Lösungen (Matrixkomposition)

1

a

```
λ1 = 1;  λ2 = -1;  λ3 = 3;
v1 = {1, 0, 0}; v2 = {1, 1, 0};  v3 = {0, 1, -1};
```

a1

```
X = Transpose[{v1, v2, v3}]; X // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

a2

```
Det[X]
```

-1

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a3

```
Dλ = {{λ1, 0, 0}, {0, λ2, 0}, {0, 0, λ3}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
Xinv = Inverse[X]; Xinv // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} 1 & -2 & -2 \\ 0 & -1 & -4 \\ 0 & 0 & 3 \end{pmatrix}$$

a4

```
Eigenvalues[A]
```

{3, -1, 1}

```
Eigenvectors[A]
```

{{0, -1, 1}, {1, 1, 0}, {1, 0, 0}}

```
Eigensystem[A]
```

{{3, -1, 1}, {{0, -1, 1}, {1, 1, 0}, {1, 0, 0}}}

a5

```
Det[A] == Det[Dλ]
```

True

b

```
A1 = Inverse[A]; A1 // MatrixForm
```

$$\begin{pmatrix} 1 & -2 & -2 \\ 0 & -1 & -\frac{4}{3} \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

```
Eigenvalues[A1]
```

$\left\{-1, 1, \frac{1}{3}\right\}$

```
Eigenvectors[A1]
```

{{1, 1, 0}, {1, 0, 0}, {0, -1, 1}}

```
Eigensystem[A1]
```

$\left\{\left\{-1, 1, \frac{1}{3}\right\}, \{\{1, 1, 0\}, \{1, 0, 0\}, \{0, -1, 1\}\}\right\}$

```
Det[A1] == Det[Inverse[Dλ]]
```

True

b

```
A2 = Transpose[A]; A2 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -2 & -4 & 3 \end{pmatrix}$$

```
Eigenvalues[A2]
```

```
{3, -1, 1}
```

```
Eigenvectors[A2]
```

```
{{0, 0, 1}, {0, 1, 1}, {-1, 1, 1}}
```

```
Eigensystem[A2]
```

```
{{3, -1, 1}, {{0, 0, 1}, {0, 1, 1}, {-1, 1, 1}}}
```

```
Det[A2] == Det[Dλ]
```

```
True
```

2

a

```
λ1 = 1;  λ2 = -1;  λ3 = 3;
v1 = {1/2, 0, 1/2}; v2 = {1, 1, 0};  v3 = {0, 1, 1};
```

a1

```
X = Transpose[{v1, v2, v3}]; X // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$$

a2

```
Det[X]
```

```
1
```

Vektoren l.u.

a3

```
Dλ = {{λ1, 0, 0}, {0, λ2, 0}, {0, 0, λ3}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
Xinv = Inverse[X]; Xinv // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -2 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix}$$

a4

```
Eigenvalues[A]
```

{3, -1, 1}

```
Eigenvectors[A]
```

{{0, 1, 1}, {1, 1, 0}, {1, 0, 1}}

```
Eigensystem[A]
```

{{3, -1, 1}, {{0, 1, 1}, {1, 1, 0}, {1, 0, 1}}}

a5

```
Det[A] == Det[Dλ]
```

True

b

```
A1 = Inverse[A]; A1 // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

```
Eigenvalues[A1]
```

$\left\{-1, 1, \frac{1}{3}\right\}$

```
Eigenvectors[A1]
```

{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}

```
Eigensystem[A1]
```

$\left\{\left\{-1, 1, \frac{1}{3}\right\}, \{\{1, 1, 0\}, \{1, 0, 1\}, \{0, 1, 1\}\}\right\}$

```
Det[A1] == Det[Inverse[Dλ]]
```

True

b

```
A2 = Transpose[A]; A2 // MatrixForm
```

$$\begin{pmatrix} 0 & -2 & -1 \\ -1 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

```
Eigenvalues[A2]
```

```
{3, -1, 1}
```

```
Eigenvectors[A2]
```

```
{{-1, 1, 1}, {-1, -1, 1}, {1, -1, 1}}
```

```
Eigensystem[A2]
```

```
{{3, -1, 1}, {{-1, 1, 1}, {-1, -1, 1}, {1, -1, 1}}}
```

```
Det[A2] == Det[Dλ]
```

```
True
```