Biegelinie

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1. Biegelinie eines einseitig horizontal eingespannten Trägers mit vertikaler Punktlast

a Definitionen, Vorbereitungen:

Balken- oder Trägerlänge xL = 5 m;
Balken, Rechtecksquerschnitt: Breite b = 5/100 m; Höhe h = 1/10 m;
Axiales Flächenträgheitsmoment Iy = b h^3/12 (Masse übernommen);
Esastizitätsmodul eE=210000 *1/(1/1000^2) N/ m^2;
Kraft F=10^3 N;

```
Remove["Global`*"]
```

a. Approximierte die Differentialgleichung, exakte Lösung der Approximation der Differentialgleichung

```
xL=5; b=5/100; h=1/10; Iy=b h^3/12; eE=210000 *1/(1/1000^2); F=10^3;  eE
```

```
210000000000
```

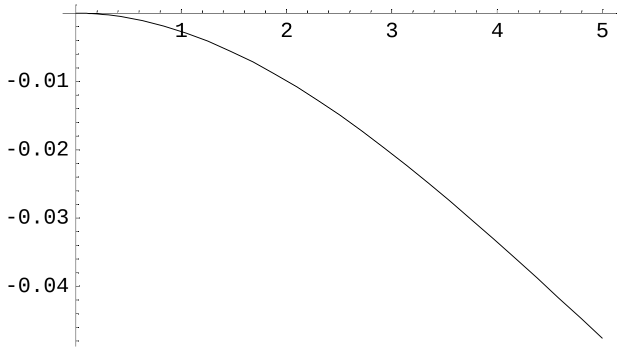
```
M[x_]:= F (xL-x);
y''[x] == -M[x]/(eE Iy)
```

$$y''[x] == \frac{1}{875}(-5+x)$$

```
solv = DSolve[{y''[x] == -M[x]/(eE Iy),y[0]==0,y'[0]==0},y,x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{-15\,x^2+x^3}{5250}\right]\right\}\right\}$$

```
k1=Plot[y[x]/.solv,{x,0,5}];
```



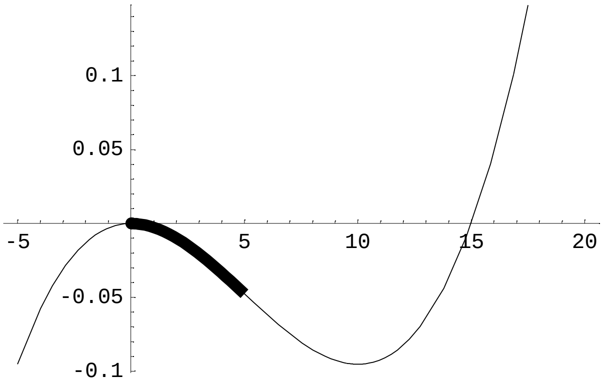
Endwert in Metern:

```
Endwert1 = (y[x]/.solv)/.x->5.
```

{-0.047619}

Globaler Funktionsverlauf

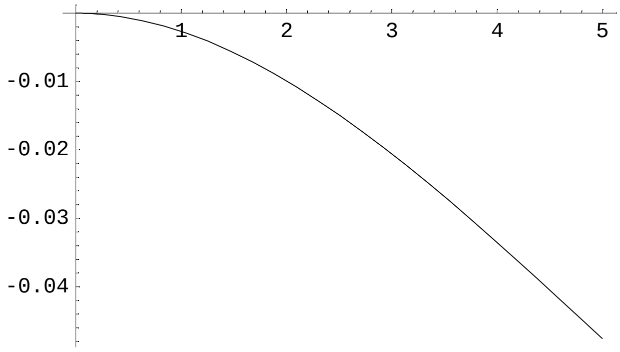
```
p1=Plot[y[x]/.solv,{x,-0,5},PlotStyle→{Thickness[.02]},DisplayFunction->Identity];

p2=Plot[y[x]/.solv,{x,-5,20},DisplayFunction->Identity];

Show[p1,p2,DisplayFunction→$DisplayFunction];
```



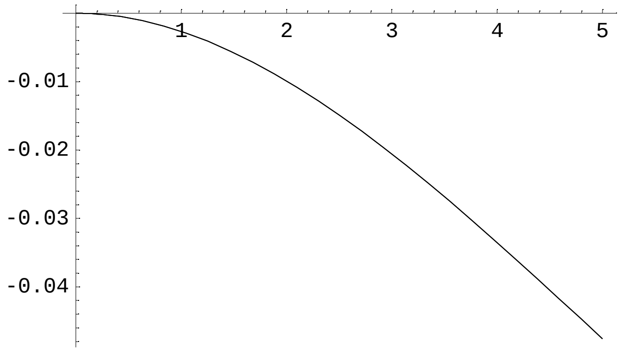
b. Exakte Differentialgleichung, numerische Lösung.
(Exakt zu schwierig zu lösen)

```
(* Remove["Global`*"] *)
Remove[x,y]
```

```
solution = NDSolve[{y^(2)[x] + M[x] / (eE Iy) (1 + (y'[x])^2)^(3/2) == 0,
   y[0] == 0, y'[0] == 0}, y, {x, 0, 5}];
k2 = Plot[y[x] /. solution, {x, 0, 5}];
```




```
Show[k1,k2];
```




```
Endwert2 = (y[x]/.solution)/.x->5.
```

{-0.0476224}

```
Abweichung1 = Endwert1 - Endwert2
```

$\{3.30876\times 10^{-6}\}$

```
Abweichung1InProzentVonEndwert1 = Abs[Abweichung1/ Endwert2 100]
```

{0.00694791}

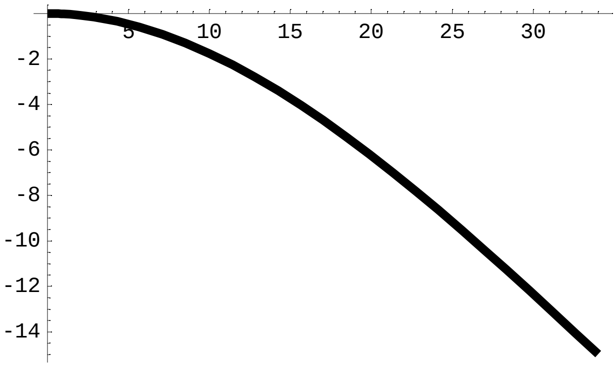
c. Vergleich der Graphen bei xL = 34

```
Remove[x,y]
```

```
xL = 34;
solv = DSolve[{y''[x] == -M[x]/(eE Iy),y[0]==0,y'[0]==0},y,x]
```

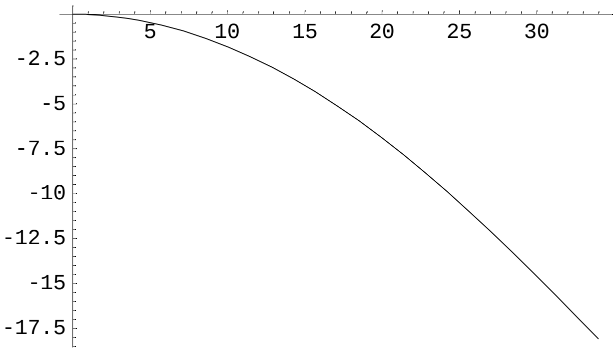
$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{-102\,x^2 + x^3}{5250}\right]\right\}\right\}$

```
k1=Plot[y[x]/.solv,{x,0,34},PlotStyle→{Thickness[.015]}];
```

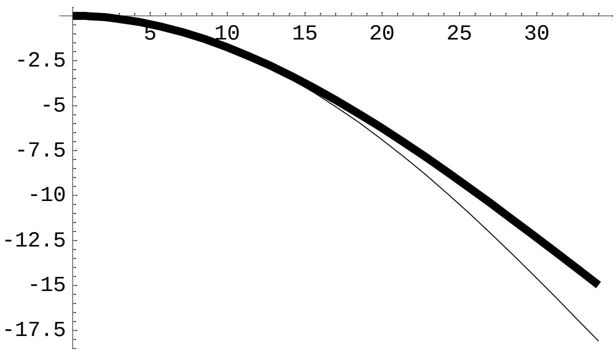



```
Remove[x,y]
```

```
solution = NDSolve[{y⁽²⁾[x] + M[x] / (eE Iy) (1 + (y'[x])^2)^(3/2) == 0,
   y[0] == 0, y′[0] == 0}, y, {x, 0, 34}];
k2 = Plot[y[x] /. solution, {x, 0, 34}];
```




```
Show[k1,k2];
```



FAZIT: Interessant ist die Grösse der ungünstigen Abweichung zwischen der exakten Lösung im Näherungsmodell und der numerischen Lösung im exakten Modell! Wer hätte das gedacht, da ja in der Praxis meist mit dem Näherungsmodell gearbeitet wird!

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2. Biegelinie eines einseitig horizontal eingespannten Trägers mit vertikaler konstanter Streckenlast

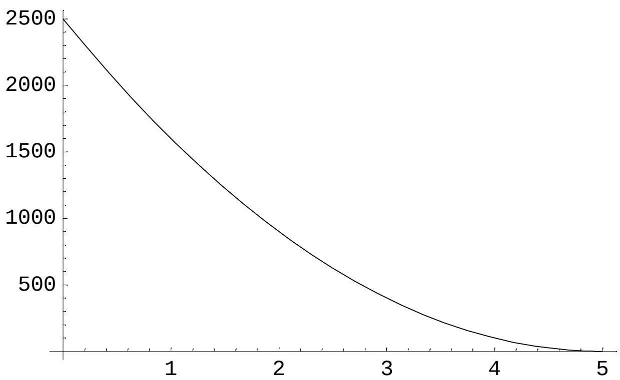
```
(* Remove["Global`*"] *)
Remove[x,y]
```

a. Approximierte Differentialgleichung, exakte Lösung

```
xL = 5;
M[x_]:= Evaluate[Integrate[F (xL-s)/xL,{s,u,xL}]/.u->x];
M[x]
```

$2500 - 1000\,x + 100\,x^2$

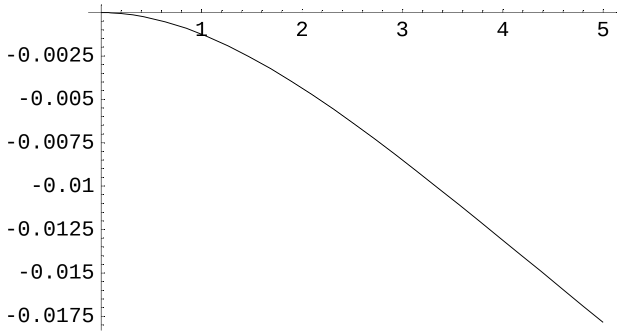
```
Plot[M[x],{x,0,5}];
```




```
solv = DSolve[{y''[x] == -M[x]/(eE Iy),y[0]==0,y'[0]==0},y,x]
```

$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{-150\,x^2 + 20\,x^3 - x^4}{105000}\right]\right\}\right\}$

```
k3=Plot[y[x]/.solv,{x,0,5}];
```



Endwert in Metern:

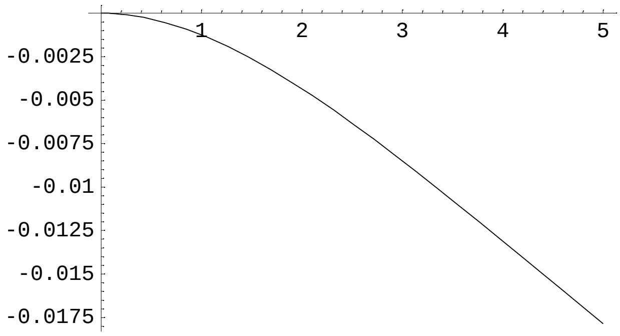
```
Endwert3 = (y[x]/.solv)/.x->5.
```

```
{-0.0178571}
```

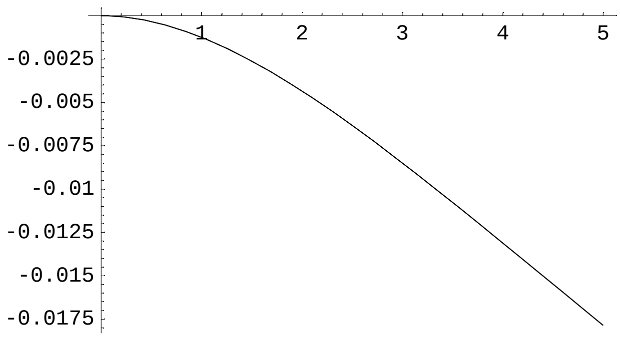
b. Exakte Differentialgleichung, numerische Lösung. (Exakt zu schwierig zu lösen)

```
(* Remove["Global`*"] *)
Remove[x,y]
```

```
solution = NDSolve[{y^(2)[x] + M[x] / (eE Iy) (1 + (y'[x])^2)^(3/2) == 0,
   y[0] == 0, y'[0] == 0}, y, {x, 0, 5}];
k4 = Plot[y[x] /. solution, {x, 0, 5}];
```




```
Show[k3,k4];
```




```
Endwert4 = (y[x]/.solution)/.x->5.
```

```
{-0.0178573}
```

```
Abweichung2 = Endwert3 - Endwert4
```

$\{1.57223 \times 10^{-7}\}$

```
Abweichung2InProzentVonEndwert3 = Abs[Abweichung2/ Endwert4 100]
```

```
{0.000880443}
```

c. Vergleich

```
Abweichung2-Abweichung1
```

$\{-3.15153\times 10^{-6}\}$

```
Abweichung1InProzentVonEndwert1-Abweichung2InProzentVonEndwert3
```

{0.00606746}

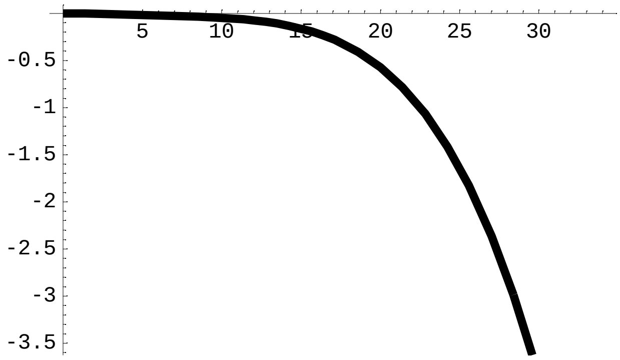
c. Vergleich der Graphen

```
Remove[x,y,solv]
```

```
xL = 34;
solv = DSolve[{y''[x] == -M[x]/(eE Iy),y[0]==0,y'[0]==0},y,x]
```

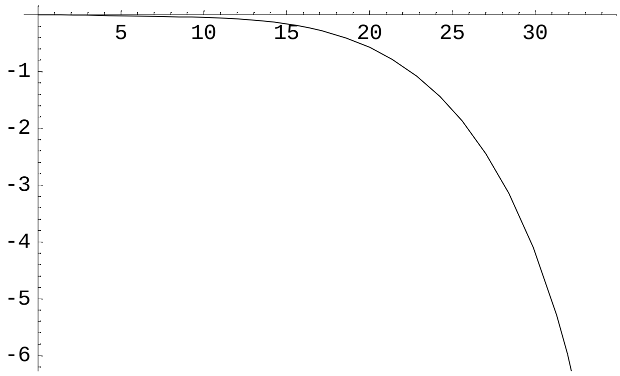
$\left\{\left\{y\to \text{Function}\left[\{x\},\ \frac{-150\,x^2+20\,x^3-x^4}{105000}\right]\right\}\right\}$

```
k3=Plot[y[x]/.solv,{x,0,34},PlotStyle→{Thickness[.015]}];
```

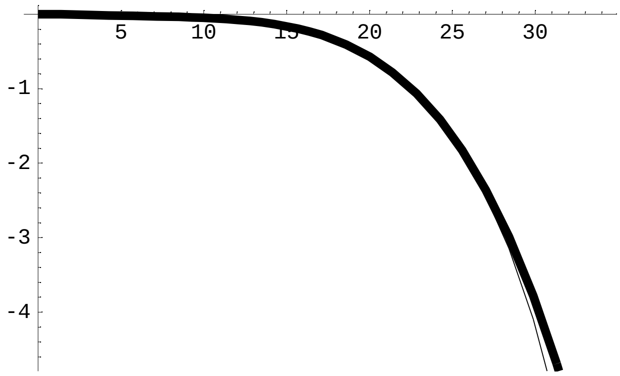



```
Remove[x,y,solution]
```

```
solution = NDSolve[{y⁽²⁾[x] + M[x] / (eE Iy) (1 + (y'[x])^2)^(3/2) == 0,
   y[0] == 0, y′[0] == 0}, y, {x, 0, 34}];
k4 = Plot[y[x] /. solution, {x, 0, 34}];
```



```
Show[k3,k4];
```



FAZIT: Interessant ist wieder die Grösse der ungünstigen Abweichung zwischen der exakten Lösung im Näherungsmodell und der numerischen Lösung im exakten Modell! Wer hätte das gedacht, da ja in der Praxis meist mit dem Näherungsmodell gearbeitet wird!

4 Lastkombinationen nach eigenen Modellen

Versuche eingene Kombinationen!