Lösungen

1

```
Remove["Global`*"]

B = {{1, 2}, {3, 4}}; B // MatrixForm
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

```
mD = {{1, 0}, {0, 2}}; mD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

a

```
B1 = Inverse[B];
B1 // MatrixForm
```

$$\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

b

```
A = B.mD.Inverse[B];
A // MatrixForm
```

$$\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}$$

c

```
Eigenvalues[A]
```

{2, 1}

```
λ1 = Eigenvalues[A][[1]]
```

2

```
λ2 = Eigenvalues[A][[2]]
```

1

d

Die Eigenwerte von A sind die Diagonalelemente von D

e

```
Eigenvectors[A]
```

```
{{1, 2}, {1, 3}}
```

```
x1 = Eigenvectors[A][[1]]
```

```
{1, 2}
```

```
x2 = Eigenvectors[A][[2]]
```

```
{1, 3}
```

```
Eigensystem[A]
```

```
{{2, 1}, {{1, 2}, {1, 3}}}
```

f

Der zweite Eigenvektor ist die erste Spalte von A, der erste Eigenvektor ist bis auf einen Faktor (=0.5) die zweite Spalte von A. Die Eigenvektoren sind nur bis auf einen Faktor bestimmt. Die Reihenfolge hängt von der Auswahl des Betrachters ab.

2

```
Remove["Global`*"]
```

```
B = {{1, 2, 3}, {2, 3, 4}, {3, 4, 4}};  B // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 4 \end{pmatrix}$$

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} -4 & 4 & -1 \\ 4 & -5 & 2 \\ -1 & 2 & -1 \end{pmatrix}$$

```
mD = {{1, 0, 0}, {0, 2, 0}, {0, 0, 3}}; mD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

a

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} -4 & 4 & -1 \\ 4 & -5 & 2 \\ -1 & 2 & -1 \end{pmatrix}$$

b

```
A = B.mD.Inverse[B];
A // MatrixForm
```

$$\begin{pmatrix} 3 & 2 & -2 \\ 4 & 2 & -2 \\ 8 & -4 & 1 \end{pmatrix}$$

c

```
IdentityMatrix[3] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
CharacteristicPolynomial[A, λ]
```

$6 - 11\lambda + 6\lambda^2 - \lambda^3$

```
Det[A - λ IdentityMatrix[3]]
```

$6 - 11\lambda + 6\lambda^2 - \lambda^3$

```
Det[mD - λ IdentityMatrix[3]]
```

$6 - 11\lambda + 6\lambda^2 - \lambda^3$

```
Det[A - λ IdentityMatrix[3]] == Det[mD - λ IdentityMatrix[3]]
```

True

d

```
Eigensystem[A]
```

{{3, 2, 1}, {{3, 4, 4}, {2, 3, 4}, {1, 2, 3}}}

```
Eigenvalues[A]
```

{3, 2, 1}

```
Eigenvectors[A]
```

{{3, 4, 4}, {2, 3, 4}, {1, 2, 3}}

```
Eigensystem[mD]
{{3, 2, 1}, {{0, 0, 1}, {0, 1, 0}, {1, 0, 0}}}
Eigenvalues[mD]
{3, 2, 1}
Eigenvectors[mD]
{{0, 0, 1}, {0, 1, 0}, {1, 0, 0}}
```

Die Eigenwektoren von mD sind die Vektoren in den Achsenrichtungen (hier auf 1 normiert).

e

```
sum = Apply[Plus, Eigenvalues[A]]
6
CoefficientList[CharacteristicPolynomial[A, λ], λ]
{6, -11, 6, -1}
CoefficientList[CharacteristicPolynomial[A, λ], λ] // Reverse
{-1, 6, -11, 6}
Reverse[CoefficientList[CharacteristicPolynomial[A, λ], λ]][[2]]
6
```

f

```
Det[A]
6
Det[mD]
6
Reverse[CoefficientList[CharacteristicPolynomial[A, λ], λ]] // Last
6
```

g

```
CharacteristicPolynomial[A, λ] // Factor
-(-3 + λ) (-2 + λ) (-1 + λ)
CharacteristicPolynomial[A, λ] /. λ → 0
6
```

```
Apply[Times, Eigenvalues[A]]
```

```
6
```

```
Apply[Times, Eigenvalues[mD]]
```

```
6
```

Die Eigenwerte von A und von mD sind dieselben. Das Produkt der Eigenwerte von mD erkennt man als Wert des charakteristischen Polynoms an der Stelle 0. Dieser Wert ist also der letzte (der konstante) Koeffizient des charakteristischen Polynoms und daher auch die Determinante von A oder von mD. Diese wird ja erzeugt, wenn man in Det(A- E) oder in Det(mD- E) das   gleich 0 setzt.

3

```
Remove["Global`*"]
```

```
xVec[x_, y_] := {x, y};
mVec = {4, 3}; r = 2;
kreis[rVec_, r_] := rVec.rVec - r^2;
```

a) Pol und Polare

```
pol = {0, 0};
kreis[(xVec[x, y] - mVec), r] == 0
```

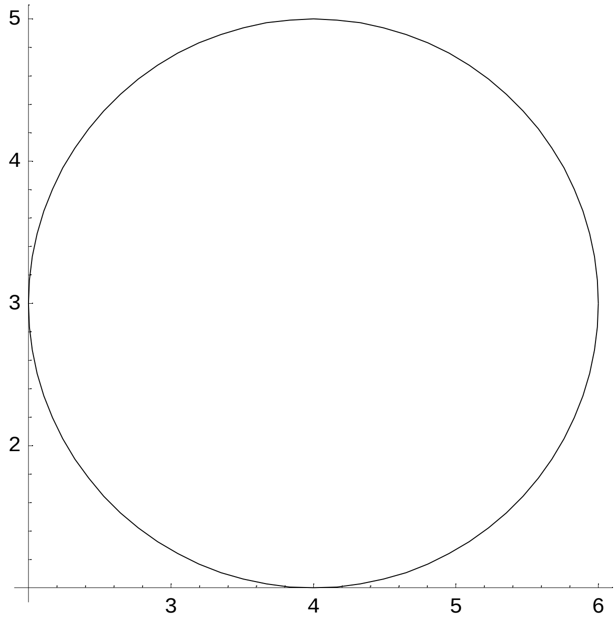
$-4 + (-4 + x)^2 + (-3 + y)^2 == 0$

```
Expand[kreis[(xVec[x, y] - mVec), r]] == 0
```

$21 - 8x + x^2 - 6y + y^2 == 0$

```
<< Graphics`ImplicitPlot`
```

```
kPl = ImplicitPlot[kreis[(xVec[x, y] - mVec), r] == 0, {x, 0, 7}];
```



```
polare[xVec_, mVec_, pol_, r_] := (xVec[x, y] - mVec).(pol - mVec) - r^2;
polare[xVec, mVec, pol, r] == 0
```

$-4 - 4 (-4 + x) - 3 (-3 + y) == 0$

```
Simplify[polare[xVec, mVec, pol, r]] == 0
```

$21 - 4 x - 3 y == 0$

```
solv = Solve[{polare[xVec, mVec, pol, r] == 0, kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}]
```

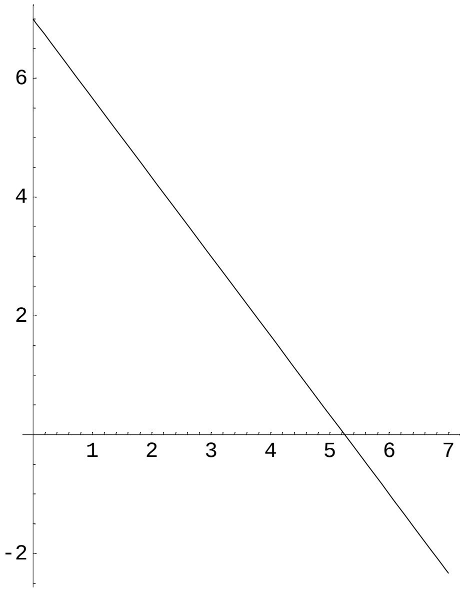
$\left\{\left\{x \to \frac{6}{25} \left(14 - \sqrt{21}\right), y \to \frac{1}{25} \left(63 + 8 \sqrt{21}\right)\right\}, \left\{x \to \frac{6}{25} \left(14 + \sqrt{21}\right), y \to \frac{1}{25} \left(63 - 8 \sqrt{21}\right)\right\}\right\}$

```
solv1 = solv // N
```

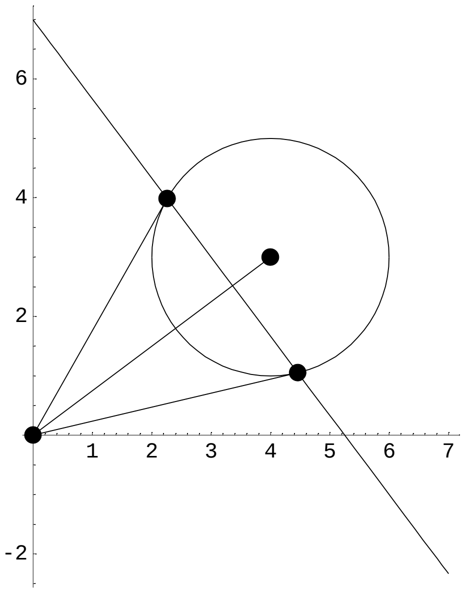
{{x → 2.26018, y → 3.98642}, {x → 4.45982, y → 1.05358}}

```
pT1 = {x, y} /. solv1[[1]];
pT2 = {x, y} /. solv1[[2]];
```

```
polarePl = ImplicitPlot[polare[xVec, mVec, pol, r] == 0, {x, 0, 7}];
```




```
Show[kPl, polarePl, Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2],
    Point[pT1], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



b) Tangente

```
tangente[xVec_, mVec_, pT_, r_] := (xVec[x, y] - mVec).(pT - mVec) - r^2;
```

```
Chop[Expand[polare[xVec, mVec, pT1, r]]] == 0
```

```
-1.73982 x + 0.986424 y == 0
```

```
Chop[Expand[polare[xVec, mVec, pT1, r]]] == 0 /. {x -> 0, y → 0}
```

```
True
```

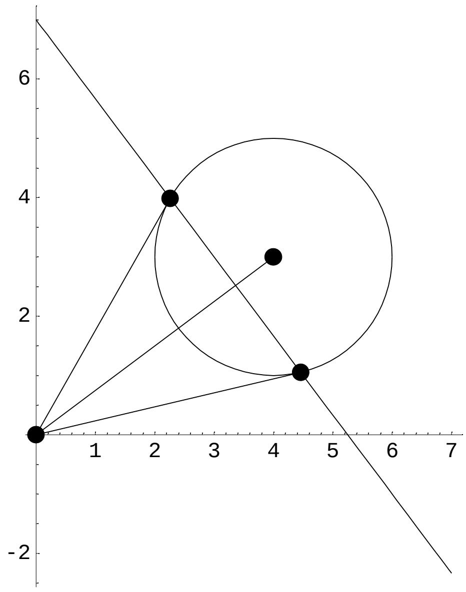
```
Chop[Expand[polare[xVec, mVec, pT2, r]]] == 0
```

```
0.459818 x - 1.94642 y == 0
```

```
Chop[Expand[polare[xVec, mVec, pT2, r]]] == 0 /. {x -> 0, y → 0}
```

```
True
```

```
Show[kPl, polarePl, Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2],
    Point[pT1], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



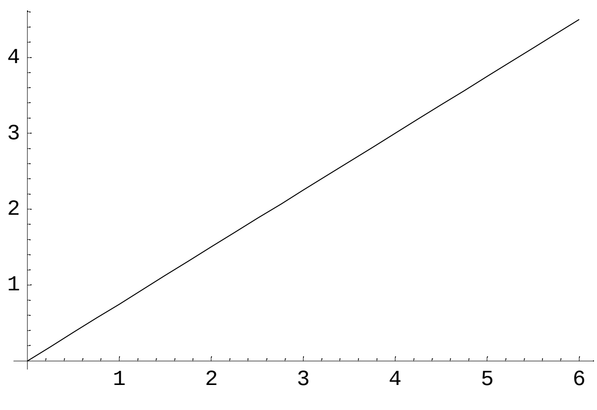
c) Mittelpunktsgerade zum Pol

```
senkr[v_] := {-v[[2]], v[[1]]};
geradePolM[xVec_, mVec_, pol_] := (xVec - mVec).senkr[pol - mVec];
```

```
Expand[geradePolM[xVec[x, y], mVec, pol]] == 0
```

```
3 x - 4 y == 0
```

```
yGer[x_] := 3 x / 4;
gerPl = Plot[yGer[x], {x, 0, 6}];
```



```
Solve[{geradePolM[xVec[x, y], mVec, pol] == 0, kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}]
```

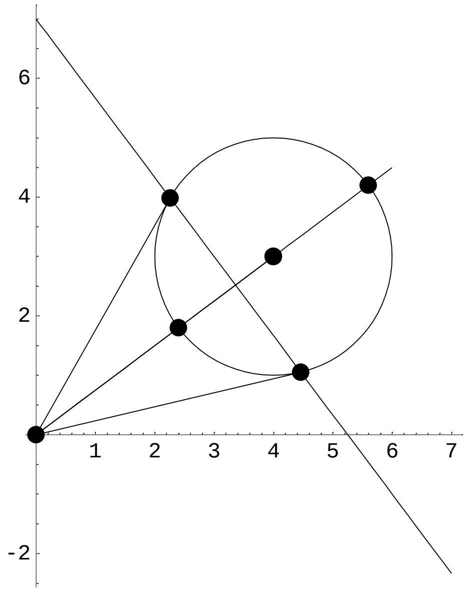
$\{\{x \to \frac{12}{5}, y \to \frac{9}{5}\}, \{x \to \frac{28}{5}, y \to \frac{21}{5}\}\}$

```
solv2 = Solve[{geradePolM[xVec[x, y], mVec, pol] == 0,
    kreis[(xVec[x, y] - mVec), r] == 0}, {x, y}] // N
```

{{x → 2.4, y → 1.8}, {x → 5.6, y → 4.2}}

```
p1 = {x, y} /. solv2[[1]];
p2 = {x, y} /. solv2[[2]];
```

```
Show[kPl, polarePl, gerPl,
  Graphics[{PointSize[0.04], Point[pol], Point[mVec], Point[pT2], Point[pT1],
    Point[p1], Point[p2], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



d) Apollonius

```
Norm[p2 - pol] / Norm[p1 - pol]
```

2.33333

```
Rationalize[Norm[p2 - pol] / Norm[p1 - pol]]
```

$\frac{7}{3}$

```
Norm[p1 - pol] / Norm[p1 - {x, yGer[x]}] == Norm[p2 - pol] / Norm[p2 - {x, yGer[x]}]
```

$$\frac{3.}{\sqrt{\text{Abs}[2.4 - x]^2 + \text{Abs}[1.8 - \frac{3x}{4}]^2}} == \frac{7.}{\sqrt{\text{Abs}[5.6 - x]^2 + \text{Abs}[4.2 - \frac{3x}{4}]^2}}$$

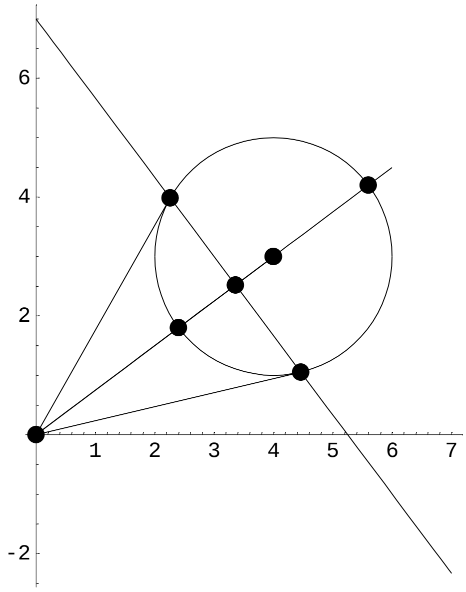
```
solv3 = Solve[(Norm[p1 - pol] / Norm[p1 - {x, yGer[x]}])^2 ==
   (Norm[p2 - pol] / Norm[p2 - {x, yGer[x]}])^2, {x}]
```

{{x → 0.}, {x → 3.36}}

```
pol2 = {x, yGer[x]} /. solv3[[2]]
```

{3.36, 2.52}

```
Show[kPl, polarePl, gerPl, Graphics[
   {PointSize[0.04], Point[pol], Point[mVec], Point[pT2], Point[pT1], Point[p1],
    Point[p2], Point[pol2], Line[{pol, mVec}], Line[{pol, pT1}], Line[{pol, pT2}]}]];
```



e) Potenzgerade

```
kreis1 = (kreis[(xVec[x, y] - mVec), r] == 0)
```

$-4 + (-4 + x)^2 + (-3 + y)^2 == 0$

```
kreis2 = (kreis[(xVec[x, y] - pol), 3 r] == 0)
```

$-36 + x^2 + y^2 == 0$

```
potenz1[x_, y_] := kreis[(xVec[x, y] - mVec), r];
potenz2[x_, y_] := kreis[(xVec[x, y] - pol), 3 r];
```

```
Expand[potenz1[x, y]] == potenz2[x, y]
```

$21 - 8 x + x^2 - 6 y + y^2 == -36 + x^2 + y^2$

```
? Reduce
```

Reduce[expr, vars] reduces the statement expr by solving equations or inequalities for vars and eliminating quantifiers. Reduce[expr, vars, dom] does the reduction over the domain dom. Common choices of dom are Reals, Integers and Complexes. Mehr…

```
Reduce[Expand[potenz1[x, y]] == potenz2[x, y], {x, y}]
```

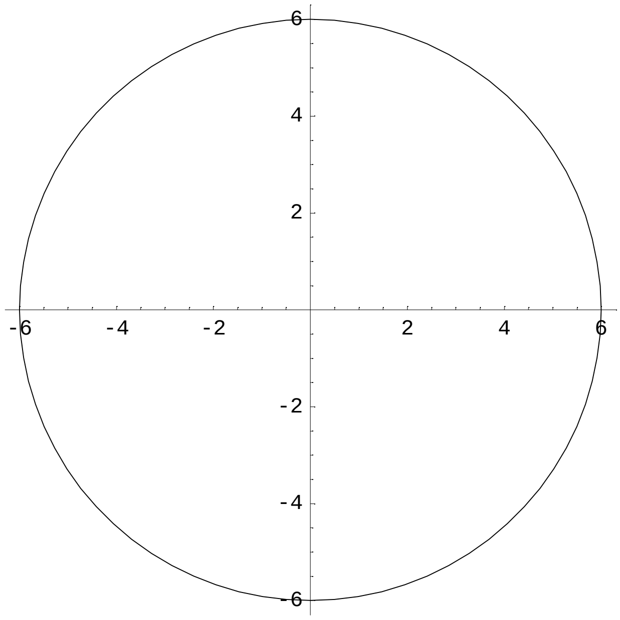
$y == \frac{19}{2} - \frac{4 x}{3}$

```
solv4 = Solve[Reduce[Expand[potenz1[x, y]] == potenz2[x, y], {x, y}], {y}] // Flatten
```

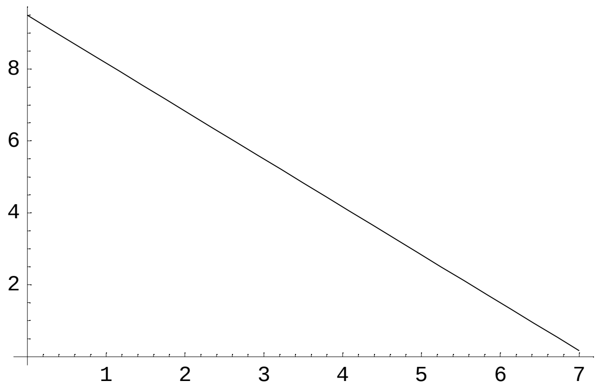
$\left\{y \to \frac{1}{6}\,(57 - 8\,x)\right\}$

```
potenzGer[x_] := y /. solv4
```

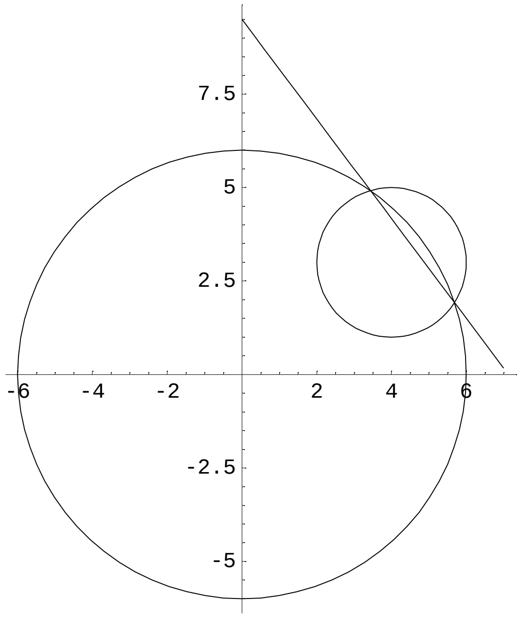
```
kPl2 = ImplicitPlot[kreis2, {x, -6, 7}];
```




```
plPotGer = Plot[potenzGer[x], {x, 0, 7}];
```



```
Show[kPl2, plPotGer, kPl];
```



Die Potenzgerade geht durch die Schnittpunkte der beiden Kreise.

4

```
Remove["Global`*"]

pA = {1, 3, 2}; pB = {4, 1, 3}; pQ = {7, 7, 7};

g[t_] := pA + t (pB - pA);  n = (pB - pA)
g[t]

{3, -2, 1}

{1 + 3 t, 3 - 2 t, 2 + t}

kG[x_, y_, z_, d_] := n.{x, y, z} + d;
kG[p_, d_] := n.p + d;
kG[x, y, z, d_] == 0

3 x - 2 y + z + d_ == 0

kG[pQ, d] == 0

14 + d == 0

solv1 = Solve[kG[pQ, d] == 0, {d}] // Flatten

{d → -14}

kG[x_, y_, z_] := n.{x, y, z} + d /. solv1;
kG[p_] := n.p + d /. solv1;

Expand[kG[g[t]]] == 0

-15 + 14 t == 0
```

```
solv2 = Solve[Expand[kG[g[t]]] == 0, {t}] // Flatten
```

$\{t \to \frac{15}{14}\}$

```
pL = g[t] /. solv2
```

$\{\frac{59}{14}, \frac{6}{7}, \frac{43}{14}\}$

```
% // N
```

{4.21429, 0.857143, 3.07143}

5

```
Remove["Global`*"]
```

```
2 x - y + 3 z - 1 == 0; pQ = {7, 7, 7};
```

```
n = {2, -1, 3};
```

```
kG[x_, y_, z_] := n.{x, y, z} - 1; kG[x, y, z]
```

$-1 + 2 x - y + 3 z$

```
kG[p_] := n.p - 1;
```

```
g[t_] := pQ + t n; g[t]
```

{7 + 2 t, 7 - t, 7 + 3 t}

```
Expand[kG[g[t]]] == 0
```

$27 + 14 t == 0$

```
solv = Solve[Expand[kG[g[t]]] == 0, {t}] // Flatten
```

$\{t \to -\frac{27}{14}\}$

```
pL = g[t] /. solv
```

$\{\frac{22}{7}, \frac{125}{14}, \frac{17}{14}\}$

```
% // N
```

{3.14286, 8.92857, 1.21429}