Lösungen

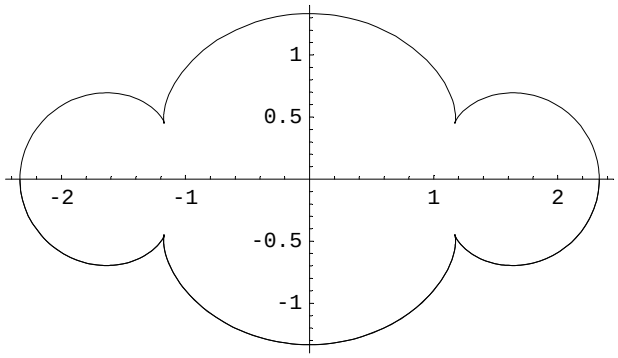
Vektorkurven und Richtungsvektoren

1. Beispiel von 2D-Kurven

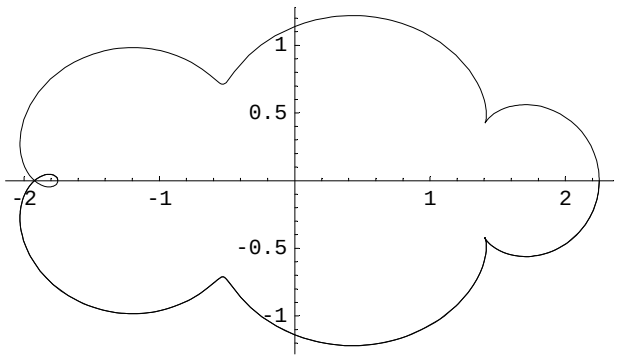
```
Remove["Global`*"]
```

```
v[t_]:={2Cos[t]+Cos[5t]/3,Sin[t]+Sin[5t]/3};
```

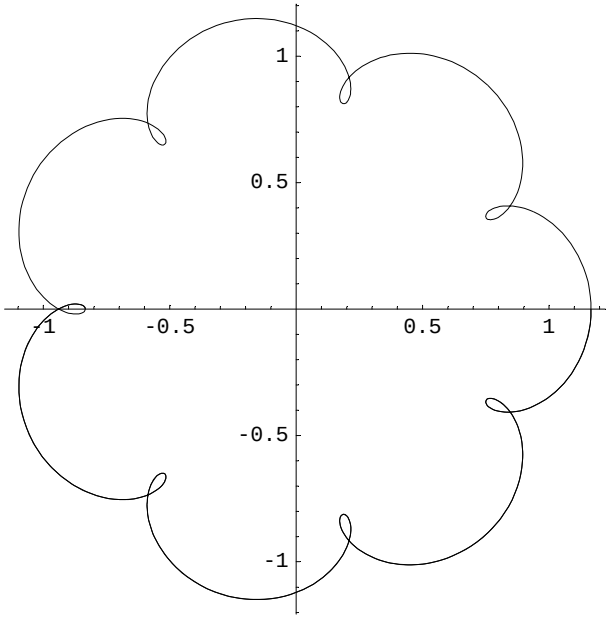
```
pl1 = ParametricPlot[v[t], {t, -Pi, 2 Pi}, AspectRatio → Automatic];
```



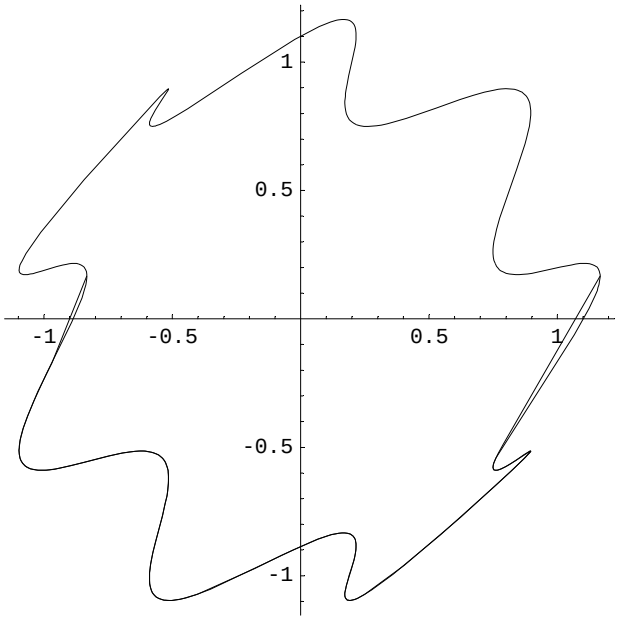

```
ParametricPlot[{2 Cos[t] + Cos[6 t] / 4, Sin[t] + Sin[6 t] / 4},
  {t, -Pi, 2 Pi}, AspectRatio → Automatic];
```



```
ParametricPlot[{Cos[t]+Cos[8t]/6,Sin[t]+Sin[8t]/6},{t,-Pi,2Pi},AspectRatio→
Automatic];
```




```
ParametricPlot[{Cos[t]+Cos[8t]/6,Sin[t]+Cos[8t]/6},{t,-Pi,2Pi},AspectRatio→
Automatic];
```



2. Ein Richtungsvektor

```
v'[t]
```

$$\left\{-2\,\text{Sin}[t]-\frac{5}{3}\,\text{Sin}[5\,t],\ \text{Cos}[t]+\frac{5}{3}\,\text{Cos}[5\,t]\right\}$$

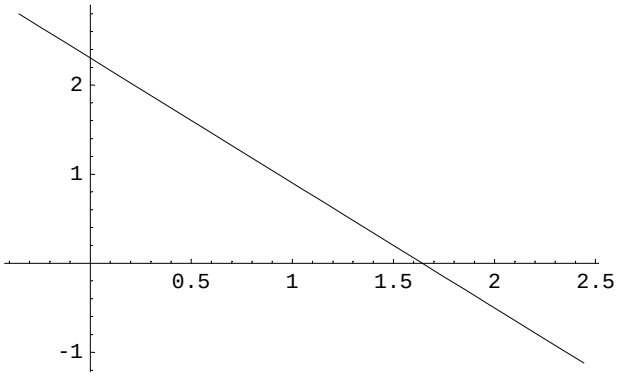
```
D[{2 Cos[t] + Cos[5 t] / 3, Sin[t] + Sin[5 t] / 3}, t]
```

$$\left\{-2\,\text{Sin}[t]-\frac{5}{3}\,\text{Sin}[5\,t],\ \text{Cos}[t]+\frac{5}{3}\,\text{Cos}[5\,t]\right\}$$

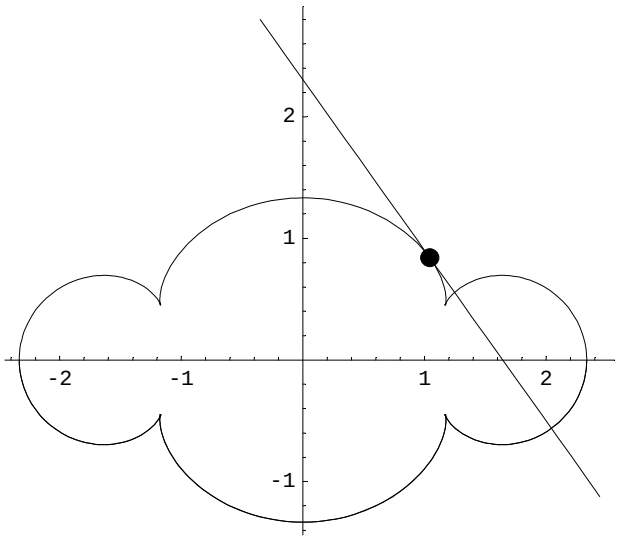
```
vv = D[{2 Cos[t] + Cos[5 t] / 3, Sin[t] + Sin[5 t] / 3}, t] /. t → 1.2
```

```
{-1.39839, 1.96264}
```

```
g[t_]:=v[1.2]+t vv;
pl2 = ParametricPlot[Evaluate[g[u]],{u,-1,1}];
```




```
Show[pl1,pl2,Graphics[{PointSize[0.03],Point[v[1.2]]}]];
```



An einer Spitze entsteht ein Extremum mindestens einer Komponente des Tangentenvektors. Es existiert dort manchmal auch keine eindeutige Tangente.
Beispiel:

```
Solve[Evaluate[D[2Cos[t]+Cos[5t]/3,t]]==0,{t}]//N
```

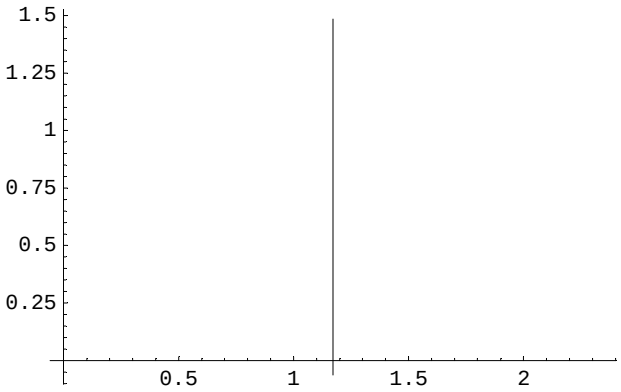
```
{{t → 0.}, {t → -2.17109}, {t → 2.17109}, {t → -0.970499}, {t → 0.970499},
 {t → -2.28687}, {t → 2.28687}, {t → -0.854718}, {t → 0.854718}}
```

```
vv=D[{2Cos[t]+Cos[5t]/3,Sin[t]+Sin[5t]/3},t]/.t→0.854718
```

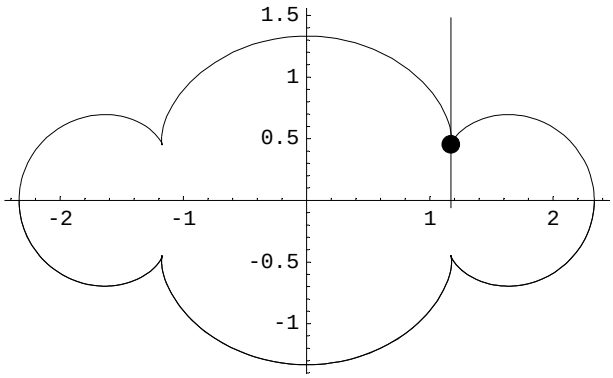
```
{-7.11327 × 10^-7, -0.0516563}
```

```
vv={0,-0.051656283416512405};
```

```
g1[t_]:=v[0.854718]+t vv;
pl3 = ParametricPlot[Evaluate[g1[t]],{t,-20,10},AspectRatio→Automatic];
```



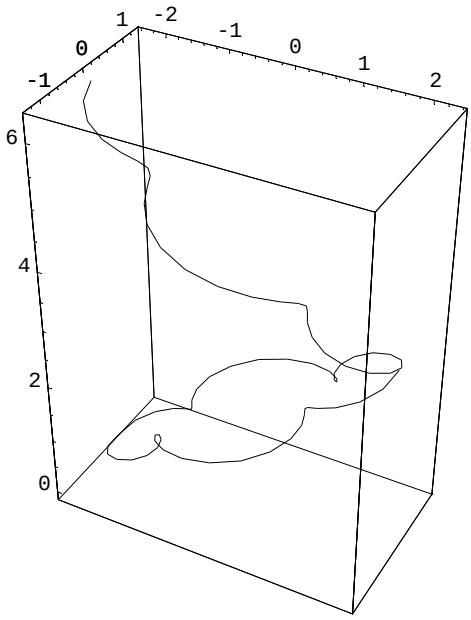

```
Show[pl1,pl3,Graphics[{PointSize[0.03],Point[v[0.854718]]}]];
```



3. Ein 3D-Plot

```
Remove["Global`*"]
```

```
ParametricPlot3D[
  {2 Cos[t] + Cos[5 t] / 3, Sin[t] + Sin[5 t] / 3, Sqrt[(t - Pi)^2]}, {t, -Pi, 2 Pi}];
```



4. Geradenspiegelung

```
Remove["Global`*"]
```

```
a = {3, 1};
λ1 = 1; λ2 = -1;
v1 = a; v2 = {-a[[2]], a[[1]]};
OP = {1, 5};
OO = {0, 0};
```

```
eV = Transpose[{v1, v2}]; eV // MatrixForm
```

$$\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$$

```
eD = {{λ1, 0}, {0, λ2}}; eD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

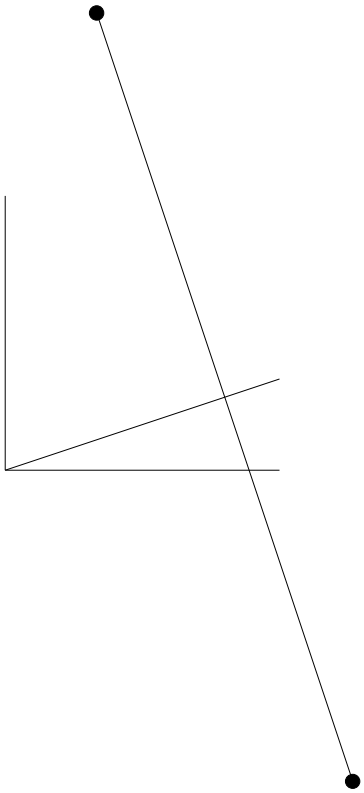
```
M = eV.eD.Inverse[eV]; M // MatrixForm
```

$$\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$$

```
OQ = M.OP
```

$$\left\{\frac{19}{5}, -\frac{17}{5}\right\}$$

```
Show[Graphics[{
    Line[{OO, a}], Line[{OO, {3, 0}}], Line[{OO, {0, 3}}]
    , PointSize[0.04], Point[OP], Point[OQ], Line[{OP, OQ}]
   }], AspectRatio → Automatic];
```



4. Geradenspiegelung

```
Remove["Global`*"]
```

```
a = {3, 1};
λ1 = 1; λ2 = -1;
v1 = a; v2 = {-a[[2]], a[[1]]};
OP = {1, 5};
OO = {0, 0};
dM5[k_] = {{Cos[k 2 Pi / 5], -Sin[k 2 Pi / 5]}, {Sin[k 2 Pi / 5], Cos[k 2 Pi / 5]}};
p[k_] := dM5[k].OP
```

```
eV = Transpose[{v1, v2}]; eV // MatrixForm
```

$$\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$$

```
eD = {{λ1, 0}, {0, λ2}}; eD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

```
M = eV.eD.Inverse[eV];  M // MatrixForm
```

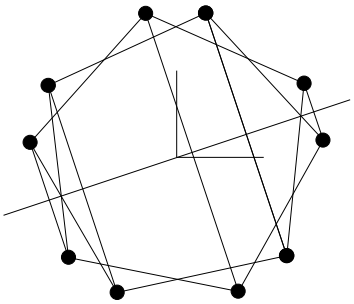
$$\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$$

```
OQ = M.OP
```

$$\left\{\frac{19}{5}, -\frac{17}{5}\right\}$$

```
q[k_] := M.p[k]
```

```
Show[Graphics[{
    Line[{-2 a, 2 a}], Line[{OO, {3, 0}}], Line[{OO, {0, 3}}]
    , PointSize[0.04], Point[OP], Point[p[1]],
    Point[p[2]], Point[p[3]], Point[p[4]], Point[p[5]],
    Point[OQ], Point[q[1]], Point[q[2]], Point[q[3]], Point[q[4]], Point[q[5]],
    Line[{OP, OQ}], Line[{p[1], q[1]}], Line[{p[2], q[2]}],
    Line[{p[3], q[3]}], Line[{p[4], q[4]}], Line[{p[5], q[5]}],
    Line[{p[1], p[2], p[3], p[4], p[5], p[1]}],
    Line[{q[1], q[2], q[3], q[4], q[5], q[1]}]
   }], AspectRatio → Automatic];
```



5. Ebenenspiegelung

```
Remove["Global`*"]
```

```
r0 = {1, 2, 3}; a = {1, 2, -1}; b = {-1, 1, 1};
(* ϕ *)
x[λ_, μ_] = r0 + λ a + μ b;
```

a Translation, Ebenenspiegelung, Rücktranslation

```
tr[p_] := p - r0;
```

```
eV = Transpose[{a, b, Cross[a, b]}]; eV // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$

```
λ1 = 1; λ2 = 1; λ3 = -1;
eD = {{λ1, 0, 0}, {0, λ2, 0}, {0, 0, λ3}}; eD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
M = eV.eD.Inverse[eV]; M // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

```
sp[p_] := M.p

rtr[p_] := p + r0;
```

b Berechnung von M siehe a

```
ϕSpiegel[p_] := rtr[M.tr[p]]
```

c Spiegelung des Tetraeders {p1,p2,p3,p4}

```
p1 = {1, 5, 2};
p2 = {2, 6, 5};
p3 = {4, 3, 1};
p4 = {-1, 1, 5};

q1 = ϕSpiegel[p1]
```

```
{2, 5, 3}
```

```
q2 = ϕSpiegel[p2]
```

```
{-1, 6, 2}
```

```
q3 = ϕSpiegel[p3]
```

```
{3, 3, 0}
```

```
q4 = ϕSpiegel[p4]
```

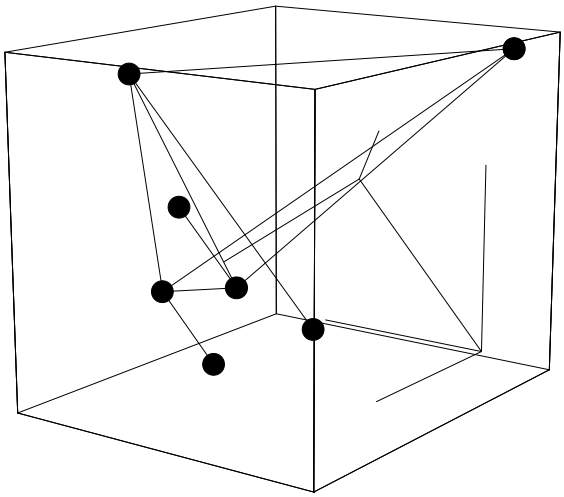
```
{-1, 1, 5}
```

c Visualisierung

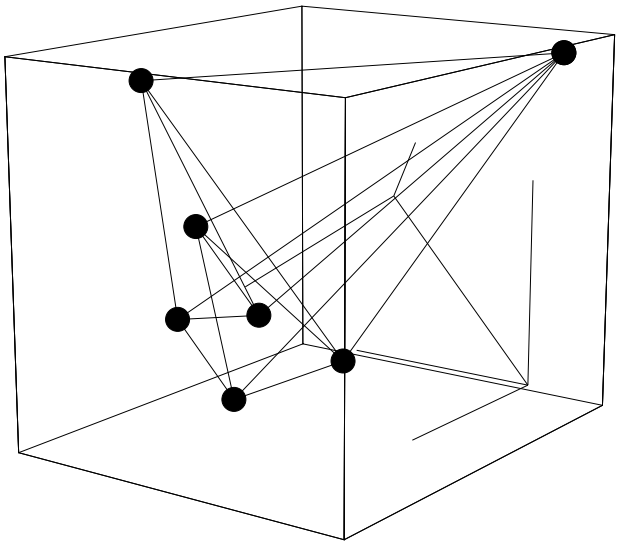
```
OO = {0, 0, 0}
```

```
{0, 0, 0}
```

```
Show[Graphics3D[{
    Line[{OO, {3, 0, 0}}], Line[{OO, {0, 3, 0}}], Line[{OO, {0, 0, 3}}],
    Line[{OO, r0}], Line[{r0, r0 + a}], Line[{r0, r0 + b}],
    PointSize[0.04],
    Point[p1], Point[p2],
    Point[p3], Point[p4],
    Point[q1], Point[q2],
    Point[q3], Point[q4],
    Line[{p1, p2, p3, p1, p4, p2, p3, p4}], (*Line[{q1,q2,q3,q1,q4,q2,q3,q4}],*)
    Line[{p1, q1}], Line[{p2, q2}], Line[{p3, q3}],
    Line[{p4, q4}]
   }], AspectRatio → Automatic, ViewPoint -> {-2.539, 3.502, 1.037}];
```



```
Show[Graphics3D[{
    Line[{OO, {3, 0, 0}}], Line[{OO, {0, 3, 0}}], Line[{OO, {0, 0, 3}}],
    Line[{OO, r0}], Line[{r0, r0 + a}], Line[{r0, r0 + b}],
    PointSize[0.04],
    Point[p1], Point[p2],
    Point[p3], Point[p4],
    Point[q1], Point[q2],
    Point[q3], Point[q4],
    Line[{p1, p2, p3, p1, p4, p2, p3, p4}], Line[{q1, q2, q3, q1, q4, q2, q3, q4}],
    Line[{p1, q1}], Line[{p2, q2}], Line[{p3, q3}],
    Line[{p4, q4}]
   }], AspectRatio → Automatic, ViewPoint -> {-2.539, 3.502, 1.037}];
```



```
Show[Graphics3D[{
    Line[{OO, {3, 0, 0}}], Line[{OO, {0, 3, 0}}], Line[{OO, {0, 0, 3}}],
    Line[{OO, r0}], Line[{r0, r0 + a}], Line[{r0, r0 + b}],
    PointSize[0.04],
    Point[p1], Point[p2],
    Point[p3], Point[p4],
    Point[q1], Point[q2],
    Point[q3], Point[q4],
    Line[{p1, p2, p3, p1, p4, p2, p3, p4}], Line[{q1, q2, q3, q1, q4, q2, q3, q4}],
    Line[{p1, q1}], Line[{p2, q2}], Line[{p3, q3}],
    Line[{p4, q4}]
   }], AspectRatio → Automatic];
```