Lösungen Teil 1

Teil 2 weiter unten!

1

In Matrixdarstellung:

```
OA={-3,5,2}; OB={1,-3,4}; AB=OB-OA;
Print[MatrixForm[Transpose[{OA}]],"  ", MatrixForm[Transpose[{OB}]],"  ",
MatrixForm[Transpose[{AB}]]];
```

$\begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 4 \\ -8 \\ 2 \end{pmatrix}$

2

```
Remove["Global`*"];

OA={2,1,3}; OB={3,0,2}; OC={5,-1,-3}; OD={3,1,-1};
AB=OB-OA; AC=OC-OA; CD=OD-OC; AA=OA-OA;
Solve[AB==x AC,{x}]
```

{}

(* AB nict parallel AC *)

```
Solve[AB==x CD,{x}]
```

$\{\{x \to -\frac{1}{2}\}\}$

(* AB parallel CD *)

```
Solve[x AB==AA,{x}]
```

$\{\{x \to 0\}\}$

(* AB parallel AA *)

```
Solve[x AB==AC,{x}]
```

{}

(* AB parallel AC *)

3

```
OA={-4,5,1}; OB={2,6,3}; OC={6,-2,-1}; OD={12,-1,1}; OE={6,1,2};
AB=OB-OA; AC=OC-OA; CD=OD-OC; DB=OB-OD;

Solve[x AB==CD,{x}]
```

{{x → 1}}

```
Solve[x AB==OE,{x}]
```

{{x → 1}}

(* AB, CD und OE sind gleich und somit parallel *)

```
Solve[x AB==AC,{x}]
```

{}

(* AB, CD und AC sind nicht parallel *)

```
Solve[x AC==DB,{x}]
```

{{x → -1}}

(* AC und DB sind antiparallel *)

4

In Matrixdarstellung:

```
a={-1,2,5}; b={0,-2,3};
Print[MatrixForm[Transpose[{3 a}]], "   ", MatrixForm[Transpose[{a-2b}]]];
```

$\begin{pmatrix} -3 \\ 6 \\ 15 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$

5

Nicht in Matrixdarstellung:

```
e = a 1/Norm[a]
```

$\left\{-\frac{1}{\sqrt{30}}, \sqrt{\frac{2}{15}}, \sqrt{\frac{5}{6}}\right\}$

```
N[%]
```

```
{-0.182574, 0.365148, 0.912871}
```

6

```
OA={2, -3, 5}; OB={7, -7, 6}; OC={27, -23, 10};
```

```
Solve[x (OB-OA)==OC-OA,{x}]
```

```
{{x → 5}}
```

(* Die Punkte liegen auf einer Geraden. *)

7

Nicht in Matrixdarstellung:

```
OC={x, y, 4};  OA={3, -4, 2}; OB={7, 2, 1};
solv = Solve[t (OB-OA)[[3]] == (OC-OA)[[3]],{t}] //Flatten
```

```
{t → -2}
```

```
OC=t (OB-OA)+OA /.solv
```

```
{-5, -16, 4}
```

8

Nicht in Matrixdarstellung:

```
OA={1,1,1};  OC={-5,3,2};  OD={-2,-4,-2};
DA=OA-OD;
OB=OC+DA
```

```
{-2, 8, 5}
```

9

Nicht in Matrixdarstellung:

```
OA={3,-2,5};  OB={7,5,10};  OE={5,4,6};
AE=OE-OA;  BE=OE-OB;
OC=OA+2 AE
```

{7, 10, 7}

```
OD=OB+2 BE
```

{3, 3, 2}

10

Nicht in Matrixdarstellung:

```
OA={1, 3, -2}; OB={5, -1, 4};
OM = 1/2 (OA+OB)
```

{3, 1, 1}

11

Nicht in Matrixdarstellung:

```
OA={-4, 2, -1}; OB={7, -1, 5}; OC={0, 2, 2};
OS = 1/3 (OA+OB+OC)
```

{1, 1, 2}

12

In Matrixdarstellung:

```
OA={0, -2, 1}; OB={-1, 5, 0}; OD={1, -1, 4};
OM= 1/2 (OC+OD);  ON= 1/2 (OM+OB);
x = ON;
Print[MatrixForm[Transpose[{x}]], " = ", MatrixForm[Transpose[{x//N}]]];
```

$$\begin{pmatrix} -\frac{1}{4} \\ \frac{11}{4} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -0.25 \\ 2.75 \\ 1.5 \end{pmatrix}$$

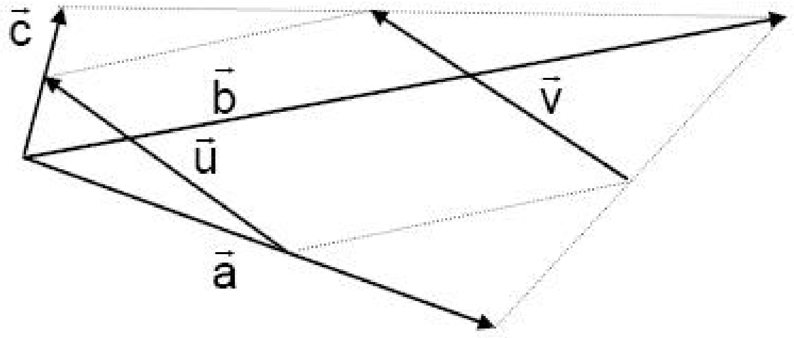
13

"vec" bedeutet "Vektor".
vec(OS) = 1/3 (vec(OA) +vec(OB) +vec(OC))

14

15

vec(u) = 0.5 (vec(c) - vec(a))
vec(v) = vec(c) + 0.5 (vec(b) - vec(c)) -
vec(a) + 0.5 (vec(b) - vec(a)) = 0.5 (vec(c) - vec(a))
Somit   vec(u) = vec(v)



Lösungen Teil 2

1

"vec" bedeutet "Vektor".
vec(AS) = 1/3 (vec(AB) +vec(AC) +vec(OC))
vec(AF) = 1/4 vec(AD) + 1/6 (vec(AB) +vec(AC))

2

```
180 Degree//N
```

3.14159

```
F[α_,r_]:= r {Cos[α Degree],Sin[α Degree]} ;
F[-30,190]+F[40,80]+F[(90+25),100]
```

$\{95\sqrt{3} + 80\,\text{Cos}[40\,°] + 100\,\text{Cos}[115\,°],\ -95 + 80\,\text{Sin}[40\,°] + 100\,\text{Sin}[115\,°]\}$

```
%//N
```

{183.567, 47.0538}

```
Norm[%]
```

189.501

3

Achtung: Hier werden die Koordinatenachsen in der natürlichen Reihenfolge gewählt. Dies kann auch anders geschehen. Dann Aendert die Reihenfolge der Koordinaten, nicht aber die Grösse.

```
K1={7,6,-5}/Norm[{7,6,-5}] 2000
```

$\left\{1400\sqrt{\frac{10}{11}},\ 1200\sqrt{\frac{10}{11}},\ -1000\sqrt{\frac{10}{11}}\right\}$

```
%//N
```

{1334.85, 1144.16, −953.463}

```
K2={0,6,-5}/Norm[{0,6,-5}] 800
```

$\left\{0, \frac{4800}{\sqrt{61}}, -\frac{4000}{\sqrt{61}}\right\}$

```
%//N
```

{0., 614.577, -512.148}

```
K1+K2
```

$\left\{1400\sqrt{\frac{10}{11}}, 1200\sqrt{\frac{10}{11}} + \frac{4800}{\sqrt{61}}, -1000\sqrt{\frac{10}{11}} - \frac{4000}{\sqrt{61}}\right\}$

```
%//N
```

{1334.85, 1758.73, -1465.61}

```
E1={-7,0,-3}/Norm[{-7,0,-3}];
E2={0,0,-3}/Norm[{0,0,-3}];
E3={0,-6,-3}/Norm[{0,-6,-3}];
solv= Solve[K1+K2 == λ1 E1+ λ2 E2+ λ3 E3,{λ1,λ2,λ3}]// Flatten
```

$\left\{\lambda2 \to \frac{200}{61}\left(32\sqrt{61} + 61\sqrt{110}\right), \lambda1 \to -400\sqrt{\frac{145}{11}}, \lambda3 \to -\frac{600}{671}\sqrt{5}\left(44\sqrt{61} + 61\sqrt{110}\right)\right\}$

```
λ1 E1 /.solv
```

$\left\{1400\sqrt{\frac{10}{11}}, 0, 600\sqrt{\frac{10}{11}}\right\}$

```
%//N
```

{1334.85, 0., 572.078}

```
Norm[%]
```

1452.27

```
λ2 E2 /.solv
```

$\left\{0, 0, -\frac{200}{61}\left(32\sqrt{61} + 61\sqrt{110}\right)\right\}$

```
%//N
```

{0., 0., -2917.05}

```
Norm[%]
```

2917.05

```
λ3 E3 /.solv
```

$\left\{0, \frac{1200}{671}\left(44\sqrt{61} + 61\sqrt{110}\right), \frac{600}{671}\left(44\sqrt{61} + 61\sqrt{110}\right)\right\}$

```
%//N
```

{0., 1758.73, 879.366}

```
Norm[%]
```

```
1966.32
```

4

```
Remove["Global`*"];

F1 = {0,0,3}/Norm[{0,0,3}];
F2 = {0,-4,3}/Norm[{0,-4,3}];
F3 = {12,-4,3}/Norm[{12,-4,3}];
x = λ1 F1 + λ2 F2 + λ3 F3
```

$$\left\{\frac{12\,\lambda 3}{13},\ -\frac{4\,\lambda 2}{5}-\frac{4\,\lambda 3}{13},\ \lambda 1+\frac{3\,\lambda 2}{5}+\frac{3\,\lambda 3}{13}\right\}$$

Die F sind normiert, alle müssen daher gleich sein.

```
x = 65 (F1 + F2 + F3)
```

```
{60, -72, 119}
```

5

Z.B. der erste, der zweite und der fünfte Vektor:

```
Det[{{2,-1,-3},{0,1,2},{1,0,2}}]
```

```
5
```

5 ungleich 0 !

6

```
a1={1,-2,3};
a2={0,1,-1};
a3={-2,0,4};
b1={0,3,1};
b2={-1,3,1};
b3={5,-6,1};
x={2,-3,2};

Det[{a1,a2,a3}]
```

```
6
```

5 ungleich 0 ==> Basis

```
Det[{b1,b2,b3}]
```

```
9
```

9 ungleich 0 ==> Basis

```
solv1 = Solve[x==λ1 a1 + λ2 a2 + λ3 a3, {λ1,λ2,λ3}]//Flatten
```

$\{\lambda 1 \to 1, \lambda 2 \to -1, \lambda 3 \to -\frac{1}{2}\}$

Das sind die Komonenten.

```
solv1 = Solve[x==λ1 b1 + λ2 b2 + λ3 b3, {λ1,λ2,λ3}]//Flatten
```

$\{\lambda 1 \to -2, \lambda 2 \to 3, \lambda 3 \to 1\}$

Das sind die Komonenten.

7

```
OA={2,-3,5};  OB={7,-7,5};  OC={0,1,7};  OD={2,3,6};
AB=OB-OA; AC=OC-OA; AD=OD-OA;
Solve[OD==OA+λ AB+μ AC,{λ,μ}]
```

{}

Nicht auf einer Ebene.

```
Det[{AB,AC,AD}]
```

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Resultat ungleich null: Nicht auf einer Ebene.