Lösungen Teil 1

1

```
OA = {3, 0, 4}; OB = {1, 1, 1}; OC = {-7, 5, 11};
Solve[(OB - OA) == λ (OC - OA), {λ}]
```

```
{}
```

Die Punkte liegen icht auf einer Geraden.

2

a

```
a1 = {1, 3, 0}; a2 = {6, 2, 0};
r1 = {2, -3, 1}; r2 = {-1, -4, 1};
Solve[a1 == λ a2, {λ}]
```

```
{}
```

Richtungsvektoren nicht parallel

```
solv = Solve[r1 + λ a1 == r2 + μ a2, {λ, μ}] // Flatten
```

$\{\lambda \to 0, \mu \to \frac{1}{2}\}$

```
Schnittpunkt = r1 + λ a1 /. solv[[1]]
```

```
{2, -3, 1}
```

b

```
a1 = {4, 3, 0}; a2 = {-8, -6, 0};
r1 = {0, 0, 0}; r2 = {10, 6, 3};
Solve[a1 == λ a2, {λ}]
```

$\{\{\lambda \to -\frac{1}{2}\}\}$

Richtungsvektoren parallel

```
solv = Solve[r1 + λ a1 == r2 + μ a2, {λ, μ}] // Flatten
```

```
{}
```

Kein Schnittpunkt

c

```
a1 = {0, 0, 1}; a2 = {2, 0, -1};
r1 = {3, 0, 5}; r2 = {1, -3, 6};
Solve[a1 == λ a2, {λ}]
```

```
{}
```

Richtungsvektoren nicht parallel

```
solv = Solve[r1 + λ a1 == r2 + μ a2, {λ, μ}] // Flatten
```

```
{}
```

Kein Schnittpunkt

d

```
a1 = {7, 2, -1}; a2 = {-14, -4, 2};
r1 = {4, -3, 2}; r2 = {-10, -7, 4};
Solve[a1 == λ a2, {λ}]
```

$\{\{\lambda \to -\frac{1}{2}\}\}$

Richtungsvektoren parallel

```
solv = Solve[r1 + λ a1 == r2 + μ a2, {λ, μ}] // Flatten
```

$\{\lambda \to -2 - 2\mu\}$

3

```
xg[s_] := -1 + s 6;
yg[s_] := -4 + s 2;
zg[s_] :=  1 + s 0;
g[s_] := {xg[s], yg[s], zg[s]};  g[s]
```

$\{-1 + 6s, -4 + 2s, 1\}$

```
xh[t_] :=  2 + t 1;
yh[t_] := -3 + t 3;
zh[t_] :=  1 + t 0;
h[t_] := {xh[t], yh[t], zh[t]};  h[t]
```

$\{2 + t, -3 + 3t, 1\}$

```
Solve[g[s] == h[t], {s, t}]
```

$\{\{s \to \frac{1}{2}, t \to 0\}\}$

```
Schnittpunkt = h[0]
```

{2, -3, 1}

```
w[t_] := Schnittpunkt +
   t ((g[1] - g[0]) / Norm[(g[1] - g[0])] + (h[1] - h[0]) / Norm[(h[1] - h[0])]);
w[
 t]
```

$$\left\{2+2\sqrt{\frac{2}{5}}\ t,\ -3+2\sqrt{\frac{2}{5}}\ t,\ 1\right\}$$

```
w[t Sqrt[5 / 2] / 2]
```

{2 + t, -3 + t, 1}

```
{w[t Sqrt[5 / 2] / 2]} // Transpose // MatrixForm
```

$$\begin{pmatrix} 2+t \\ -3+t \\ 1 \end{pmatrix}$$

4

```
OA = {3, -2, 2}; OB = {-3, 5, 8};
OU = {2, 1, -3}; OV = {1, 5, 4};
OW = {6, -2, -1};
g[t_] := OA + t (OB - OA);
g[t]
```

{3 - 6 t, -2 + 7 t, 2 + 6 t}

```
Φ[λ_, μ_] := OU + λ (OV - OU) + μ (OW - OU);
Φ[λ, μ]
```

{2 - λ + 4 μ, 1 + 4 λ - 3 μ, -3 + 7 λ + 2 μ}

```
solv = Solve[g[t]  == Φ[λ, μ], {t, λ, μ}] // Flatten
```

{t → -3, λ → -3, μ → 4}

```
Schnittpunkt = g[t]  /. solv
```

{21, -23, -16}

5

a

```
Φ[λ_, μ_] := {1, 2, -3} + λ {-1, 4, 7} + μ {4, -3, 2};
Φ[λ, μ]
```

{1 - λ + 4 μ, 2 + 4 λ - 3 μ, -3 + 7 λ + 2 μ}

```
Ψ[ν_, ξ_] := {5, -2, -3} + ν {22, -23, -4} + ξ {13, 0, 29};
Ψ[ν, ξ]
```

$\{5 + 22\nu + 13\xi,\ -2 - 23\nu,\ -3 - 4\nu + 29\xi\}$

```
solv = Solve[Ψ[ν, ξ] == Φ[λ, μ], {ν, ξ, λ, μ}] // Flatten
```

{}

Kein Schnittpunkt: Ebenen parallel

b

```
Φ[λ_, μ_] := {5, 1, 8} + λ {1, 0, 3} + μ {2, 1, -1};
Φ[λ, μ]
```

$\{5 + \lambda + 2\mu,\ 1 + \mu,\ 8 + 3\lambda - \mu\}$

```
Ψ[ν_, ξ_] := {11, 7, -16} + ν {6, 2, -1} + ξ {-1, -1, 3};
Ψ[ν, ξ]
```

$\{11 + 6\nu - \xi,\ 7 + 2\nu - \xi,\ -16 - \nu + 3\xi\}$

```
solv = Solve[Ψ[ν, ξ] == Φ[λ, μ], {ν, ξ, λ, μ}] // Flatten
```

$\left\{\nu \to -\frac{6}{7} + \frac{\mu}{7},\ \xi \to \frac{30}{7} - \frac{5\mu}{7},\ \lambda \to -\frac{24}{7} - \frac{3\mu}{7}\right\}$

```
(Φ[λ, μ] /. solv) // Simplify
```

$\left\{\frac{11(1+\mu)}{7},\ 1 + \mu,\ -\frac{16}{7}(1+\mu)\right\}$

```
(Ψ[ν, ξ] /. solv) // Simplify
```

$\left\{\frac{11(1+\mu)}{7},\ 1 + \mu,\ -\frac{16}{7}(1+\mu)\right\}$

Schnittgerade, hier dargestellt mit Parameter

```
(1 - t^6) / (1 - t) // Simplify
```

$1 + t + t^2 + t^3 + t^4 + t^5$

6

```
Remove["Global`*"]
```

```
Φ[λ_,μ_]:={2,3,1}+λ{4,-2,3}+μ{1,0,-2};
Φ[λ,μ]
```

$\{2 + 4\lambda + \mu,\ 3 - 2\lambda,\ 1 + 3\lambda - 2\mu\}$

```
solv=Solve[{x,y,z}==Φ[λ,μ],{λ,μ,x}]//Flatten
```

$\left\{\lambda \to \frac{3-y}{2},\ \mu \to \frac{1}{4}(11 - 3y - 2z),\ x \to \frac{1}{4}(43 - 11y - 2z)\right\}$

```
(x1-x==0 /.solv)
```

$$x1+\frac{1}{4}(-43+11\,y+2\,z)==0$$

```
(%/.x1->x)//Simplify
```

$$4\,x+11\,y+2\,z==43$$

7

```
Remove["Global`*"]
```

```
Φ[{x_,y_,z_}]:=3x-7z-21;
```

```
P1={0,0,z}; P2={0,y,0}; P3={x,0,0};
```

```
solv1=Solve[Φ[P1]==0,{z}]//Flatten
```

$\{z\to -3\}$

```
P1={0,0,z}/.solv1
```

$\{0,\ 0,\ -3\}$

```
solv2=Solve[Φ[P2]==0,{y}]//Flatten
```

$\{\}$

```
P2={0,y,0}/.solv2
```

$\{0,\ y,\ 0\}$

y ist beliebig. Setze y=1

```
y=1;
```

```
solv3=Solve[Φ[P3]==0,{x}]//Flatten
```

$\{x\to 7\}$

```
P3={x,0,0}/.solv3
```

$\{7,\ 0,\ 0\}$

```
Φ[λ_,μ_]:= P1+λ(P2-P1)+μ(P3-P1);
{Φ[λ,μ]}//Transpose//MatrixForm
```

$$\begin{pmatrix} 7\,\mu \\ \lambda \\ -3+3\,\lambda+3\,\mu \end{pmatrix}$$

8

```
Remove["Global`*"]
```

a

```
PA={4,3,-2}; PB={-3,1,2}; PC={1,0,2};
Φ[λ_,μ_]:= PA+λ(PB-PA)+μ(PC-PA);
{Φ[λ,μ]}//Transpose//MatrixForm
```

$$\begin{pmatrix} 4-7\lambda-3\mu \\ 3-2\lambda-3\mu \\ -2+4\lambda+4\mu \end{pmatrix}$$

b

```
PA={2,-3,0}; PB={-4,6,2}; PC={0,0,9};
Φ[λ_,μ_]:= PA+λ(PB-PA)+μ(PC-PA);
{Φ[λ,μ]}//Transpose//MatrixForm
```

$$\begin{pmatrix} 2-6\lambda-2\mu \\ -3+9\lambda+3\mu \\ 2\lambda+9\mu \end{pmatrix}$$

Lösungen Teil 2

1

```
Remove["Global`*"]

OA={-1,0,5}; OB={3,-4,7}; OC={2,2,3};
α = ArcCos[(OB-OA).(OC-OA)/(Norm[OB-OA] Norm[OC-OA])]
```

$$\frac{\pi}{2}$$

```
%/Degree//N
```

90.

```
β = ArcCos[(OA-OB).(OC-OB)/(Norm[OA-OB] Norm[OC-OB])]
```

$$\text{ArcCos}\left[\frac{6}{\sqrt{53}}\right]$$

```
%/Degree//N
```

34.4962

```
γ = ArcCos[(OA-OC).(OB-OC)/(Norm[OA-OC] Norm[OB-OC])]
```

$$\text{ArcCos}\left[\sqrt{\frac{17}{53}}\right]$$

```
%/Degree//N
```

55.5038

Kontrolle

```
(α + β + γ) / Degree // N
```

180.

2

```
Remove["Global`*"]

OP={1,2,0}; Φ[{x_,y_,z_}]:=x-y+2z-3; nVec={1,-1,2};
h[λ_]:=OP+λ nVec;
solv=Solve[Φ[h[λ]]==0,{λ}]//Flatten
```

$\left\{\lambda \to \frac{2}{3}\right\}$

```
OS=h[λ]/.solv
```

$\left\{\frac{5}{3}, \frac{4}{3}, \frac{4}{3}\right\}$

```
OPgespiegelt=OP+2(OS-OP)
```

$\left\{\frac{7}{3}, \frac{2}{3}, \frac{8}{3}\right\}$

```
d=Norm[2(OS-OP)]
```

$4 \sqrt{\frac{2}{3}}$

```
Norm[OP-OPgespiegelt]
```

$4 \sqrt{\frac{2}{3}}$

```
% //N
```

3.26599

3

```
Remove["Global`*"]

OP={p1,p2,p3}; a={a1,a2,a3}; b={b1,b2,b3};
g1[λ_]:=OP+ λ a;
g2[μ_]:=OP+ μ b;

s1[λ_]:=OP+ λ (a/Norm[a]+b/Norm[b]);
s2[μ_]:=OP+ μ (a/Norm[a]-b/Norm[b]);
```

```
((a/Norm[a]+b/Norm[b]).(a/Norm[a]-b/Norm[b])//Simplify)/.{Abs[a1]^2->a1^2,Abs[a2]^2
->a2^2,Abs[a3]^2->a3^2,Abs[b1]^2->b1^2,Abs[b2]^2->b2^2,Abs[b3]^2->b3^2}
```

$$(a1^2\, b1^2 + a2^2\, b1^2 + a1^2\, b2^2 + a2^2\, b2^2 + a1^2\, b3^2 + a2^2\, b3^2 - a1^2\,(b1^2 + b2^2 + b3^2) - a2^2\,(b1^2 + b2^2 + b3^2))\,/\,((a1^2 + a2^2 + a3^2)\,(b1^2 + b2^2 + b3^2))$$

```
%//Expand
```

0

4

```
Remove["Global`*"]
```

```
a={3,8,x}; b={3,-8,x};
a.b == Norm[a] Norm[b] Cos[60 Degree]
```

$$-55 + x^2 == \frac{1}{2}\,(73 + \text{Abs}[x]^2)$$

```
Solve[a.b == Norm[a] Norm[b] Cos[60 Degree],{x}]
```

$$\{\{x \to -\sqrt{183}\},\ \{x \to \sqrt{183}\}\}$$

```
%//N
```

$\{\{x \to -13.5277\},\ \{x \to 13.5277\}\}$

5

```
Remove["Global`*"]
```

```
OA={2,-4,-9}; OB={0,6,1}; OX[s_]:={3,12,16}+s {3,10,11};
Simplify[(OA-OB).(OX[s]-OB)]==0
```

$-204\,(1 + s) == 0$

```
solv=Solve[(OA-OB).(OX[s]-OB)==0,{s}]//Flatten
```

$\{s \to -1\}$

```
OC=OX[s]/.solv
```

{0, 2, 5}

```
OD=OC+(OA-OB)
```

{2, -8, -5}

6

```
Remove["Global`*"]
```

```
HNFΦ[x_,y_,z_]:=(x+2y+3z-5)/Sqrt[1^2+2^2+3^2];
HNFΨ[x_,y_,z_]:=(x+2y+3z+2)/Sqrt[1^2+2^2+3^2];
```

```
HNFΦ[0,0,0]
```

$$-\frac{5}{\sqrt{14}}$$

```
HNFΨ[0,0,0]
```

$$\sqrt{\frac{2}{7}}$$

```
HNFΨ[0,0,0]-HNFΦ[0,0,0]
```

$$\sqrt{\frac{2}{7}}+\frac{5}{\sqrt{14}}$$

```
N[%]
```

1.87083

7

```
Remove["Global`*"]
```

```
NFFΦ[x_,y_,z_]:=(3x-y+3z-1)/Sqrt[3^2+(-1)^2+2^2];
HNFΨ[x_,y_,z_]:=(-6x+2y-4z-7)/Sqrt[(-6)^2+2^2+(-4)^2];
Print[NFFΦ[x,y,z], "  ",HNFΨ[x,y,z]]
```

$$\frac{-1+3x-y+3z}{\sqrt{14}} \quad \frac{-7-6x+2y-4z}{2\sqrt{14}}$$

Normalenvektoren nicht paralle!

```
Solve[{NFFΦ[x,y,z]==0, HNFΨ[x,y,z]==0},{x,y,z}]
```

$$\left\{\left\{x\to -\frac{25}{6}+\frac{y}{3},\ z\to \frac{9}{2}\right\}\right\}$$

```
N[%]
```

$\{\{x \to -4.16667 + 0.333333\,y,\ z \to 4.5\}\}$

Schnittgerade! Abstand = 0.

8

```
Remove["Global`*"]
```

```
OP={-1,0,3};
HNFΦ[x_,y_,z_]:=(-x+2y+5z+2)/Sqrt[(-1)^2+2^2+5^2]; HNFΦ[{x_,y_,z_}]:= HNFΦ[x,y,z];
HNFΦ[x,y,z]
```

$$\frac{2-x+2y+5z}{\sqrt{30}}$$

```
HNFΦ[OP]
```

$$3\sqrt{\frac{6}{5}}$$

```
N[%]
```

3.28634

9

```
Remove["Global`*"]

HNFΦ[x_,y_,z_]:=(3x-5y-4z-10)/Sqrt[3^2+(-5)^2+(-4)^2];
HNFΦ[x,y,z]
```

$$\frac{-10+3x-5y-4z}{5\sqrt{2}}$$

```
HNFΦ1[x_,y_,z_]:=(3x-5y-4z-10)/Sqrt[3^2+(-5)^2+(-4)^2]+4;
HNFΦ1[0,0,0]
```

$$4-\sqrt{2}$$

```
N[%]
```

2.58579

```
HNFΦ2[x_,y_,z_]:=(3x-5y-4z-10)/Sqrt[3^2+(-5)^2+(-4)^2]-4;
HNFΦ2[0,0,0]
```

$$-4-\sqrt{2}$$

```
N[%]
```

-5.41421

10

```
Remove["Global`*"]

HNFΦ[x_,y_,z_]:=(-x-2y+z-2)/Sqrt[(-1)^2+(-2)^2+(1)^2];
HNFΦ[x,y,z]
```

$$\frac{-2-x-2y+z}{\sqrt{6}}$$

```
OA={3,4,2}; OB={1,-1,5}; OC={3,0,1}; nVec=Cross[(OB-OA),(OC-OA)]
```

{17, -2, 8}

```
HNFΨ[x_,y_,z_]:=({x,y,z}.nVec+dD)/Norm[nVec]; HNFΨ[{x_,y_,z_}]:=HNFΨ[x,y,z];
HNFΨ[x,y,z]
```

$$\frac{dD + 17\,x - 2\,y + 8\,z}{\sqrt{357}}$$

```
solv=Solve[HNFΨ[OA]==0,{dD}]//Flatten
```

{dD → -59}

```
HNFΨ[x_,y_,z_]:=({x,y,z}.nVec+dD)/Norm[nVec]/. solv;
HNFΨ[x,y,z]
```

$$\frac{-59 + 17\,x - 2\,y + 8\,z}{\sqrt{357}}$$

```
Simplify[Sqrt[357] HNFΨ[x,y,z]-Sqrt[6] HNFΦ[x,y,z]]==0
```

$-57 + 18\,x + 7\,z == 0$

```
Simplify[Sqrt[357] HNFΨ[x,y,z]+Sqrt[6] HNFΦ[x,y,z]]==0
```

$-61 + 16\,x - 4\,y + 9\,z == 0$

11

Der Umkreismittelpunkt liegt auf den Senkrechten durch die Seitenmittelpunkte.

```
Remove["Global`*"]
```

```
OA={a1,a2,a3}; OB={b1,b2,b3}; OC={c1,c2,c3};
OM[λ_,μ_]:=OA+λ (OB-OA)+μ (OC-OA)//Simplify;
OM[λ,μ]
```

{b1 λ + c1 μ - a1 (-1 + λ + μ), b2 λ + c2 μ - a2 (-1 + λ + μ), b3 λ + c3 μ - a3 (-1 + λ + μ)}

OM ist Lösung des Gleichungssystems:
r^2 = |OM-OA|^2, r^2 = |OM-Ob|^2, r^2 = |OM-OC|^2
(3 Gelichungen, Unbekannte r,   ,   )

```
{r^2==(OM[λ,μ]-OA), r^2==(OM[λ,μ]-OB), r^2==(OM[λ,μ]-OC)}
```

{r^2 == {-a1 + b1 λ + c1 μ - a1 (-1 + λ + μ),
  -a2 + b2 λ + c2 μ - a2 (-1 + λ + μ), -a3 + b3 λ + c3 μ - a3 (-1 + λ + μ)},
 r^2 == {-b1 + b1 λ + c1 μ - a1 (-1 + λ + μ), -b2 + b2 λ + c2 μ - a2 (-1 + λ + μ),
  -b3 + b3 λ + c3 μ - a3 (-1 + λ + μ)}, r^2 == {-c1 + b1 λ + c1 μ - a1 (-1 + λ + μ),
  -c2 + b2 λ + c2 μ - a2 (-1 + λ + μ), -c3 + b3 λ + c3 μ - a3 (-1 + λ + μ)}}

```
Solve[{r^2==(OM[λ,μ]-OA).(OM[λ,μ]-OA), r^2==(OM[λ,μ]-OB).(OM[λ,μ]-OB),
r^2==(OM[λ,μ]-OC).(OM[λ,μ]-OC)},{λ,μ,r}]//Simplify
```

$$\{\{r \to -\frac{1}{2}\sqrt{\,}\,(((a1^2 + a2^2 + a3^2 - 2\, a1\, b1 + b1^2 - 2\, a2\, b2 + b2^2 - 2\, a3\, b3 + b3^2)$$
$$(a1^2 + a2^2 + a3^2 - 2\, a1\, c1 + c1^2 - 2\, a2\, c2 + c2^2 - 2\, a3\, c3 + c3^2)$$
$$(b1^2 + b2^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2 - 2\, b3\, c3 + c3^2))\,/$$
$$(a1^2\, b2^2 + a1^2\, b3^2 - 2\, a1\, b2^2\, c1 - 2\, a1\, b3^2\, c1 + b2^2\, c1^2 + b3^2\, c1^2 - 2\, a1^2\, b2\, c2 +$$
$$2\, a1\, b1\, b2\, c2 + 2\, a1\, b2\, c1\, c2 - 2\, b1\, b2\, c1\, c2 + a1^2\, c2^2 - 2\, a1\, b1\, c2^2 + b1^2\, c2^2 +$$
$$b3^2\, c2^2 + a3^2\, (b1^2 + b2^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2) - 2\, a1^2\, b3\, c3 +$$
$$2\, a1\, b1\, b3\, c3 + 2\, a1\, b3\, c1\, c3 - 2\, b1\, b3\, c1\, c3 - 2\, b2\, b3\, c2\, c3 + a1^2\, c3^2 -$$
$$2\, a1\, b1\, c3^2 + b1^2\, c3^2 + b2^2\, c3^2 + a2^2\, (b1^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b3\, c3 + c3^2) -$$
$$2\, a2\, (-b1\, b2\, c1 + b2\, c1^2 + a1\, (b1 - c1)\, (b2 - c2) + b1^2\, c2 + b3^2\, c2 - b1\, c1\, c2 +$$
$$a3\, (b2 - c2)\, (b3 - c3) - b2\, b3\, c3 - b3\, c2\, c3 + b2\, c3^2) - 2\, a3\, (b3\, c1^2 - b2\, b3\, c2 +$$
$$b3\, c2^2 + a1\, (b1 - c1)\, (b3 - c3) + b1^2\, c3 + b2^2\, c3 - b2\, c2\, c3 - b1\, c1\, (b3 + c3))))),$$
$$\lambda \to ((b1^2 - a2\, b2 + b2^2 - a3\, b3 + b3^2 - b1\, c1 + a1\, (-b1 + c1) + a2\, c2 - b2\, c2 + a3\, c3 - b3\, c3)$$
$$(a1^2 + a2^2 + a3^2 - 2\, a1\, c1 + c1^2 - 2\, a2\, c2 + c2^2 - 2\, a3\, c3 + c3^2))\,/$$
$$(2\, (a1^2\, b2^2 + a1^2\, b3^2 - 2\, a1\, b2^2\, c1 - 2\, a1\, b3^2\, c1 + b2^2\, c1^2 + b3^2\, c1^2 - 2\, a1^2\, b2\, c2 +$$
$$2\, a1\, b1\, b2\, c2 + 2\, a1\, b2\, c1\, c2 - 2\, b1\, b2\, c1\, c2 + a1^2\, c2^2 - 2\, a1\, b1\, c2^2 + b1^2\, c2^2 +$$
$$b3^2\, c2^2 + a3^2\, (b1^2 + b2^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2) - 2\, a1^2\, b3\, c3 +$$
$$2\, a1\, b1\, b3\, c3 + 2\, a1\, b3\, c1\, c3 - 2\, b1\, b3\, c1\, c3 - 2\, b2\, b3\, c2\, c3 + a1^2\, c3^2 -$$
$$2\, a1\, b1\, c3^2 + b1^2\, c3^2 + b2^2\, c3^2 + a2^2\, (b1^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b3\, c3 + c3^2) -$$
$$2\, a2\, (-b1\, b2\, c1 + b2\, c1^2 + a1\, (b1 - c1)\, (b2 - c2) + b1^2\, c2 + b3^2\, c2 - b1\, c1\, c2 +$$
$$a3\, (b2 - c2)\, (b3 - c3) - b2\, b3\, c3 - b3\, c2\, c3 + b2\, c3^2) - 2\, a3\, (b3\, c1^2 - b2\, b3\, c2 +$$
$$b3\, c2^2 + a1\, (b1 - c1)\, (b3 - c3) + b1^2\, c3 + b2^2\, c3 - b2\, c2\, c3 - b1\, c1\, (b3 + c3))))),$$
$$\mu \to ((a1^2 + a2^2 + a3^2 - 2\, a1\, b1 + b1^2 - 2\, a2\, b2 + b2^2 - 2\, a3\, b3 + b3^2)$$
$$(a3\, b3 + a1\, (b1 - c1) - b1\, c1 + c1^2 + a2\, (b2 - c2) - b2\, c2 + c2^2 - a3\, c3 - b3\, c3 + c3^2))\,/$$
$$(2\, (a1^2\, b2^2 + a1^2\, b3^2 - 2\, a1\, b2^2\, c1 - 2\, a1\, b3^2\, c1 + b2^2\, c1^2 + b3^2\, c1^2 - 2\, a1^2\, b2\, c2 +$$
$$2\, a1\, b1\, b2\, c2 + 2\, a1\, b2\, c1\, c2 - 2\, b1\, b2\, c1\, c2 + a1^2\, c2^2 - 2\, a1\, b1\, c2^2 + b1^2\, c2^2 +$$
$$b3^2\, c2^2 + a3^2\, (b1^2 + b2^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2) - 2\, a1^2\, b3\, c3 +$$
$$2\, a1\, b1\, b3\, c3 + 2\, a1\, b3\, c1\, c3 - 2\, b1\, b3\, c1\, c3 - 2\, b2\, b3\, c2\, c3 + a1^2\, c3^2 -$$
$$2\, a1\, b1\, c3^2 + b1^2\, c3^2 + b2^2\, c3^2 + a2^2\, (b1^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b3\, c3 + c3^2) -$$
$$2\, a2\, (-b1\, b2\, c1 + b2\, c1^2 + a1\, (b1 - c1)\, (b2 - c2) + b1^2\, c2 + b3^2\, c2 - b1\, c1\, c2 +$$
$$a3\, (b2 - c2)\, (b3 - c3) - b2\, b3\, c3 - b3\, c2\, c3 + b2\, c3^2) - 2\, a3\, (b3\, c1^2 - b2\, b3\, c2 +$$
$$b3\, c2^2 + a1\, (b1 - c1)\, (b3 - c3) + b1^2\, c3 + b2^2\, c3 - b2\, c2\, c3 - b1\, c1\, (b3 + c3)))))\},$$
$$\{r \to \frac{1}{2}\sqrt{\,}\,(((a1^2 + a2^2 + a3^2 - 2\, a1\, b1 + b1^2 - 2\, a2\, b2 + b2^2 - 2\, a3\, b3 + b3^2)$$
$$(a1^2 + a2^2 + a3^2 - 2\, a1\, c1 + c1^2 - 2\, a2\, c2 + c2^2 - 2\, a3\, c3 + c3^2)$$
$$(b1^2 + b2^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2 - 2\, b3\, c3 + c3^2))\,/$$
$$(a1^2\, b2^2 + a1^2\, b3^2 - 2\, a1\, b2^2\, c1 - 2\, a1\, b3^2\, c1 + b2^2\, c1^2 + b3^2\, c1^2 - 2\, a1^2\, b2\, c2 +$$
$$2\, a1\, b1\, b2\, c2 + 2\, a1\, b2\, c1\, c2 - 2\, b1\, b2\, c1\, c2 + a1^2\, c2^2 - 2\, a1\, b1\, c2^2 + b1^2\, c2^2 +$$
$$b3^2\, c2^2 + a3^2\, (b1^2 + b2^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2) - 2\, a1^2\, b3\, c3 +$$
$$2\, a1\, b1\, b3\, c3 + 2\, a1\, b3\, c1\, c3 - 2\, b1\, b3\, c1\, c3 - 2\, b2\, b3\, c2\, c3 + a1^2\, c3^2 -$$
$$2\, a1\, b1\, c3^2 + b1^2\, c3^2 + b2^2\, c3^2 + a2^2\, (b1^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b3\, c3 + c3^2) -$$
$$2\, a2\, (-b1\, b2\, c1 + b2\, c1^2 + a1\, (b1 - c1)\, (b2 - c2) + b1^2\, c2 + b3^2\, c2 - b1\, c1\, c2 +$$
$$a3\, (b2 - c2)\, (b3 - c3) - b2\, b3\, c3 - b3\, c2\, c3 + b2\, c3^2) - 2\, a3\, (b3\, c1^2 - b2\, b3\, c2 +$$
$$b3\, c2^2 + a1\, (b1 - c1)\, (b3 - c3) + b1^2\, c3 + b2^2\, c3 - b2\, c2\, c3 - b1\, c1\, (b3 + c3))))),$$
$$\lambda \to ((b1^2 - a2\, b2 + b2^2 - a3\, b3 + b3^2 - b1\, c1 + a1\, (-b1 + c1) + a2\, c2 - b2\, c2 + a3\, c3 - b3\, c3)$$
$$(a1^2 + a2^2 + a3^2 - 2\, a1\, c1 + c1^2 - 2\, a2\, c2 + c2^2 - 2\, a3\, c3 + c3^2))\,/$$
$$(2\, (a1^2\, b2^2 + a1^2\, b3^2 - 2\, a1\, b2^2\, c1 - 2\, a1\, b3^2\, c1 + b2^2\, c1^2 + b3^2\, c1^2 - 2\, a1^2\, b2\, c2 +$$
$$2\, a1\, b1\, b2\, c2 + 2\, a1\, b2\, c1\, c2 - 2\, b1\, b2\, c1\, c2 + a1^2\, c2^2 - 2\, a1\, b1\, c2^2 + b1^2\, c2^2 +$$
$$b3^2\, c2^2 + a3^2\, (b1^2 + b2^2 - 2\, b1\, c1 + c1^2 - 2\, b2\, c2 + c2^2) - 2\, a1^2\, b3\, c3 +$$
$$2\, a1\, b1\, b3\, c3 + 2\, a1\, b3\, c1\, c3 - 2\, b1\, b3\, c1\, c3 - 2\, b2\, b3\, c2\, c3 + a1^2\, c3^2 -$$
$$2\, a1\, b1\, c3^2 + b1^2\, c3^2 + b2^2\, c3^2 + a2^2\, (b1^2 + b3^2 - 2\, b1\, c1 + c1^2 - 2\, b3\, c3 + c3^2) -$$
$$2\, a2\, (-b1\, b2\, c1 + b2\, c1^2 + a1\, (b1 - c1)\, (b2 - c2) + b1^2\, c2 + b3^2\, c2 - b1\, c1\, c2 +$$
$$a3\, (b2 - c2)\, (b3 - c3) - b2\, b3\, c3 - b3\, c2\, c3 + b2\, c3^2) - 2\, a3\, (b3\, c1^2 - b2\, b3\, c2 +$$
$$b3\, c2^2 + a1\, (b1 - c1)\, (b3 - c3) + b1^2\, c3 + b2^2\, c3 - b2\, c2\, c3 - b1\, c1\, (b3 + c3))))),$$
$$\mu \to ((a1^2 + a2^2 + a3^2 - 2\, a1\, b1 + b1^2 - 2\, a2\, b2 + b2^2 - 2\, a3\, b3 + b3^2)$$

$$(a3\,b3 + a1\,(b1 - c1) - b1\,c1 + c1^2 + a2\,(b2 - c2) - b2\,c2 + c2^2 - a3\,c3 - b3\,c3 + c3^2))\,/\,(2\,(a1^2\,b2^2 + a1^2\,b3^2 - 2\,a1\,b2^2\,c1 - 2\,a1\,b3^2\,c1 + b2^2\,c1^2 + b3^2\,c1^2 - 2\,a1^2\,b2\,c2 + 2\,a1\,b1\,b2\,c2 + 2\,a1\,b2\,c1\,c2 - 2\,b1\,b2\,c1\,c2 + a1^2\,c2^2 - 2\,a1\,b1\,c2^2 + b1^2\,c2^2 + b3^2\,c2^2 + a3^2\,(b1^2 + b2^2 - 2\,b1\,c1 + c1^2 - 2\,b2\,c2 + c2^2) - 2\,a1^2\,b3\,c3 + 2\,a1\,b1\,b3\,c3 + 2\,a1\,b3\,c1\,c3 - 2\,b1\,b3\,c1\,c3 - 2\,b2\,b3\,c2\,c3 + a1^2\,c3^2 - 2\,a1\,b1\,c3^2 + b1^2\,c3^2 + b2^2\,c3^2 + a2^2\,(b1^2 + b3^2 - 2\,b1\,c1 + c1^2 - 2\,b3\,c3 + c3^2) - 2\,a2\,(-b1\,b2\,c1 + b2\,c1^2 + a1\,(b1 - c1)\,(b2 - c2) + b1^2\,c2 + b3^2\,c2 - b1\,c1\,c2 + a3\,(b2 - c2)\,(b3 - c3) - b2\,b3\,c3 - b3\,c2\,c3 + b2\,c3^2) - 2\,a3\,(b3\,c1^2 - b2\,b3\,c2 + b3\,c2^2 + a1\,(b1 - c1)\,(b3 - c3) + b1^2\,c3 + b2^2\,c3 - b2\,c2\,c3 - b1\,c1\,(b3 + c3))))\}\}$$

```
OMC=OA+(OB-OA)/2 //Simplify
```

$$\left\{\frac{a1+b1}{2},\ \frac{a2+b2}{2},\ \frac{a3+b3}{2}\right\}$$

```
OMA=OB+(OC-OB)/2 //Simplify
```

$$\left\{\frac{b1+c1}{2},\ \frac{b2+c2}{2},\ \frac{b3+c3}{2}\right\}$$

b

```
OA={-2,5,-5}; OB={3,-1,5}; OC={0,3,-1};
OM[λ_,μ_]:=OA+λ (OB-OA)+μ (OC-OA)//Simplify;
OM[λ,μ]
```

$$\{-2 + 5\,\lambda + 2\,\mu,\ 5 - 6\,\lambda - 2\,\mu,\ -5 + 10\,\lambda + 4\,\mu\}$$

```
solv=Solve[{r^2==(OM[λ,μ]-OA).(OM[λ,μ]-OA), r^2==(OM[λ,μ]-OB).(OM[λ,μ]-OB),
r^2==(OM[λ,μ]-OC).(OM[λ,μ]-OC)},{λ,μ,r}]//Simplify //Flatten
```

$$\left\{r \to -\sqrt{\frac{29463}{10}},\ \lambda \to \frac{297}{5},\ \mu \to -\frac{3059}{20},\ r \to \sqrt{\frac{29463}{10}},\ \lambda \to \frac{297}{5},\ \mu \to -\frac{3059}{20}\right\}$$

```
N[%]
```

$$\{r \to -54.2798,\ \lambda \to 59.4,\ \mu \to -152.95,\ r \to 54.2798,\ \lambda \to 59.4,\ \mu \to -152.95\}$$

```
OM[λ,μ]/.solv
```

$$\left\{-\frac{109}{10},\ -\frac{91}{2},\ -\frac{114}{5}\right\}$$

```
N[%]
```

$$\{-10.9,\ -45.5,\ -22.8\}$$

Lösungen Teil 3

1

```
Remove["Global`*"]
```

```
a={1,0,3}; b={2,-2,3};
Cross[a,b]
```

{6, 3, -2}

```
Norm[Cross[a,b]]
```

7

2

```
Remove["Global`*"]

OP={2,4,-5};
g[s_]:={1,0,-6}+s {-3,5,2};

(OP-g[s]).{-3,5,2}==0
```

$5\,(4-5\,s)+2\,(1-2\,s)-3\,(1+3\,s)==0$

```
solv=Solve[(OP-g[s]).{-3,5,2}==0,{s}]//Flatten
```

$\left\{s\to\frac{1}{2}\right\}$

```
g[s]/.solv
```

$\left\{-\frac{1}{2},\ \frac{5}{2},\ -5\right\}$

```
N[%]
```

{-0.5, 2.5, -5.}

3

```
Remove["Global`*"]

OA={2,-1,5}; OB={3,3,0}; OC={-4,3,-2}; OD={-1,-3,2};
Volumen = 1/6 Det[{OC-OA, OB-OA, OD-OA}]
```

$\frac{67}{3}$

```
N[%]
```

22.3333

```
Oberflaeche = 1/2 (Norm[Cross[OB-OA,OC-OA]]+ Norm[Cross[OC-OA,OD-OA]]+
Norm[Cross[OD-OA,OB-OA]]+  Norm[Cross[OC-OB,OD-OB]])
```

$\frac{1}{2}\left(2\sqrt{227}+2\sqrt{598}+\sqrt{1261}+\sqrt{2217}\right)$

```
N[%]
```

22.3333

4

```
Remove["Global`*"]

OA={0,5,0}; OB={5,0,6};
OC={5,13-4,0}; OD={0,13,6};
```

d=Volumen/Grundfläche

```
d = Det[{OB-OA,OC-OA,OD-OC}]
```

510

```
G = Cross[OB-OA,OD-OC]//Norm
```

$\sqrt{6541}$

```
N[%]
```

80.8764

```
d/G
```

$\frac{510}{\sqrt{6541}}$

```
N[%]
```

6.30591

5

```
Remove["Global`*"]

kF={30,-60,30}; a={1,2,0}; b={-2,0,2};
kF1[λ_,μ_]:= λ a+ μ b; kF2[ν_]:= ν Cross[a,b];
solv=Solve[kF==kF1[λ,μ]+kF2[ν],{λ,μ,ν}]//Flatten
```

$\{\lambda \to -20, \mu \to -5, \nu \to 10\}$

```
kF1[λ,μ]/.solv
```

$\{-10, -40, -10\}$

```
kF2[ν]/.solv
```

$\{40, -20, 40\}$

6

```
Remove["Global`*"]
```

```
OP={2,0,-3}; Φ[λ_,μ_]:={2,11,-16}+λ{-3,0,2}+μ{0,1,-3};
g[t_]:= OP + t Cross[{-3,0,2},{0,1,-3}]; g[t]
```

{2 - 2 t, -9 t, -3 - 3 t}

7

```
Remove["Global`*"]
```

```
OP={2,0,-1};
Φ[x_,y_,z_]:={x,y,z}.{2,-1,3}+4;
Ψ[x_,y_,z_]:={x,y,z}.{1,1,-2}-3;
Γ[λ_,μ_]:= OP + λ  {2,-1,3}+ μ {1,1,-2}; Γ[λ,μ]
```

$\{2+2\lambda+\mu,\ -\lambda+\mu,\ -1+3\lambda-2\mu\}$

```
{Γ[λ,μ]}//Transpose//MatrixForm
```

$$\begin{pmatrix} 2+2\lambda+\mu \\ -\lambda+\mu \\ -1+3\lambda-2\mu \end{pmatrix}$$

8

```
Remove["Global`*"]
```

a

```
OO={0,0,0}; OA={12,4,0}; OB={2,14,0}; h=10;
OM=(OA+OB)/2
```

{7, 9, 0}

```
nVec = h Cross[OA,OB]/Norm[Cross[OA,OB]]
```

{0, 0, 10}

```
OC=OM+nVec
```

{7, 9, 10}

b

```
ACB = ArcCos[(OA-OC).(OB-OC)/(Norm[OA-OC] Norm[OB-OC])]
```

$\mathrm{ArcCos}\left[\frac{1}{3}\right]$

```
ACB//N
```

1.23096

```
ACB/Degree//N
```

70.5288

c

```
nOAC=Cross[OA,OC]
```

{40, -120, 80}

```
nOBC=Cross[OB,OC]
```

{140, -20, -80}

```
α = ArcCos[nOAC.nOBC / Norm[nOAC] / Norm[nOBC]]
```

$\text{ArcCos}\left[\frac{1}{\sqrt{231}}\right]$

```
α//N
```

1.50495

```
αDeg=α/Degree//N
```

86.2275

```
Winkel=180-αDeg
```

93.7725

c

```
FlaecheDach= ( Norm[nOAC]+Norm[nOBC] )/2
```

$\frac{1}{2}\left(40\sqrt{14} + 20\sqrt{66}\right)$

```
FlaecheDach//N
```

156.074

d

```
Det[{OA,OB,OC}]/6
```

$\frac{800}{3}$

```
%//N
```

266.667