

Lösungen

Aufgabe 1

```
Remove["Global`*"]
```

a

```
z1 = 2 + 4 I; z2 = -4 - 6 I;
```

```
1 / z1
```

$$\frac{1}{10} - \frac{i}{5}$$

```
N[%]
```

```
0.1 - 0.2 i
```

```
1 / z2
```

$$-\frac{1}{13} + \frac{3i}{26}$$

```
N[%]
```

```
-0.0769231 + 0.115385 i
```

```
Conjugate[z1]
```

```
2 - 4 i
```

```
Re[z1]
```

```
2
```

b

```
Solve[z1 z + 1 / z2 == Conjugate[z1] / Re[z1], {z}]
```

$$\left\{ \left\{ z \rightarrow -\frac{41}{130} - \frac{111i}{260} \right\} \right\}$$

```
Flatten[Solve[z1 z + 1 / z2 == Conjugate[z1] / Re[z1], {z}], 2]
```

$$\left\{ z \rightarrow -\frac{41}{130} - \frac{111i}{260} \right\}$$

```
zRes = z /. Flatten[Solve[z1 z + 1 / z2 == Conjugate[z1] / Re[z1], {z}], 2]; zRes
```

$$-\frac{41}{130} - \frac{111i}{260}$$

```
N[%]
```

```
-0.315385 - 0.426923 i
```

a, b

```
p[z_] := Point[{Re[z], Im[z]}]
```

```
Show[Graphics[
```

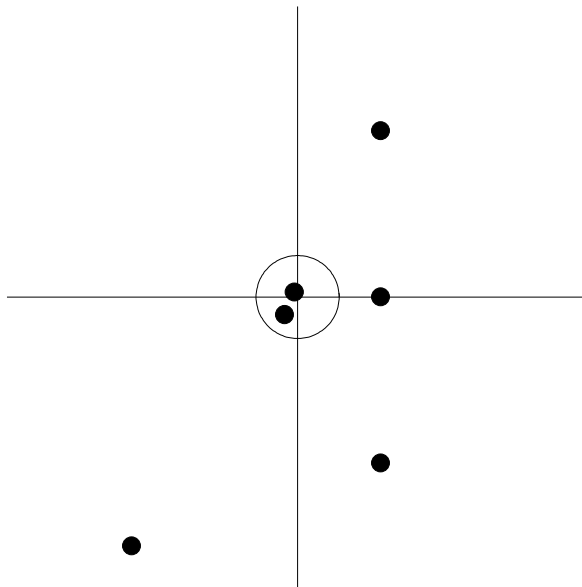
```
{PointSize[0.03],
```

```
p[z1], p[z2], p[z2^(-1)], p[Conjugate[z1]], p[Re[z1]], p[zRes],
```

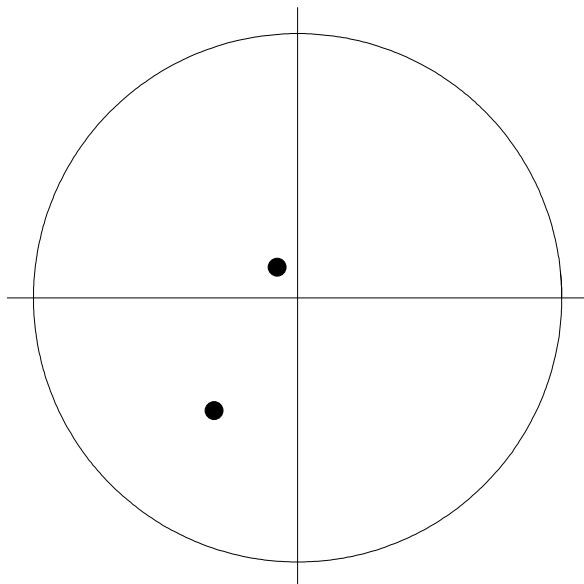
```
Line[{{0, -7}, {0, 7}}, Line[{{-7, 0}, {7, 0}}],
```

```
Circle[{0, 0}, 1]
```

```
}], AspectRatio -> 0.1];
```



```
Show[Graphics[
  {PointSize[0.03],
   p[z2^(-1)], p[zRes],
   Line[{{0, -1.1}, {0, 1.1}}, Line[{{-1.1, 0}, {1.1, 0}}],
   Circle[{0, 0}, 1]
  }], AspectRatio -> 0.1];
```



c

```
Abs[z1] / Abs[z2]
```

$$\sqrt{\frac{5}{13}}$$

```
Abs[z1 / z2]
```

$$\sqrt{\frac{5}{13}}$$

```
N[%]
```

```
0.620174
```

d, e

```
Solve[w^6 == z2, {w}]
```

```
{w -> -(-4 - 6 i)^{1/6}, {w -> (-4 - 6 i)^{1/6}, {w -> -(-4 - 6 i)^{1/6} (-1)^{1/3},
 {w -> (-4 - 6 i)^{1/6} (-1)^{1/3}, {w -> -(-4 - 6 i)^{1/6} (-1)^{2/3}, {w -> (-4 - 6 i)^{1/6} (-1)^{2/3}}
```

```
solv1 = Solve[w^6 == z2, {w}] // N // Flatten
```

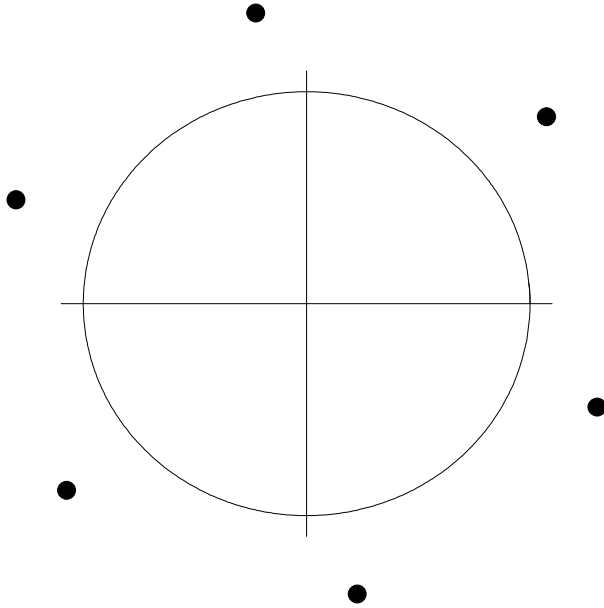
```
{w -> -1.30095 + 0.489384 i, w -> 1.30095 - 0.489384 i, w -> -1.07429 - 0.881964 i,
 w -> 1.07429 + 0.881964 i, w -> 0.226656 - 1.37135 i, w -> -0.226656 + 1.37135 i}
```

```
w[n_] := (w /. solv1[[n]])
```

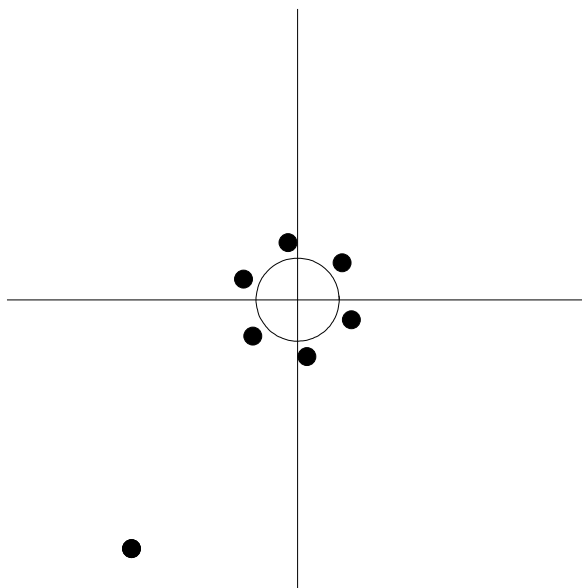
```
w[4]
```

```
1.07429 + 0.881964 i
```

```
Show[Graphics[
  Join[{PointSize[0.03],
    Line[{{0, -1.1}, {0, 1.1}}, Line[{{-1.1, 0}, {1.1, 0}}],
    Circle[{0, 0}, 1]
  }, Table[p[w[n]], {n, 6}]], AspectRatio -> 01];
```



```
Show[Graphics[
  Join[{PointSize[0.03], p[z2],
    Line[{{0, -7}, {0, 7}}, Line[{{-7, 0}, {7, 0}}],
    Circle[{0, 0}, 1], p[z2]
  }, Table[p[w[n]], {n, 6}]], AspectRatio -> 1];
```



Aufgabe 2

```
Remove["Global`*"]
```

a, b

$$zA = -2 + i; \quad zB = 4 - i; \quad zC = 2 + 6i;$$

$$\varphi = 5 / 11 \text{ Pi}$$

$$\frac{5 \pi}{11}$$

$$zA1 = zA E^{(I \varphi)}$$

$$(-2 + i) e^{\frac{5 i \pi}{11}}$$

N[%]

$$-1.27445 - 1.83733 i$$

$$zB1 = zB E^{(I \varphi)}$$

$$(4 - i) e^{\frac{5 i \pi}{11}}$$

N[%]

$$1.55908 + 3.81697 i$$

$$zC1 = zC E^{(I \varphi)}$$

$$(2 + 6i) e^{\frac{5 i \pi}{11}}$$

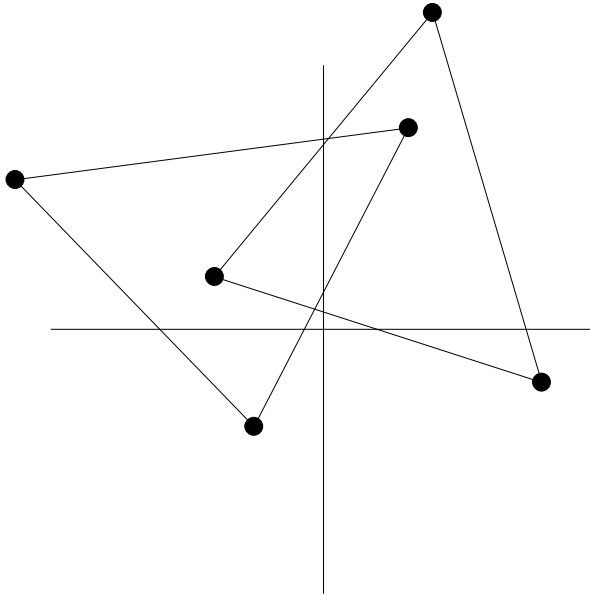
N[%]

$$-5.6543 + 2.83353 i$$

```

p[z_] := Point[{Re[z], Im[z]}];
q[z_] := {Re[z], Im[z]};
Show[Graphics[
  {PointSize[0.03],
   p[zA], p[zA1], p[zB], p[zB1], p[zC], p[zC1],
   Line[{q[zA], q[zB]}], Line[{q[zA], q[zC]}], Line[{q[zC], q[zB]}],
   Line[{q[zA1], q[zB1]}], Line[{q[zA1], q[zC1]}], Line[{q[zC1], q[zB1]}],
   Line[{0, -5}, {0, 5}], Line[{-5, 0}, {5, 0}]}], AspectRatio -> 0.1];

```

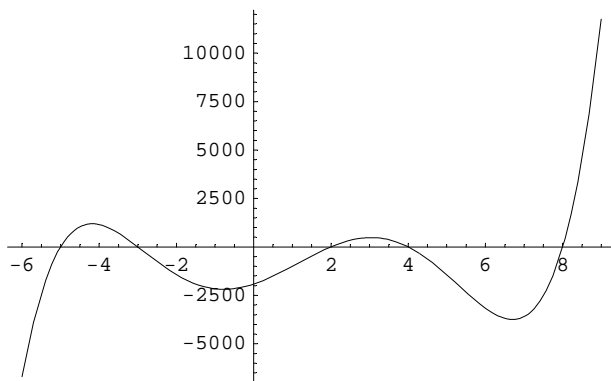


Aufgabe 3

```
f[x_] := 2 (x - 2) (x + 3) (x - 4) (x + 5) (x - 8)
```

a

```
Plot[f[x], {x, -6, 9}];
```



b

```
Solve[f[x] == 0, {x}]
{{x -> -5}, {x -> -3}, {x -> 2}, {x -> 4}, {x -> 8}}
```

c

5 NS

```
Remove["Global`*"]
```

Aufgabe 4

```
Cos[6 φ] // TrigExpand
Cos[φ]^6 - 15 Cos[φ]^4 Sin[φ]^2 + 15 Cos[φ]^2 Sin[φ]^4 - Sin[φ]^6
```

Aufgabe 5

```
Remove["Global`*"]
<<Calculus`FourierTransform`
FourierTrigSeries[Cos[φ 2 Pi]^6, φ, 10]
5/16 + 15/32 Cos[4 π φ] + 3/16 Cos[8 π φ] + 1/32 Cos[12 π φ]
FourierTrigSeries[Cos[φ 2 Pi]^6, φ, 10] // Simplify
Cos[2 π φ]^6
```

Aufgabe 6

```
Apart[(3 x^2 - 2 x + 1) / (x^2 + 2 x + 1)]
3 + 6/(1 + x)^2 - 8/(1 + x)
```

Aufgabe 7

```
Apart[1 / ((x - 2)^2 (x^2 + 2 x + 3))]
1/(11 (-2 + x)^2) - 6/(121 (-2 + x)) + (13 + 6 x)/(121 (3 + 2 x + x^2))
```

Aufgabe 8

```
Apart[1 / ((x - 2) (x + 3))]
```

$$\frac{1}{5(-2+x)} - \frac{1}{5(3+x)}$$

Aufgabe 9

```
t = Solve[w^7 == 2 + 5 I, {w}] // Flatten
```

```
{w -> (2 + 5 i)^{1/7}, w -> -(-1)^{1/7} (2 + 5 i)^{1/7}, w -> (-1)^{2/7} (2 + 5 i)^{1/7}, w -> -(-1)^{3/7} (2 + 5 i)^{1/7},
w -> (-1)^{4/7} (2 + 5 i)^{1/7}, w -> -(-1)^{5/7} (2 + 5 i)^{1/7}, w -> (-1)^{6/7} (2 + 5 i)^{1/7}}
```

```
tab = Table[w[k] = w /. t[[k]], {k, 1, 7}]
```

```
{(2 + 5 i)^{1/7}, -(-1)^{1/7} (2 + 5 i)^{1/7}, (-1)^{2/7} (2 + 5 i)^{1/7}, -(-1)^{3/7} (2 + 5 i)^{1/7},
(-1)^{4/7} (2 + 5 i)^{1/7}, -(-1)^{5/7} (2 + 5 i)^{1/7}, (-1)^{6/7} (2 + 5 i)^{1/7}}
```

```
Sum[tab[[k]], {k, 1, 7}] // N // Chop
```

```
0
```

```
Sum[tab[[k]], {k, 1, 7}] // ComplexExpand // FullSimplify
```

```
0
```

```
Remove["Global`*"]
```

Aufgabe 10

```
Apart[x^3 / ((x - 1) (x + 2) (x + 4))]
```

$$1 + \frac{1}{15(-1+x)} + \frac{4}{3(2+x)} - \frac{32}{5(4+x)}$$

```
Apart[x^2 / ((x - 1)^2 (x + 2))]
```

$$\frac{1}{3(-1+x)^2} + \frac{5}{9(-1+x)} + \frac{4}{9(2+x)}$$

```
Apart[x^2 / ((x - 1)^4)]
```

$$\frac{1}{(-1+x)^4} + \frac{2}{(-1+x)^3} + \frac{1}{(-1+x)^2}$$

```
Apart[1 / (x (x - 1))]
```

$$\frac{1}{-1+x} - \frac{1}{x}$$