

Lösungen I / 8

1

Rechnung mit Vektorprodukt:

```
vO = {0,0,0}; vA = {4,3,0}; vB = {6,5,0}; vC = {2,8,0};
p =
{Cross[vA,vB],Cross[vB,vC],Cross[vC,vA],Cross[vA,vB]+Cross[vB,vC]+Cross[vC,vA]}/2
{{0, 0, 1}, {0, 0, 19}, {0, 0, -13}, {0, 0, 7}}
```

Umlausinn ändern: Negativ, $1+19-13=7$

Werte ohne Vorzeichen:

```
Drop[Map[Norm,p],{4}]
{1, 19, 13}
```

2

```
k1={1,5,6};k1=k1/Norm[k1];k2={2,5,7};k2=k2/Norm[k2];k1+k2
```

$$\left\{ \sqrt{\frac{2}{39} + \frac{1}{\sqrt{62}}}, \frac{5}{\sqrt{62}} + \frac{5}{\sqrt{78}}, 3\sqrt{\frac{2}{31}} + \frac{7}{\sqrt{78}} \right\}$$

```
%//N
```

```
{0.353456, 1.20114, 1.55459}
```

```
a={3,1,0};b={0,2,1};c={1,0,4};
```

```
solv=Solve[k1+k2 == λ a + μ b + ν c, {λ,μ,ν}] //Flatten
```

$$\left\{ \lambda \rightarrow \frac{39\sqrt{62} + 217\sqrt{78}}{60450}, \mu \rightarrow \frac{78\sqrt{62} + 59\sqrt{78}}{1950}, \nu \rightarrow \frac{858\sqrt{62} + 899\sqrt{78}}{60450} \right\}$$

```
%//N
```

```
{λ → 0.0367838, μ → 0.582178, ν → 0.243104}
```

```
λ a/.solv[[1]]
```

$$\left\{ \frac{39\sqrt{62} + 217\sqrt{78}}{20150}, \frac{39\sqrt{62} + 217\sqrt{78}}{60450}, 0 \right\}$$

```
%//N
```

```
{0.110351, 0.0367838, 0.}
```

Norm[%]

0.11632

Norm[%%]

$$\frac{39\sqrt{62} + 217\sqrt{78}}{6045\sqrt{10}}$$

3**a**

$$\mathbf{vA}=\{1, 5, 6\}; \mathbf{vB}=\{2, 5, 7\}; \mathbf{vC}=\{3, 1, 0\}; \mathbf{vD}=\{1, 0, 4\};$$

$$V=\text{Det}[\{\mathbf{vB}-\mathbf{vA}, \mathbf{vC}-\mathbf{vA}, \mathbf{vD}-\mathbf{vA}\}]/6$$

$$-\frac{16}{3}$$

N[%]

-5.33333

b

$$(\text{Norm}[\text{Cross}[\mathbf{vB}-\mathbf{vA}, \mathbf{vD}-\mathbf{vA}]] + \text{Norm}[\text{Cross}[\mathbf{vC}-\mathbf{vB}, \mathbf{vD}-\mathbf{vB}]] + \text{Norm}[\text{Cross}[\mathbf{vB}-\mathbf{vA}, \mathbf{vC}-\mathbf{vA}]] + \text{Norm}[\text{Cross}[\mathbf{vC}-\mathbf{vA}, \mathbf{vD}-\mathbf{vA}]])/2$$

$$\frac{1}{2} (17\sqrt{6} + \sqrt{710})$$

N[%]

34.1436

c

$$\text{winkel}=\text{ArcCos}[(\mathbf{vA}-\mathbf{vB}) \cdot (\mathbf{vC}-\mathbf{vB}) / (\text{Norm}[\mathbf{vA}-\mathbf{vB}] \text{Norm}[\mathbf{vC}-\mathbf{vB}])]$$

$$\text{ArcCos}\left[\sqrt{\frac{3}{11}}\right]$$

%/N

1.02133

%/Degree

58.5178

4

a

```

vA={4,5,-6};vB={6,2,-5};vC={2,16,1};
vX[λ_]:=vB+λ(vC-vB);
(vC-vB).(vA-vX[λ])==0

14 (3 - 14 λ) + 6 (-1 - 6 λ) - 4 (-2 + 4 λ) == 0

solv= Solve[(vC-vB).(vA-vX[λ])==0,{λ]//Flatten

{λ →  $\frac{11}{62}$ }

%/N

{λ → 0.177419}

vX0=vX[λ]/.solv

{ $\frac{164}{31}$ ,  $\frac{139}{31}$ ,  $-\frac{122}{31}$ }

%/N

{5.29032, 4.48387, -3.93548}

```

b ==> ausserhalb - x-Koordinaten vergleichen....

```

g[μ_]:=vA+ μ (vA-vX0);
g[μ][[3]]==0

-6 -  $\frac{64 μ}{31}$  == 0

solv=Solve[g[μ][[3]]==0,{μ]//Flatten

{μ →  $-\frac{93}{32}$ }

g[μ]/.solv

{ $\frac{31}{4}$ ,  $\frac{7}{2}$ , 0}

%/N

{7.75, 3.5, 0.}

```