

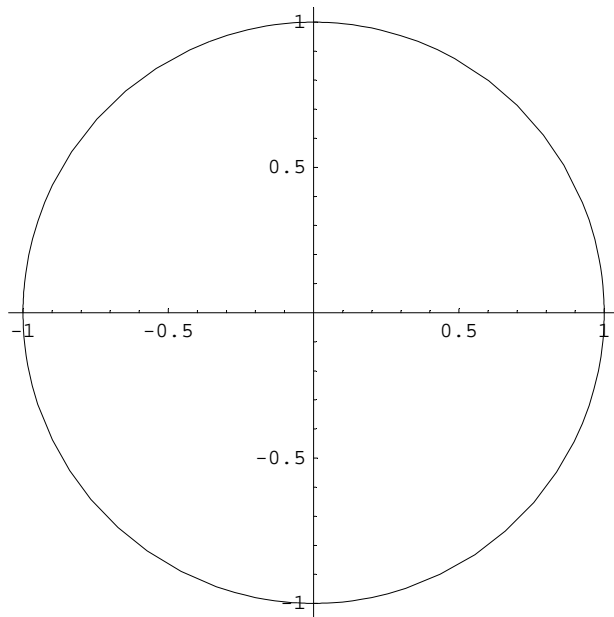
Lösungen

1

a

a1

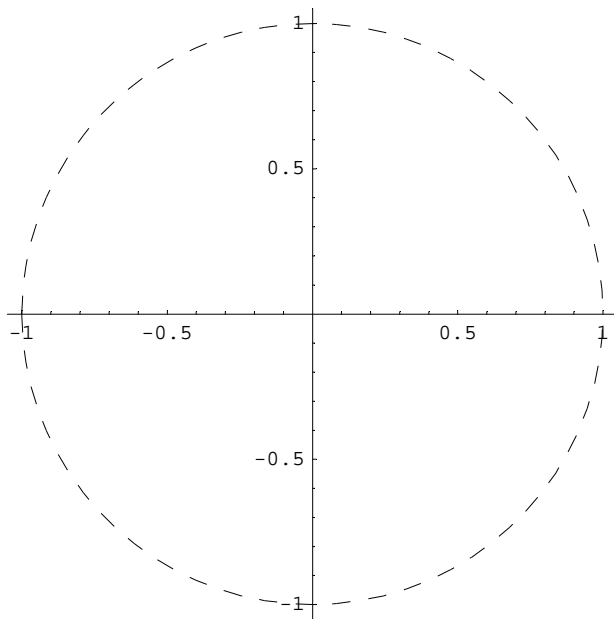
```
r = 1;  
xVec0[t_] := r {Cos[t], Sin[t]};  
  
ParametricPlot[xVec0[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```



a2

```
<< Graphics`ImplicitPlot`  
  
fig0 = ({x, y} . {x, y} - r^2 == 0);
```

```
ImplicitPlot[fig0, {x, -1, 1}, PlotStyle -> {GrayLevel[0], Dashing[ {.03} ]}];
```

**b**

```
D1 = {{1, 0}, {0, 2}}; D1 // MatrixForm
```

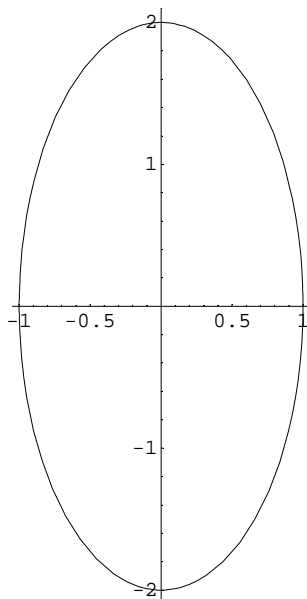
$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

b1

```
xVec1a[t_] := D1.{Cos[t], Sin[t]}; xVec1a[t]
```

```
{Cos[t], 2 Sin[t]}
```

```
ParametricPlot[xVec1a[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```

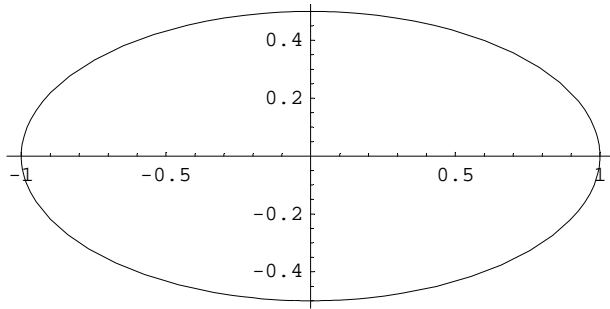


b2

```

xVec1b[t_] := Inverse[D1].{Cos[t], Sin[t]}; xVec1b[t]
{Cos[t],  $\frac{\text{Sin}[t]}{2}$ }
ParametricPlot[xVec1b[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];

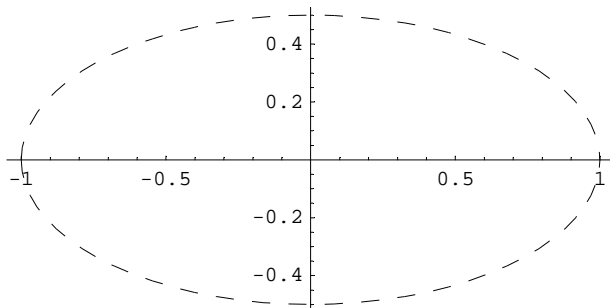
```

**b3**

```

(D1.{x, y}).(D1.{x, y}) - r^2 == 0
-1 + x^2 + 4 y^2 == 0
fig1 = ((D1.{x, y}).(D1.{x, y}) - r^2 == 0);
ImplicitPlot[fig1, {x, -1, 1}, PlotStyle -> {GrayLevel[0], Dashing[ {.03} ]}];

```

**c**

```

φ = -Pi / 6;
M = {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}};
M // MatrixForm

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

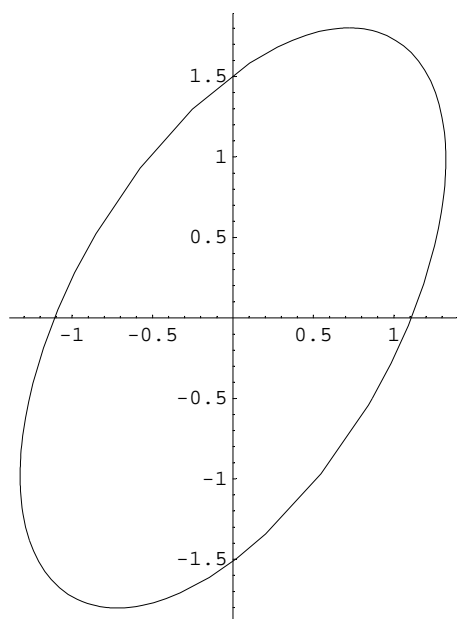
c1

```

xVec2a[t_] := M.xVec1a[t]; xVec2a[t]
{  $\frac{1}{2} \sqrt{3} \text{Cos}[t] + \text{Sin}[t]$ ,  $-\frac{\text{Cos}[t]}{2} + \sqrt{3} \text{Sin}[t]$  }

```

```
ParametricPlot[xVec2a[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```



```
Eigensystem[M.D1]
```

```
{{1/4 (3 sqrt(3) + i sqrt(5)), 1/4 (3 sqrt(3) - i sqrt(5))}, {{1/2 (sqrt(3) - i sqrt(5)), 1}, {1/2 (sqrt(3) + i sqrt(5)), 1}}}
```

```
Eigensystem[M.D1] // N
```

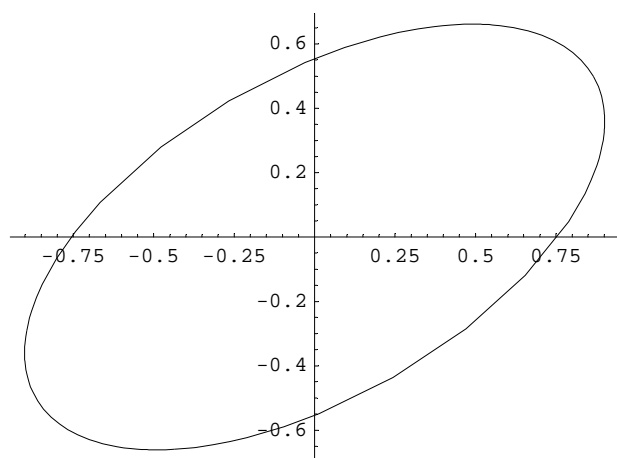
```
{{1.29904 + 0.559017 i, 1.29904 - 0.559017 i},  
{0.866025 - 1.11803 i, 1.}, {0.866025 + 1.11803 i, 1.}}
```

c2

```
xVec2b[t_] := Inverse[M].xVec1b[t]; xVec2b[t]
```

```
{1/2 sqrt(3) Cos[t] - Sin[t]/4, Cos[t]/2 + 1/4 sqrt(3) Sin[t]}
```

```
ParametricPlot[xVec2b[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```



```
Eigensystem[Inverse[M].Inverse[D1]]
```

```
{{1/8 (3 sqrt(3) + i sqrt(5)), 1/8 (3 sqrt(3) - i sqrt(5))}, {{1/4 (sqrt(3) + i sqrt(5)), 1}, {1/4 (sqrt(3) - i sqrt(5)), 1}}}
```

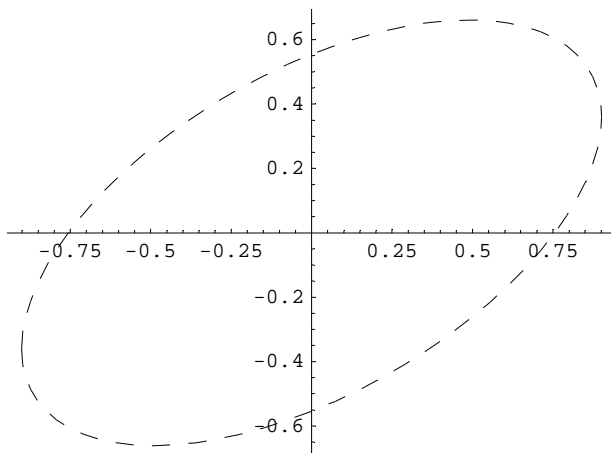
```
Eigensystem[Inverse[M].Inverse[D1]] // N
{{0.649519 + 0.279508 i, 0.649519 - 0.279508 i},
 {{0.433013 + 0.559017 i, 1.}, {0.433013 - 0.559017 i, 1.}}
```

c3

```
fig2 = ((D1.M.{x, y}).(D1.M.{x, y}) - r^2 == 0)
```

$$-1 + \left(\frac{\sqrt{3} x}{2} + \frac{y}{2} \right)^2 + (-x + \sqrt{3} y)^2 = 0$$

```
ImplicitPlot[fig2, {x, -1, 1}, PlotStyle -> {GrayLevel[0], Dashing[ {.03} ]}];
```



```
Eigensystem[D1.M]
```

```
{{1/4 (3 sqrt(3) + i sqrt(5)), 1/4 (3 sqrt(3) - i sqrt(5))}, {{1/4 (sqrt(3) - i sqrt(5)), 1}, {1/4 (sqrt(3) + i sqrt(5)), 1}}}
```

```
Eigensystem[D1.M] // N
```

```
{{1.29904 + 0.559017 i, 1.29904 - 0.559017 i},
 {{0.433013 - 0.559017 i, 1.}, {0.433013 + 0.559017 i, 1.}}
```