Lösungen

1

```
Remove["Global`*"]
```

a

```
X = Transpose[{{2, 1}, {-1, 2}}]; X // MatrixForm
```

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

```
Dλ = {{2, 0}, {0, 1}}; Dλ // MatrixForm
```

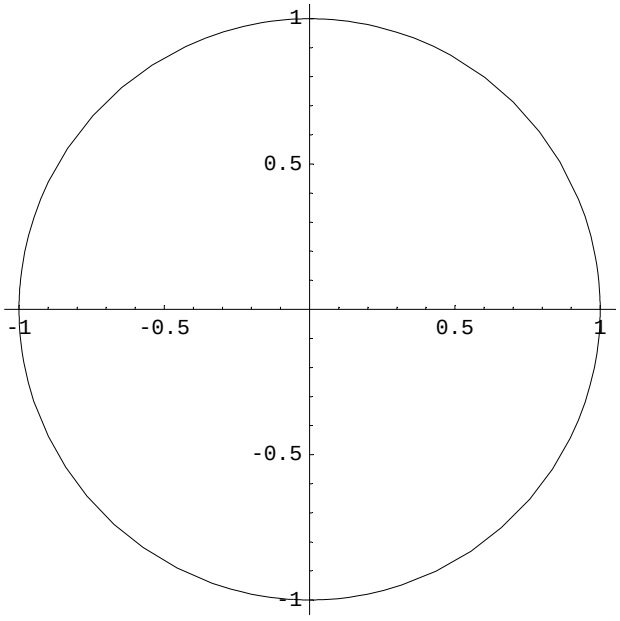
$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} \frac{9}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{6}{5} \end{pmatrix}$$

b

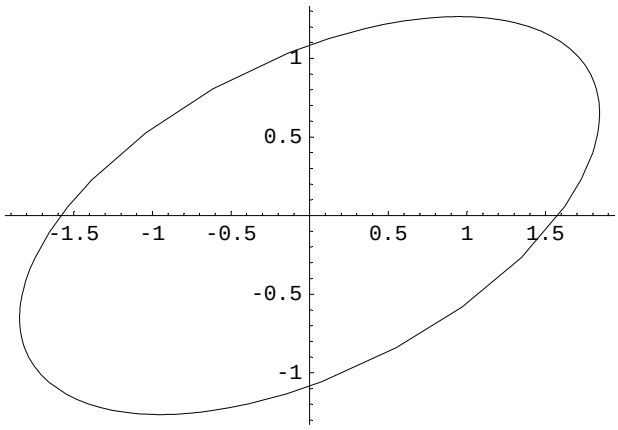
```
v1[t_] := {Cos[t], Sin[t]};
pl1 = ParametricPlot[v1[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```



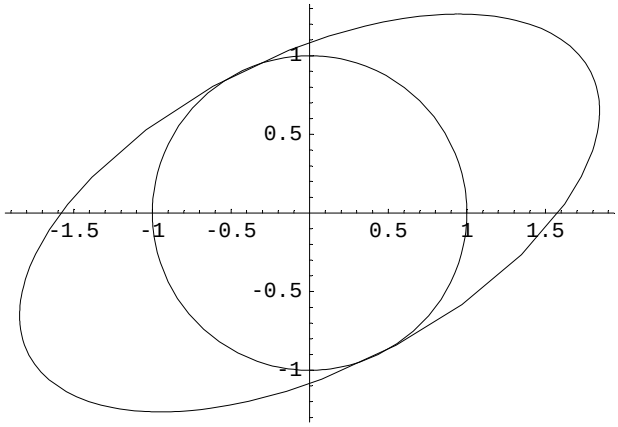
```
v2[t_] := A. v1[t]; v2[t]
```

$$\left\{\frac{9\,\text{Cos}[t]}{5}+\frac{2\,\text{Sin}[t]}{5},\ \frac{2\,\text{Cos}[t]}{5}+\frac{6\,\text{Sin}[t]}{5}\right\}$$

```
pl2 = ParametricPlot[v2[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```




```
Show[pl1, pl2];
```

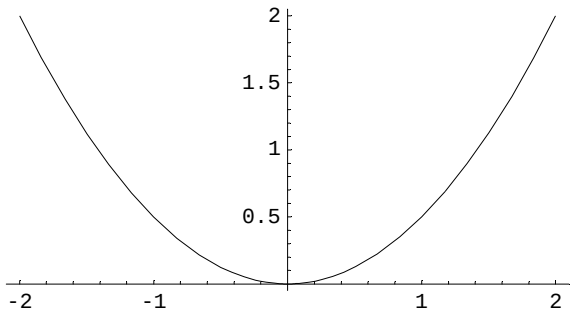


C

```
v3[t_] := {t, 1 / 2 t^2}; v3[t]
```

$$\left\{t,\ \frac{t^2}{2}\right\}$$

```
pl3 = ParametricPlot[v3[t], {t, -2, 2}, AspectRatio → Automatic];
```



```
Dreh[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; Dreh[φ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

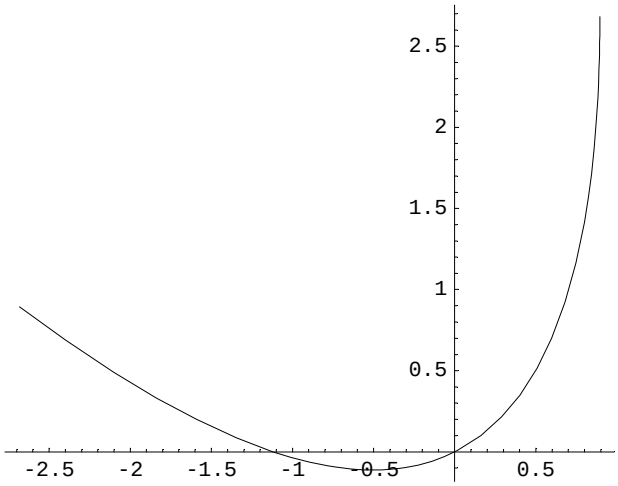
```
φ = ArcTan[1 / 2]
```

$$\text{ArcTan}\left[\frac{1}{2}\right]$$

```
v4[t_] := Dreh[φ].v3[t] ; v4[t]
```

$$\left\{\frac{2\,t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}},\ \frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}}\right\}$$

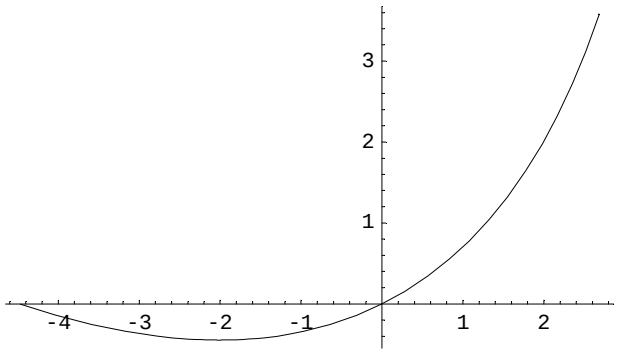
```
pl4 = ParametricPlot[v4[t], {t, -2, 2}, AspectRatio → Automatic];
```



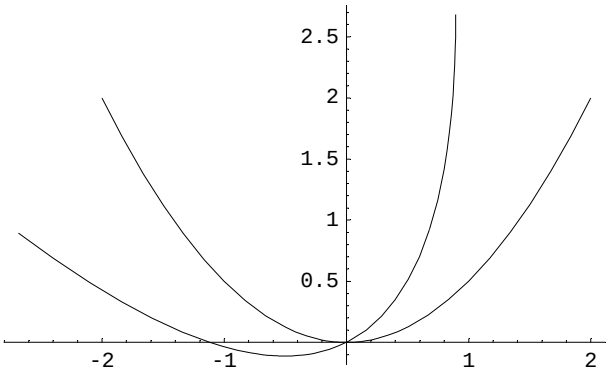

```
v5[t_] := A.v4[t]; v5[t]
```

$$\left\{\frac{9}{5}\left(\frac{2\,t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}}\right) + \frac{2}{5}\left(\frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}}\right),\ \frac{2}{5}\left(\frac{2\,t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}}\right) + \frac{6}{5}\left(\frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}}\right)\right\}$$

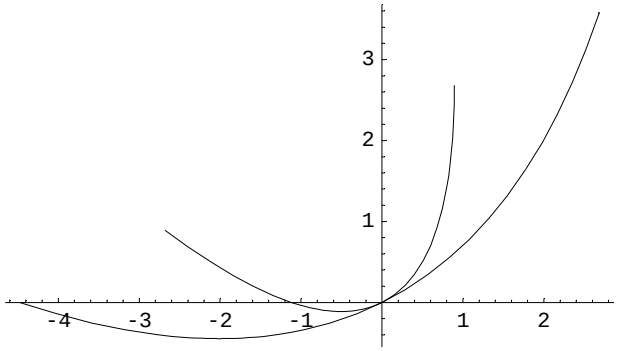
```
pl5 = ParametricPlot[v5[t], {t, -2, 2}, AspectRatio → Automatic];
```



```
Show[pl3, pl4];
```




```
Show[pl4, pl5];
```



2

```
Remove["Global`*"]
```

a

```
X[φ_] := Transpose[{{Cos[φ], Sin[φ]}, {-Sin[φ], Cos[φ]}}]; X[φ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

```
λ2 = 1.5;
Dλ[λ_] := {{λ, 0}, {0, λ2}}; Dλ[λ] // MatrixForm
```

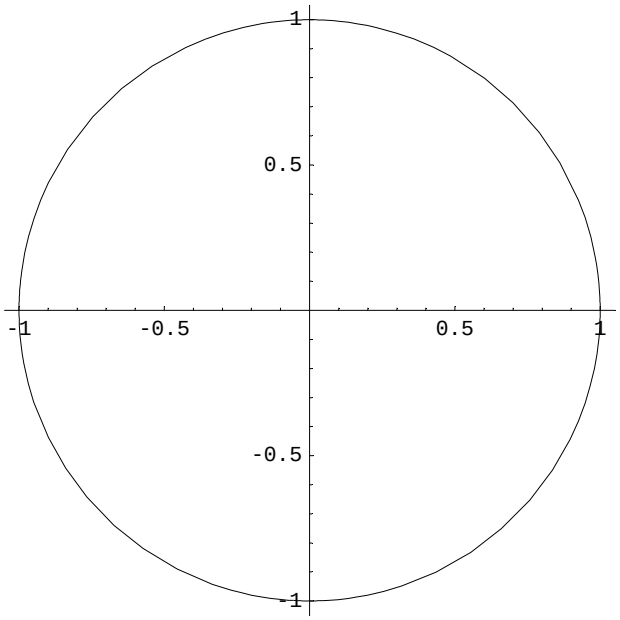
$$\begin{pmatrix} \lambda & 0 \\ 0 & 1.5 \end{pmatrix}$$

```
A[φ_, λ_] := X[φ].Dλ[λ].Inverse[X[φ]]; A[φ, λ] // MatrixForm
```

$$\begin{pmatrix} \frac{\lambda \text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} + \frac{1.5\,\text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} & -\frac{1.5\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} + \frac{\lambda\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} \\ -\frac{1.5\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} + \frac{\lambda\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} & \frac{1.5\,\text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} + \frac{\lambda\,\text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2} \end{pmatrix}$$

b

```
v1[t_] := {Cos[t], Sin[t]};
pl1 = ParametricPlot[v1[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```

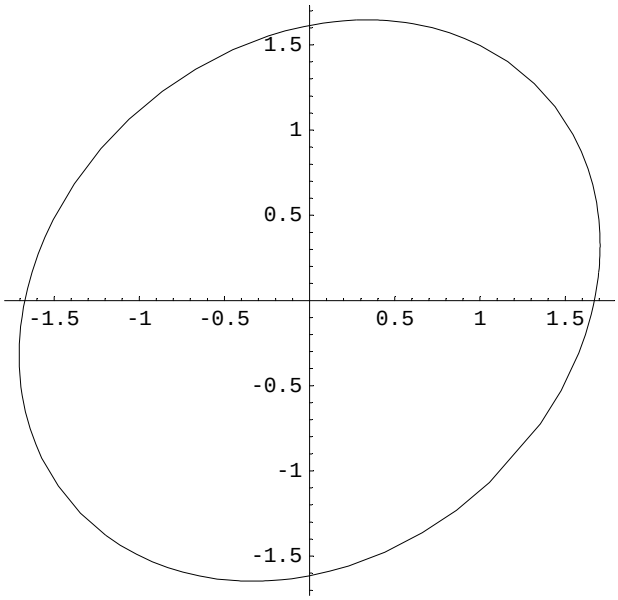



```
v2[t_, φ_, λ_] := A[φ, λ].v1[t];  v2[t, φ, λ]
```

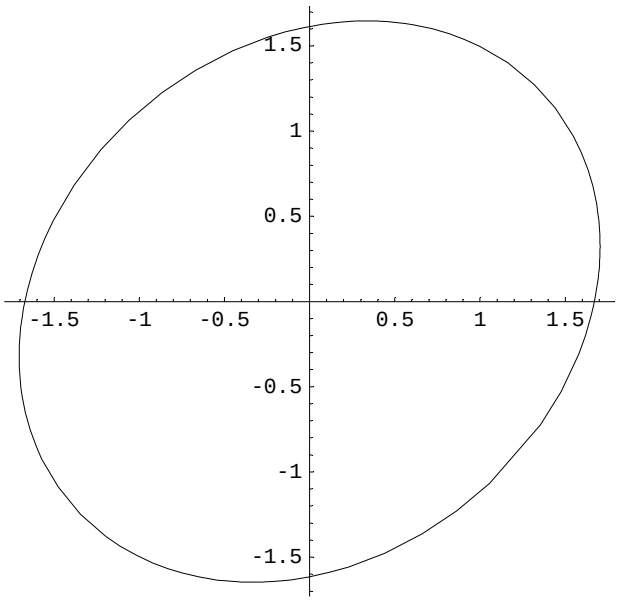
$$\left\{\text{Sin}[t]\left(-\frac{1.5\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}+\frac{\lambda\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}\right)+\right.$$
$$\text{Cos}[t]\left(\frac{\lambda\,\text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}+\frac{1.5\,\text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}\right),$$
$$\text{Cos}[t]\left(-\frac{1.5\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}+\frac{\lambda\,\text{Cos}[\varphi]\,\text{Sin}[\varphi]}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}\right)+$$
$$\left.\text{Sin}[t]\left(\frac{1.5\,\text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}+\frac{\lambda\,\text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2+\text{Sin}[\varphi]^2}\right)\right\}$$

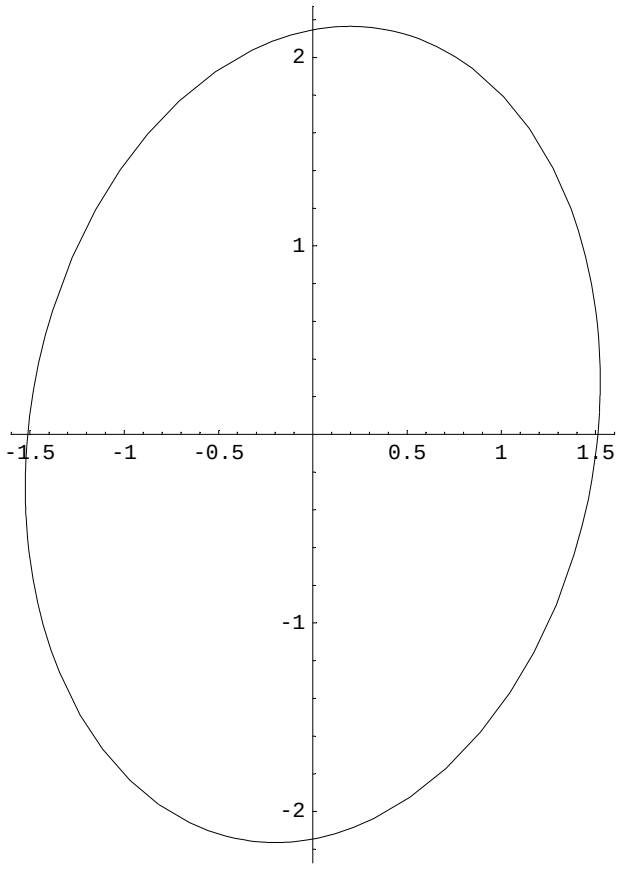
```
pl2[φ_, λ_] :=
  ParametricPlot[Evaluate[v2[t, φ, λ]], {t, 0, 2 Pi}, AspectRatio → Automatic];
```

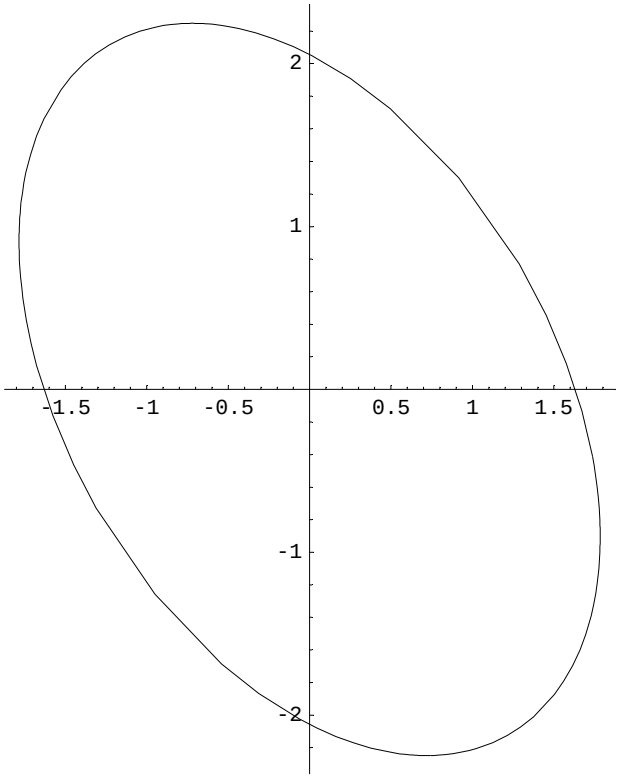
```
pl2[2 Pi / 9, 1 + Sqrt[2 Pi / 9]];
```

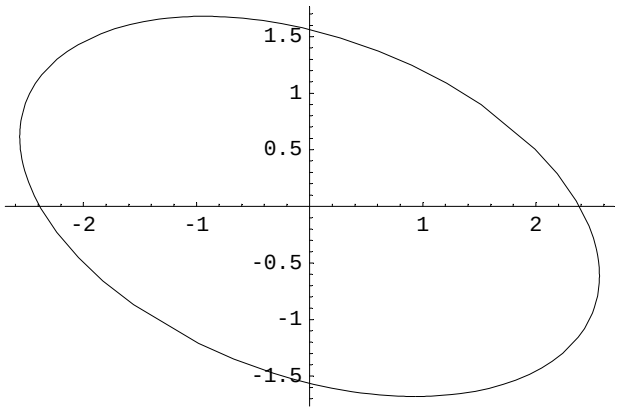


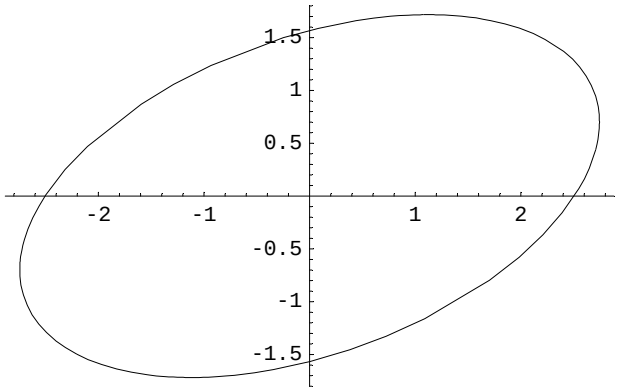

```
Show[Table[pl2[φ, 1 + Sqrt[φ]], {φ, 2 Pi / 9, 2 Pi, 2 Pi / 9}]];
```

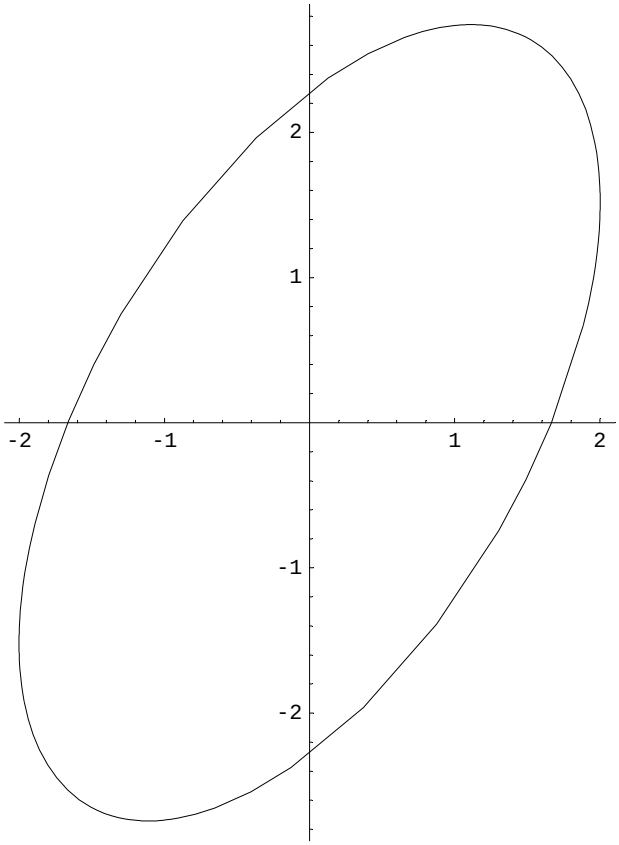


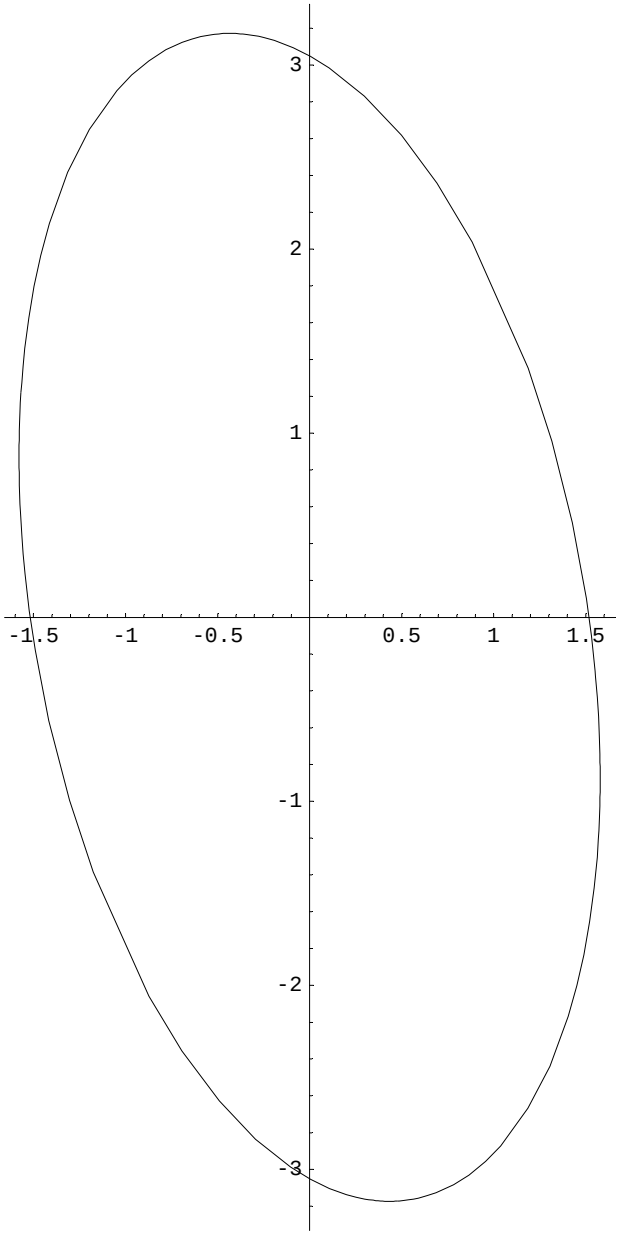

2
1
-1
-2
-1.5
-1
-0.5
0.5
1
1.5

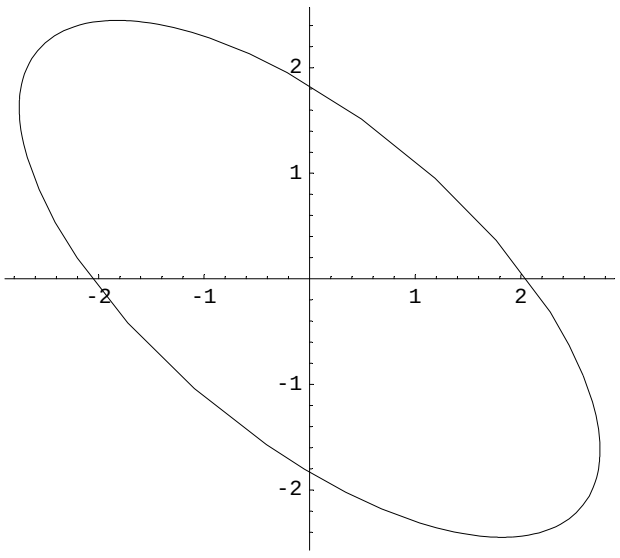

2
1
-1
-2
-1.5
-1
-0.5
0.5
1
1.5

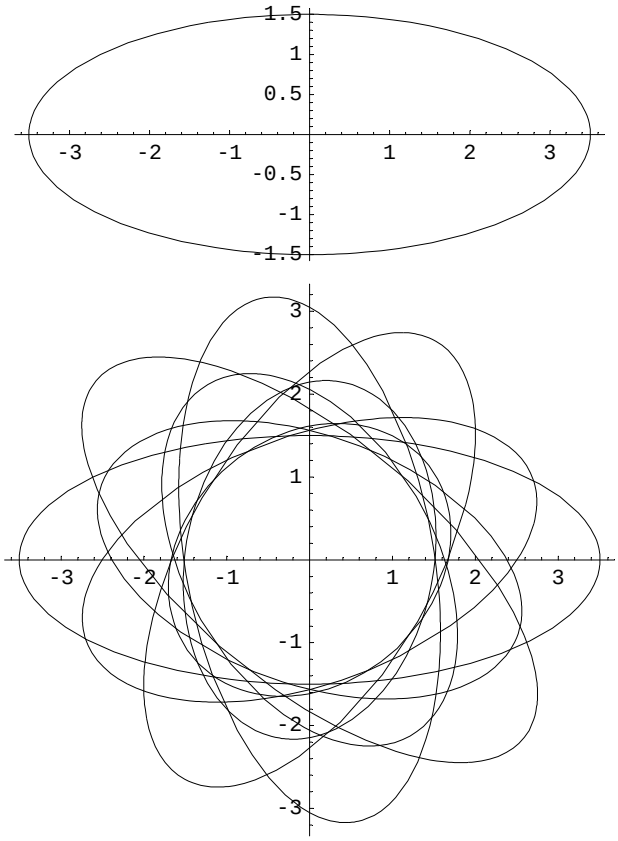

1.5
1
0.5
-2
-1
1
2
-0.5
-1
-1.5


1.5
1
0.5
-2
-1
1
2
-0.5
-1
-1.5


2
1
-2
-1
1
2
-1
-2


3
2
1
-1
-2
-3
-1.5
-1
-0.5
0.5
1
1.5


2
1
-1
-2
-2
-1
1
2



Die eingehüllte Kurve ist der zentrische Kreis mit dem Radius lambda2 = 1.5.