Lösungen

1

```
Remove["Global`*"]

u={0,0,1};
eu=u 1/Norm[u];
uv=100 eu;
a={1,0,-1};
ea=a 1/Norm[a];
b={0,1,-1};
eb=b 1/Norm[b];
c={-1,-1,-1};
ec=c 1/Norm[c];
d={-1,-3,-1};
ed=d 1/Norm[d];
dv=300 ed;
v[δ_]:=δ ed;
```

a

Nein. Eine Vorspannkraft in einem Bein muss bekannt sein. Ein Vektor kann man im Raum nur nach 3 Basisvektoren zerlegen.

b

```
uv == α ea + β eb + γ ec + dv
```

$$\{0, 0, 100\} == \left\{-\frac{300}{\sqrt{11}} + \frac{\alpha}{\sqrt{2}} - \frac{\gamma}{\sqrt{3}}, -\frac{900}{\sqrt{11}} + \frac{\beta}{\sqrt{2}} - \frac{\gamma}{\sqrt{3}}, -\frac{300}{\sqrt{11}} - \frac{\alpha}{\sqrt{2}} - \frac{\beta}{\sqrt{2}} - \frac{\gamma}{\sqrt{3}}\right\}$$

```
Solve[uv == α ea+ β eb+ γ ec + dv ,{α,β,γ}]
```

$$\left\{\left\{\alpha \to -\frac{100}{33}\left(11\sqrt{2} + 6\sqrt{22}\right), \beta \to \frac{100}{33}\left(-11\sqrt{2} + 12\sqrt{22}\right), \gamma \to -\frac{100\left(11 + 15\sqrt{11}\right)}{11\sqrt{3}}\right\}\right\}$$

```
N[%]
```

$$\{\{\alpha \to -132.421, \beta \to 123.42, \gamma \to -318.852\}\}$$

c

```
Norm[ea]==Norm[eb]
```

```
True
```

ea und eb sind gleich lang!

```
Solve[uv == α ea+ β eb+ γ ec + v[δ] ,{α,β,γ}]
```

$$\left\{\left\{\alpha \to -\frac{2}{33}\left(550\sqrt{2}+\sqrt{22}\,\delta\right),\ \beta \to \frac{4}{33}\left(-275\sqrt{2}+\sqrt{22}\,\delta\right),\ \gamma \to -\frac{5\left(220+\sqrt{11}\,\delta\right)}{11\sqrt{3}}\right\}\right\}$$

```
N[%]//ExpandAll
```

$$\{\{\alpha \to -47.1405 - 0.284268\,\delta,\ \beta \to -47.1405 + 0.568535\,\delta,\ \gamma \to -57.735 - 0.870388\,\delta\}\}$$

Damit die Kräfte in ea und eb gleich gross sind, muss   und   gleich gross sein. Dann muss   = 0 gelten. Dann muss gelten:

```
-47.1405 Norm[ea] == -57.735 Norm[ec]
```

```
False
```

```
{-47.1405 Norm[ea], -57.735 Norm[ec]}
```

```
{-47.1405, -57.735}
```

Dann sind aber die Kräfte in ea und in ec sehr verschieden.

d

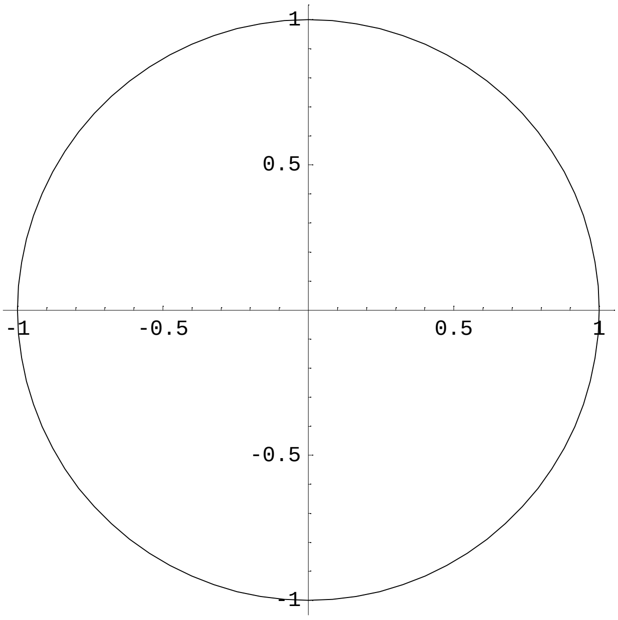
Dann stellt sich die Vorspannung auf natürliche Weise durch die Verformungskräfte selbst ein, bis ein Gleichgewicht herrscht.

2

```
Remove["Global`*"]
```

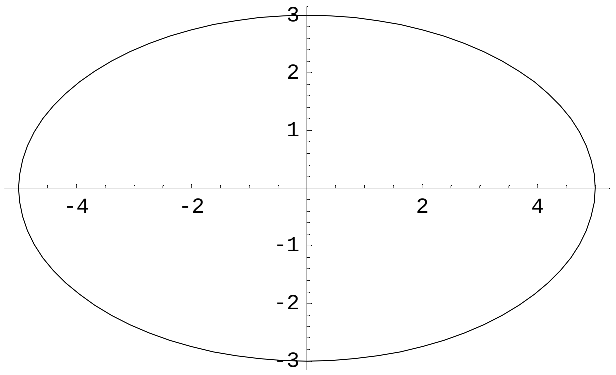
a

```
k[x_,y_]:=x^2+y^2-1;
<< Graphics`ImplicitPlot`;
ImplicitPlot[k[x,y] == 0, {x, -1,1}];
```



b

```
el[x_,y_]:=x^2/5^2+y^2/3^2-1;
<< Graphics`ImplicitPlot`;
ImplicitPlot[el[x,y] == 0, {x, -5,5}];
```



c

```
Dreh[φ_]:={{Cos[φ],-Sin[φ]},{Sin[φ],Cos[φ]}}; Dreh[φ]// MatrixForm
```

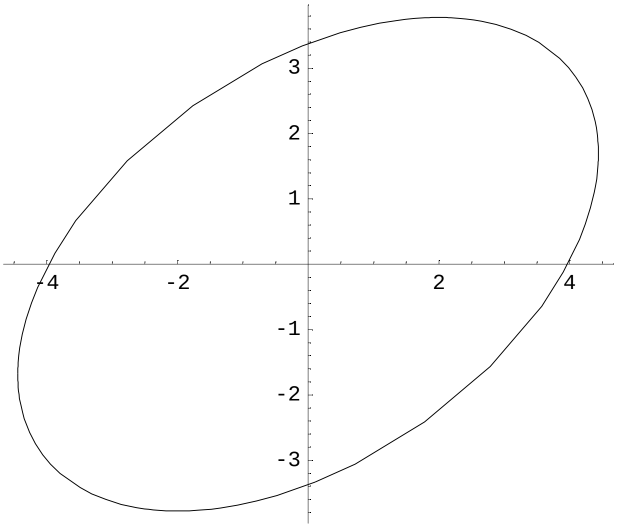
$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

```
Dreh[35 Degree].{5 Cos[u],3 Sin[u]}// MatrixForm
```

$$\begin{pmatrix} 5\,\text{Cos}[35\,°]\,\text{Cos}[u] - 3\,\text{Sin}[35\,°]\,\text{Sin}[u] \\ 5\,\text{Cos}[u]\,\text{Sin}[35\,°] + 3\,\text{Cos}[35\,°]\,\text{Sin}[u] \end{pmatrix}$$

```
el2Param[u_]:=Dreh[35 Degree].{5 Cos[u],3 Sin[u]};
```

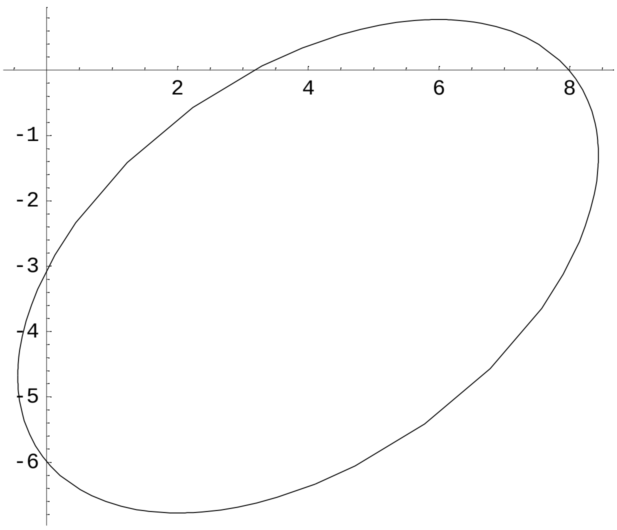
```
ParametricPlot[el2Param[u], {u,0,2Pi},AspectRatio->Automatic];
```



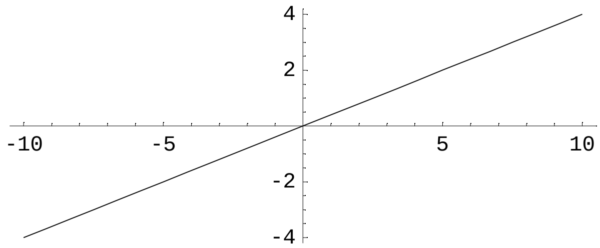
d

```
el3Param[u_]:=Dreh[35 Degree].{5 Cos[u],3 Sin[u]}+{4,-3};
```

```
p1=ParametricPlot[el3Param[u], {u,0,2Pi},AspectRatio->Automatic];
```



```
p2=ParametricPlot[t {5,2}, {t,-2,2},AspectRatio->Automatic];
```



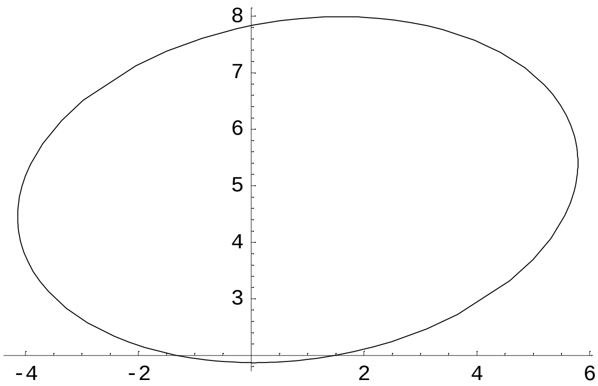

```
ArcTan[2/5]//N
```

0.380506

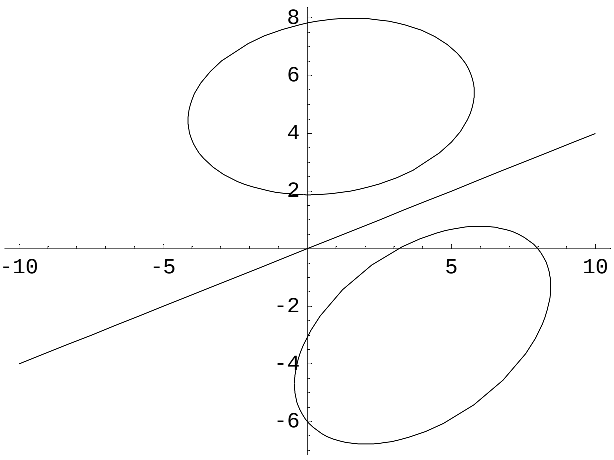
```
φ0=-ArcTan[2/5]
```

$-\text{ArcTan}\left[\frac{2}{5}\right]$

```
el4Param[u_]:=Dreh[φ0].el3Param[u];
el5Param[u_]:={el4Param[u][[1]],-el4Param[u][[2]]};
el6Param[u_]:=Dreh[-φ0].el5Param[u];
p3=ParametricPlot[el6Param[u], {u,0,2Pi},AspectRatio->Automatic];
```




```
Show[p1,p2,p3];
```



3

```
Remove["Global`*"]
```

a

```
OM={3,4,2};
r=2;
Φ[x_,y_,z_]:=x+y+2z+6;
OL={10,12,15};
Ku[x_,y_,z_]:=({x,y,z}-OM).({x,y,z}-OM)-r^2;
Ku[{x_,y_,z_}]:=Ku[x,y,z];
Ku[x,y,z]
```

$-4+(-3+x)^2+(-4+y)^2+(-2+z)^2$

```
Ku[x,y,z]//Expand
```

$25-6x+x^2-8y+y^2-4z+z^2$

```
OS=OL;
a=OM-OS;
```

Gesucht ist noch ein Punkt P0 auf der Kugeloberfläche, welcher Tangentialpunkt von S aus gesehen ist. Für ihn gelten die folgenden Gleichungen, wenn man annimmt, dass P0 dieselbe z-Koordiante haben soll wie M:

```
{z==OM[[3]], OP[x_,y_,z_]:={x,y,z};OP[x,y,z], Ku[OP[x,y,z]]==0}
```

$\{z==2, \{x, y, z\}, -4+(-3+x)^2+(-4+y)^2+(-2+z)^2==0\}$

```
(OP[x,y,z]-OM).(OP[x,y,z]-OS)==0
```

$(-10+x)(-3+x)+(-12+y)(-4+y)+(-15+z)(-2+z)==0$

```
Expand[(OP[x,y,z]-OM).(OP[x,y,z]-OS)]==0
```

$108-13x+x^2-16y+y^2-17z+z^2==0$

```
Solve[{z==OM[[3]], Ku[x,y,z]==0,(OP[x,y,z]-OM).(OP[x,y,z]-OS)==0},{x,y,z}]
```

$$\left\{\left\{x\to\frac{1}{113}\left(367-16\sqrt{109}\right), y\to\frac{2}{113}\left(242+7\sqrt{109}\right), z\to 2\right\}, \left\{x\to\frac{1}{113}\left(367+16\sqrt{109}\right), y\to\frac{2}{113}\left(242-7\sqrt{109}\right), z\to 2\right\}\right\}$$

Wenn man mit einer etwas grösseren z-Koordinate probiert, so werden die Zahlen etwas einfacher:

```
solv=Solve[{z==3, Ku[x,y,z]==0,(OP[x,y,z]-OM).(OP[x,y,z]-OS)==0},{x,y,z}]
```

$$\left\{\left\{x\to\frac{4}{113}\left(69-2\sqrt{258}\right), y\to\frac{1}{113}\left(380+7\sqrt{258}\right), z\to 3\right\}, \left\{x\to\frac{4}{113}\left(69+2\sqrt{258}\right), y\to\frac{1}{113}\left(380-7\sqrt{258}\right), z\to 3\right\}\right\}$$

```
N[%]
```

$\{\{x\to 1.30532, y\to 4.35785, z\to 3.\}, \{x\to 3.57964, y\to 2.36782, z\to 3.\}\}$

```
OP0=OP[x,y,z]/.solv[[2]]
```

$$\left\{\frac{4}{113}\left(69+2\sqrt{258}\right), \frac{1}{113}\left(380-7\sqrt{258}\right), 3\right\}$$

```
N[%]
```

{3.57964, 2.36782, 3.}

```
Ke[x_,y_,z_]:= (a.(OP0-OS))^2 (OP[x,y,z]-OS).(OP[x,y,z]-OS)- (a.(OP[x,y,z]-OS))^2
(OP0-OS).(OP0-OS);
Ke[x,y,z]
```

$$-\left(144+\left(-12+\frac{1}{113}\left(380-7\sqrt{258}\right)\right)^2+\left(-10+\frac{4}{113}\left(69+2\sqrt{258}\right)\right)^2\right)$$
$$(-7(-10+x)-8(-12+y)-13(-15+z))^2+$$
$$\left(156-8\left(-12+\frac{1}{113}\left(380-7\sqrt{258}\right)\right)-7\left(-10+\frac{4}{113}\left(69+2\sqrt{258}\right)\right)\right)^2$$
$$((-10+x)^2+(-12+y)^2+(-15+z)^2)$$

```
Ke[x,y,z]//N//Expand
```

$$16958.-140668.\,x+63662.\,x^2-249088.\,y-31136.\,x\,y+$$
$$59492.\,y^2+290788.\,z-50596.\,x\,z-57824.\,y\,z+30302.\,z^2$$

```
Schatten1[y_,z_]:=Ke[0,y,z];
Schatten1[y,z]//N//Simplify
```

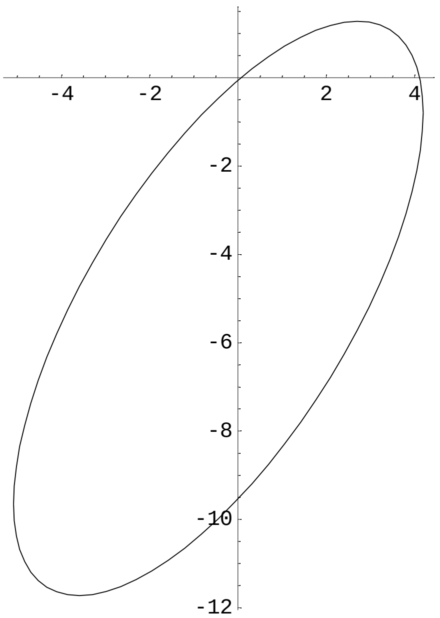
$$59492.\,y^2+y\,(-249088.-57824.\,z)+30302.\,(0.0586762+z)\,(9.53765+z)$$

```
Schatten1[y,z]//Simplify
```

$$278\,(61+214\,y^2+1046\,z+109\,z^2-16\,y\,(56+13\,z))$$

```
<< Graphics`ImplicitPlot`
```

```
ImplicitPlot[Schatten1[y,z] == 0, {y, -6,5}];
```



b

Plot der Risse: z=0

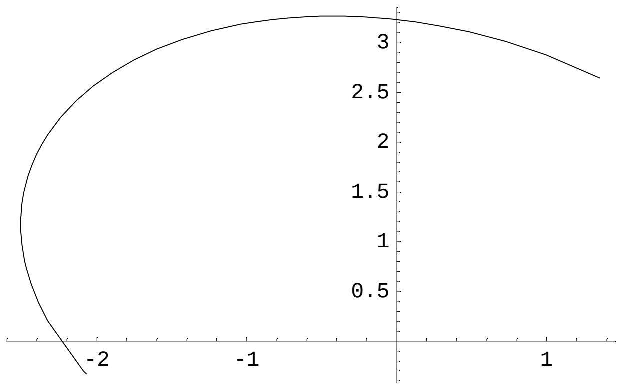
```
solv3=Solve[{Ke[x,y,z]==0,Φ[x,y,z]==0},{x,y}]//Flatten
```

$$\left\{x \to \frac{1}{555}\left(-1815-553\,z-\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right),\ y \to \frac{1}{555}\left(-1515-557\,z+\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right),\ x \to \frac{1}{555}\left(-1815-553\,z+\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right),\ y \to \frac{1}{555}\left(-1515-557\,z-\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right)\right\}$$

```
par3[z_]:={x,y}/.solv3;
par3[z]
```

$$\left\{\frac{1}{555}\left(-1815-553\,z-\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right),\ \frac{1}{555}\left(-1515-557\,z+\sqrt{278}\,\sqrt{-14385-11190\,z-1657\,z^2}\right)\right\}$$

```
ParametricPlot[par3[z],{z,-5,-1.8}];
```



Plot der Risse: y=0

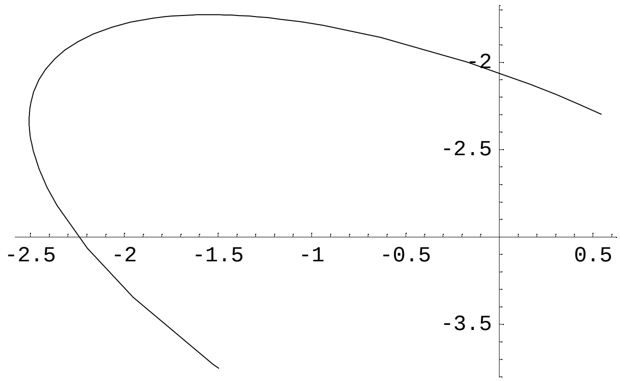
```
solv2=Solve[{Ke[x,y,z]==0,Φ[x,y,z]==0},{x,z}]//Flatten
```

$$\left\{x \to \frac{312-275\,y-2\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389},\ z \to \frac{-4323-557\,y+\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389},\ x \to \frac{312-275\,y+2\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389},\ z \to \frac{-4323-557\,y-\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389}\right\}$$

```
par2[y_]:={x,z}/.solv2;
par2[y]
```

$$\left\{\frac{312-275\,y-2\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389},\ \frac{-4323-557\,y+\sqrt{278}\,\sqrt{10560+2184\,y-1657\,y^2}}{1389}\right\}$$

```
ParametricPlot[par2[y],{y,-5,3}];
```



Plot der Risse: x=0

```
solv1=Solve[{Ke[x,y,z]==0,Φ[x,y,z]==0},{y,z}]//Flatten
```

$$\left\{y\to\frac{936-275\,x-2\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381},\right.$$
$$z\to\frac{-4611-553\,x+\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381},$$
$$y\to\frac{936-275\,x+2\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381},$$
$$\left.z\to\frac{-4611-553\,x-\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381}\right\}$$

```
par1[x_]:={y,z}/.solv1;
par1[x]
```

$$\left\{\frac{936-275\,x-2\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381},\right.$$
$$\left.\frac{-4611-553\,x+\sqrt{278}\,\sqrt{11200+312\,x-1657\,x^2}}{1381}\right\}$$

```
ParametricPlot[par1[x],{x,-3,3}];
```