

# Lösungen

## 1. Schwingungen

### Materialbereitstellung

```

Remove["Global`*"];
T = 5;
A[1] = 2;
A[2] = 3;
α[1] = π/8;
α[2] = 3 π/6;
ω = 2 π/T;
φ[t_,k_]:= ω * t + α[k];
f[t_,k_]:= A[k] Cos[φ[t,k]];
φ[A1_,φ1_,A2_,φ2_]:= ArcTan[(A1 Sin[φ1]+A2 Sin[φ2])/(A1 Cos[φ1]+A2 Cos[φ2])];
A[A1_,A2_,φ1_,φ2_]:= Sqrt[A1^2+A2^2+2 A1 A2 Cos[φ2-φ1]];
f[A_,φ_]:= A Cos[φ]

```

a

```
A[A[1],A[1],φ[t,1],φ[t,2]] //Simplify
```

$$\sqrt{8 + 8 \cos\left[\frac{3\pi}{8}\right]}$$

```
N[%]
```

3.32588

```
φ[A[1],A[1],φ[t,1],φ[t,2]] //Simplify
```

$$-\text{ArcTan}\left[\frac{\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[2]}{-80 \cos[2] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right]}\right]$$

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N[%]
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$$-1. \text{ArcTan}\left[\frac{72.7438 + 3.14159 (5. + 16. t) \cos[1.25664 t]}{33.2917 + 3.14159 (5. + 16. t) \sin[1.25664 t]}\right]$$

```
f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]] //Simplify
```

$$\frac{2 \sqrt{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}{\sqrt{1 + \frac{(\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[2])^2}{(-80 \cos[2] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right])^2}}}$$

**f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->0 //Simplify**

$$-32 \cos[2] \sqrt{\frac{2 (1 + \cos[\frac{3\pi}{8}])}{256 + \pi^2 + 32 \pi \sin[2]}}$$

**N[%]**

1.17157

**f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify**

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{120} \pi (15+8 \pi) \cos\left[\frac{\pi^2}{15}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{120} \pi (15+8 \pi) \sin\left[\frac{\pi^2}{15}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

**N[%]**

1.61738

**f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify**

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{40} \pi (5+4 \pi) \cos\left[\frac{\pi^2}{10}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{40} \pi (5+4 \pi) \sin\left[\frac{\pi^2}{10}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

**N[%]**

2.0275

**f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify**

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{120} \pi (15+16 \pi) \cos\left[\frac{2\pi^2}{15}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{120} \pi (15+16 \pi) \sin\left[\frac{2\pi^2}{15}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

**N[%]**

2.4661

**b**

**A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify**

$$\sqrt{13 + 12 \cos\left[\frac{3\pi}{8}\right]}$$

**N[%]**

4.19431

**φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify**

$$-\text{ArcTan}\left[\frac{\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[3]}{-80 \cos[3] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right]}\right]$$

**N[%]**

$$-1. \operatorname{ArcTan}\left[\frac{11.2896 + 3.14159 (5. + 16. t) \operatorname{Cos}[1.25664 t]}{79.1994 + 3.14159 (5. + 16. t) \operatorname{Sin}[1.25664 t]}\right]$$

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify**

$$\frac{\sqrt{13 + 12 \operatorname{Cos}\left[\frac{3\pi}{8}\right]}}{\sqrt{1 + \frac{(\pi (5+16 t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3])^2}{(-80 \operatorname{Cos}[3] + \pi (5+16 t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right])^2}}}}$$

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify**

$$-16 \operatorname{Cos}[3] \sqrt{\frac{13 + 12 \operatorname{Cos}\left[\frac{3\pi}{8}\right]}{256 + \pi^2 + 32 \pi \operatorname{Sin}[3]}}$$

**N[%]**

3.96999

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify**

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (15+8 \pi) \operatorname{Cos}\left[\frac{\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120} \pi (15+8 \pi) \operatorname{Sin}\left[\frac{\pi^2}{15}\right]\right)^2}}}}$$

**N[%]**

3.86067

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify**

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{40} \pi (5+4 \pi) \operatorname{Cos}\left[\frac{\pi^2}{10}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{40} \pi (5+4 \pi) \operatorname{Sin}\left[\frac{\pi^2}{10}\right]\right)^2}}}}$$

**N[%]**

3.97943

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify**

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (15+16 \pi) \operatorname{Cos}\left[\frac{2\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120} \pi (15+16 \pi) \operatorname{Sin}\left[\frac{2\pi^2}{15}\right]\right)^2}}}}$$

**N[%]**

4.11581

**c Sin[ ] = Cos[ - /2]**

$$\alpha[1] = \pi/8 - \text{Pi}/2;$$

**A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify**

$$\sqrt{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}$$

**N[%]**

1.38327

**φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify**

$$-\operatorname{ArcTan}\left[\frac{\pi(-15 + 16t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3]}{-80 \operatorname{Cos}[3] + \pi(-15 + 16t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right]}\right]$$

**N[%]**

$$-1. \operatorname{ArcTan}\left[\frac{11.2896 + 3.14159(-15. + 16. t) \operatorname{Cos}[1.25664 t]}{79.1994 + 3.14159(-15. + 16. t) \operatorname{Sin}[1.25664 t]}\right]$$

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify**

$$\frac{\sqrt{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}{\sqrt{1 + \frac{(\pi(-15+16t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3])^2}{(-80 \operatorname{Cos}[3] + \pi(-15+16t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right])^2}}}}$$

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify**

$$\sqrt{\frac{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}{1 + \frac{1}{256} (3\pi \operatorname{Sec}[3] - 16 \operatorname{Tan}[3])^2}}$$

**N[%]**

1.26028

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify**

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{120}\pi(-45+8\pi) \operatorname{Cos}\left[\frac{\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120}\pi(-45+8\pi) \operatorname{Sin}\left[\frac{\pi^2}{15}\right]\right)^2}}{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}}}$$

**N[%]**

1.37911

**f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify**

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{40}\pi(-15+4\pi) \operatorname{Cos}\left[\frac{\pi^2}{10}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{40}\pi(-15+4\pi) \operatorname{Sin}\left[\frac{\pi^2}{10}\right]\right)^2}}{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}}}$$

**N[%]**

1.37679

`f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify`

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (-45+16 \pi) \cos\left[\frac{2 \pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-45+16 \pi) \sin\left[\frac{2 \pi^2}{15}\right]\right)^2}}}$$

`N[%]`

1.36797

## d 1 neu definiert

`α[1] = -π/8;`

`A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify`

$$\sqrt{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}$$

`N[%]`

2.89962

`φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify`

$$-\text{ArcTan}\left[\frac{\pi (-5 + 16 t) \cos\left[\frac{2 \pi t}{5}\right] + 80 \sin[3]}{-80 \cos[3] + \pi (-5 + 16 t) \sin\left[\frac{2 \pi t}{5}\right]}\right]$$

`N[%]`

$$-1. \text{ArcTan}\left[\frac{11.2896 + 3.14159 (-5. + 16. t) \cos[1.25664 t]}{79.1994 + 3.14159 (-5. + 16. t) \sin[1.25664 t]}\right]$$

`f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify`

$$\frac{\sqrt{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}}{\sqrt{1 + \frac{(\pi (-5+16 t) \cos\left[\frac{2 \pi t}{5}\right] + 80 \sin[3])^2}{(-80 \cos[3] + \pi (-5+16 t) \sin\left[\frac{2 \pi t}{5}\right])^2}}}}$$

`f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify`

$$-16 \cos[3] \sqrt{\frac{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}{256 + \pi^2 - 32 \pi \sin[3]}}$$

`N[%]`

2.89512

`f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify`

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (-15+8 \pi) \cos\left[\frac{\pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-15+8 \pi) \sin\left[\frac{\pi^2}{15}\right]\right)^2}}}$$

N[%]

2.82601

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t-&gt;Pi/4 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{40} \pi (-5+4\pi) \cos\left[\frac{\pi^2}{10}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{40} \pi (-5+4\pi) \sin\left[\frac{\pi^2}{10}\right]\right)^2}}{13+12 \cos\left[\frac{5\pi}{8}\right]}}$$

N[%]

2.81547

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t-&gt;Pi/3 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{120} \pi (-15+16\pi) \cos\left[\frac{2\pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-15+16\pi) \sin\left[\frac{2\pi^2}{15}\right]\right)^2}}{13+12 \cos\left[\frac{5\pi}{8}\right]}}$$

N[%]

2.85414

## 2. Komplexe Zahlen

### Materialbereitstellung

Remove["Global`\*"];

z1 = 2+I;

z2 = 3-2I;

z3 = -4+3I;

z4 = -6-5I;

a

z = z1 + z2

5 - i

b

z = z2 - z3

7 - 5 i

**c**

$$z = z_1 + 2 z_2 - 3 z_3 + 4 z_4$$
$$-4 - 32 i$$

**d**

$$z = z_1 - 2 z_2 + 3 z_3 - 4 z_4$$
$$8 + 34 i$$

**e**

```
Remove[z];  
Solve[3 z1 - 2 z2 + 4 z3 + 6 z == 5 z2 - 3 z, {z}]
```

$$\left\{ \left\{ z \rightarrow \frac{31}{9} - \frac{29 i}{9} \right\} \right\}$$

```
N[%]
```

$$\left\{ \left\{ z \rightarrow 3.44444 - 3.22222 i \right\} \right\}$$

**f**

```
Solve[2 (z1 - 3 z2) + 4 (z3 - z ) - 5 (z4 + 2 z - z2) == 8 z1 - 8 z, {z}]
```

$$\left\{ \left\{ z \rightarrow -\frac{1}{6} + \frac{11 i}{2} \right\} \right\}$$

```
N[%]
```

$$\left\{ \left\{ z \rightarrow -0.166667 + 5.5 i \right\} \right\}$$