Lösungen

Teil I: Wiederholung Matrizen und Eigenwertprobleme

I/1 Matrizenrechnung

```
Remove["Global`*"]
```

a

```
A = {{1,2,3},{3,5,6},{4,5,6}}; A//MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 6 \\ 4 & 5 & 6 \end{pmatrix}$$

```
A1=Inverse[A]; A1 //MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -2 & 2 & -1 \\ \frac{5}{3} & -1 & \frac{1}{3} \end{pmatrix}$$

```
Det[A]
```

-3

```
Det[A1]
```

$-\frac{1}{3}$

b

```
e1={1,0,0}; e2={0,1,0}; e3={0,0,1};
A.e1
```

{1, 3, 4}

```
A.e2
```

{2, 5, 5}

```
A.e3
```

{3, 6, 6}

Spalten von A

c

```
A1.e1
```

$\{0, -2, \frac{5}{3}\}$

```
A1.e2
```

$\{-1, 2, -1\}$

```
A1.e3
```

$\{1, -1, \frac{1}{3}\}$

Spalten der Inversen von A

I/2 Eigenwerte, Eigenvektoren

```
Remove["Global`*"]
```

a

```
A = {{1, 2}, {2, 3}}; A // MatrixForm
```

$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

```
Det[A]
```

-1

b1

```
A1=Inverse[A]; A1//MatrixForm
```

$\begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}$

```
EM=IdentityMatrix[2]; EM//MatrixForm
```

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

```
Det[A-λ EM]
```

$-1 - 4\lambda + \lambda^2$

A xVec =   xVec=   EM xVec bedeutet (A-   EM) xVec = 0Vec und dass hier nicht nur die Null-Lösung existiert. Das heisst Det[A-   EM]=0. Diese Gleichung ist lösbar (charakteristische Gleichung)!

```
solv1=Solve[Det[A-λ EM]==0,{λ}]//Flatten
```

$\{\lambda \to 2 - \sqrt{5}, \lambda \to 2 + \sqrt{5}\}$

```
N[%]
```

{λ → -0.236068, λ → 4.23607}

```
λ1 = λ/.solv1[[1]]
```

$2 - \sqrt{5}$

```
λ2 = λ/.solv1[[2]]
```

$2 + \sqrt{5}$

b2

```
solv2=Solve[A.{x,y}==λ1{x,y},{x,y}]//Flatten
```

$\{x \to -\frac{1}{2}\left(1 + \sqrt{5}\right) y\}$

```
x1Vec = ({x,y}/.solv2)/.y->1
```

$\{\frac{1}{2}\left(-1 - \sqrt{5}\right), 1\}$

```
Simplify[A.x1Vec] == Simplify[λ1 x1Vec]
```

True

b3

```
N[λ1]
```

-0.236068

```
solv2a=Solve[A.{x,y}== -0.236068 {x,y},{x,y}]//Flatten
```

{x → 0., y → 0.}

b4

```
solv3=Solve[A.{x,y}==λ2{x,y},{x,y}]//Flatten
```

$\{x \to -\frac{1}{2}\left(1 - \sqrt{5}\right) y\}$

```
x2Vec = ({x,y}/.solv3)/.y->1
```

$\{\frac{1}{2}\left(-1 + \sqrt{5}\right), 1\}$

```
Simplify[A.x2Vec] == Simplify[λ2 x2Vec]
```

True

b5

```
N[λ2]
```

4.23607

```
Solve[A.{x,y}==4.23607 {x,y},{x,y}]//Flatten
```

{x → 0., y → 0.}

Teil II: Eigenwertprobleme und Matrixzerlegung

Definitionen

```
U[n_] := Inverse[S[n]];
M[n_] := S[n].Ei[n].U[n];
pr[n_] := Module[{}, Print["det(S) = ", Det[U[n]]];
  Print["Gegebene Matrix S = ", S[n] // MatrixForm];
  Print["Inverse Matrix U = ", U[n] // MatrixForm];
  Print["Diagonalmatrix D = ", Ei[n] // MatrixForm];
  Print["Resultat M = ", M[n] // MatrixForm];
  Print["Eigensystem von M = ", Eigensystem[M[n]]]]
```

II/1

```
Ei[1] = {{1, 0, 0}, {0, 2, 0}, {0, 0, 3}};
S[1] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[1]
```

det(S) = $\frac{1}{4}$

Gegebene Matrix S = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Inverse Matrix U = $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$

Diagonalmatrix D = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Resultat M = $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} & -1 \\ \frac{1}{2} & \frac{3}{2} & -1 \\ -\frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix}$

Eigensystem von M = {{3, 2, 1}, {{-1, -1, 1}, {1, -1, 1}, {1, 1, 1}}}

```
M[1] // TeXForm
```

II/2

```
Ei[2] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
S[2] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[2]
```

det(S) = $\frac{1}{4}$

Gegebene Matrix S = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Inverse Matrix U = $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$

Diagonalmatrix D = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Resultat M = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Eigensystem von M = {{1, 1, 1}, {{0, 0, 1}, {0, 1, 0}, {1, 0, 0}}}

II/3

```
Ei[4] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 2}};
S[4] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[4]
```

det(S) = $\frac{1}{4}$

Gegebene Matrix S = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Inverse Matrix U = $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$

Diagonalmatrix D = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

Resultat M = $\begin{pmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{3}{2} \end{pmatrix}$

Eigensystem von M = {{2, 1, 1}, {{-1, -1, 1}, {1, 0, 1}, {0, 1, 0}}}

```
Eigensystem[M[4]]
```

{{2, 1, 1}, {{-1, -1, 1}, {1, 0, 1}, {0, 1, 0}}}

II/4

```
Ei[3] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}};
S[3] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[3]
```

det(S) = $\frac{1}{4}$

Gegebene Matrix S = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Inverse Matrix U = $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$

Diagonalmatrix D = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Resultat M = $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$

Eigensystem von M = {{1, 1, 0}, {{1, 0, 1}, {0, 1, 0}, {-1, -1, 1}}}

```
Eigensystem[M[3]]
```

{{1, 1, 0}, {{1, 0, 1}, {0, 1, 0}, {-1, -1, 1}}}

II/5 Abbildung eines Kreises

```
Ei[5] = {{1, 0}, {0, 2}};
S[5] = {{1, 1}, {1, -1}};
pr[5]
```

det(S) = $-\frac{1}{2}$

Gegebene Matrix S = $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Inverse Matrix U = $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

Diagonalmatrix D = $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

Resultat M = $\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$

Eigensystem von M = {{2, 1}, {{-1, 1}, {1, 1}}}

```
M[5].{x, y}
```

$\left\{\frac{3x}{2} - \frac{y}{2}, -\frac{x}{2} + \frac{3y}{2}\right\}$

```
u[x_, y_] := (M[5].{x, y})[[1]];
u[x, y]
```

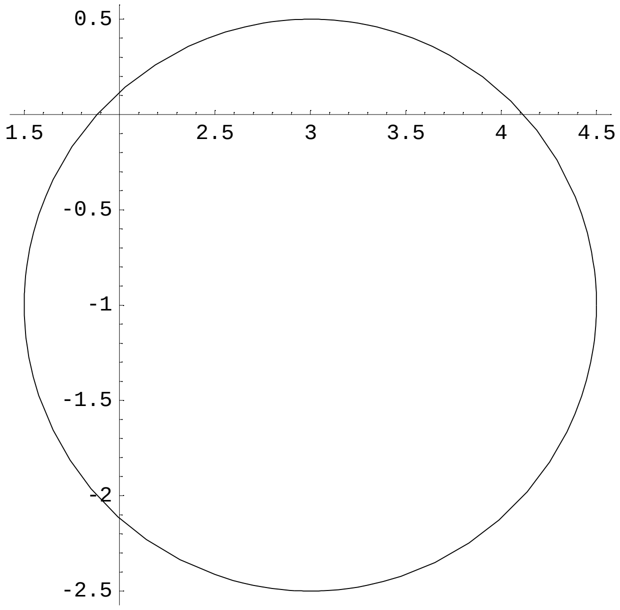
$$\frac{3x}{2} - \frac{y}{2}$$

```
v[x_, y_] := (M[5].{x, y})[[2]];
v[x, y]
```

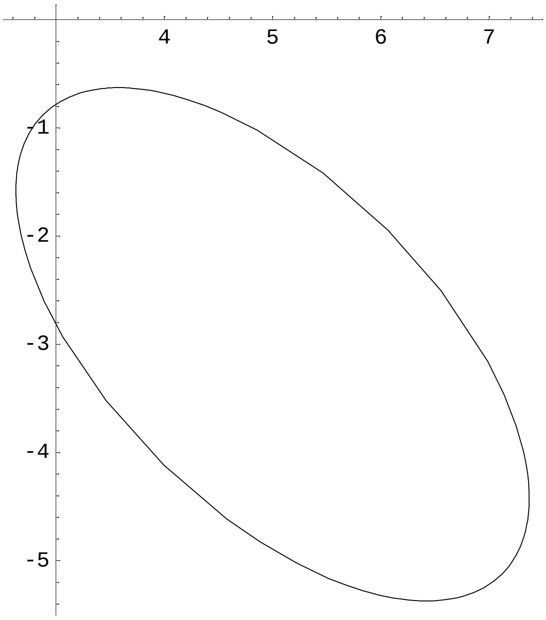
$$-\frac{x}{2} + \frac{3y}{2}$$

```
x[r_, φ_] := r Cos[φ] + 3;
y[r_, φ_] := r Sin[φ] - 1;
```

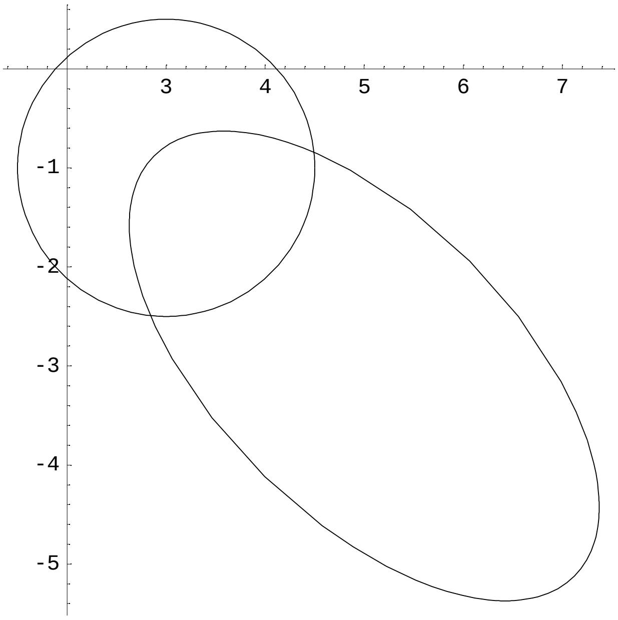
```
r = 1.5;
p1 = ParametricPlot[{x[r, φ], y[r, φ]}, {φ, 0, 2 Pi}, AspectRatio → Automatic];
```



```
p2 = ParametricPlot[{u[x[r, φ], y[r, φ]], v[x[r, φ], y[r, φ]]},
   {φ, 0, 2 Pi}, AspectRatio → Automatic];
```




```
Show[p1, p2];
```



Ellipse!