
y = u[x1]
```

$$\frac{32}{9}$$

```
y == g[x1, a1]
```

```
True
```

```
flaeche = x1 y / 2
```

$$\frac{128}{27}$$

```
% // N
```

```
4.74074
```

C

```
D[f[x, y], x] == 0
```

$$4 - 2 x == 0$$

$$D[f[x, y], y] == 0$$

$$2(6 - y)y - y^2 == 0$$

$$\text{Solve}[\text{Evaluate}\{D[f[x, y], x] == 0, D[f[x, y], y] == 0\}, \{x, y\}]$$

$$\{\{x \rightarrow 2, y \rightarrow 0\}, \{x \rightarrow 2, y \rightarrow 4\}\}$$

Punkte (2, 4) sowie Eckpunkte.