Lösungen

1

```
Remove["Global`*"]
```

a

Da der 2. Eigenwert 1 ist, besitzt die Abbildung besitzt eine Fixgerade durch den Urspung mit der Richtung des 2. Eigenvektors.

b

```
x1 = {2, 1}; x2 = {-1, 1};
X = Transpose[{x1, x2}]; X // MatrixForm
```

$$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

```
λ1 = -3 / 2 ;  λ2 = 1 ;
```

```
Dλ = {{λ1, 0}, {0, λ2}}; Dλ // MatrixForm
```

$$\begin{pmatrix} -\frac{3}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{5}{6} & \frac{1}{6} \end{pmatrix}$$

c

Die Diagonalisierung ist in b) benutzt worden.

```
Eigensystem[A]
```

$$\left\{\left\{-\frac{3}{2}, 1\right\}, \{\{2, 1\}, \{-1, 1\}\}\right\}$$

Der 2. Eigenvektor ist hier verkürzt angegeben.

d

```
mE = IdentityMatrix[2]; mE // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
Det[A - λ mE]
```

$$-\frac{3}{2} + \frac{\lambda}{2} + \lambda^2$$

```
Factor[Det[A - λ mE]]
```

$$\frac{1}{2} (-1 + \lambda) (3 + 2 \lambda)$$

```
Det[Dλ - λ mE]
```

$$-\frac{3}{2} + \frac{\lambda}{2} + \lambda^2$$

A und D haben dasselbe charakteristische Polynom.

```
Pλ[λ_] := (λ - 1) (λ - 3 / 2); Pλ[λ] // Expand
```

$$\frac{3}{2} - \frac{5 \lambda}{2} + \lambda^2$$

e

```
Det[A]
```

$$-\frac{3}{2}$$

```
Pλ[0]
```

$$\frac{3}{2}$$

```
λ1 λ2
```

$$-\frac{3}{2}$$

```
λ1 + λ2
```

$$-\frac{1}{2}$$

f

```
v1 = x1;  v2 = {3, 1};
XB = Transpose[{v1, v2}]; XB // MatrixForm
```

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

```
B = XB.Dλ.Inverse[XB]; B // MatrixForm
```

$$\begin{pmatrix} 6 & -15 \\ \frac{5}{2} & -\frac{13}{2} \end{pmatrix}$$

g

```
Det[B - λ mE]
```

$$-\frac{3}{2} + \frac{\lambda}{2} + \lambda^2$$

B hat dasselbe charakteristische Polynom wie A. Daraus liest man ab, dass B und A dieselbe Spur und dieselbe Determinante haben.

h

```
A.B // MatrixForm
```

$$\begin{pmatrix} -\frac{49}{6} & \frac{125}{6} \\ -\frac{55}{12} & \frac{137}{12} \end{pmatrix}$$

```
B.A // MatrixForm
```

$$\begin{pmatrix} \frac{17}{2} & -\frac{25}{2} \\ \frac{15}{4} & -\frac{21}{4} \end{pmatrix}$$

```
Eigenvalues[A.B]
```

$$\left\{\frac{9}{4}, 1\right\}$$

```
Eigenvalues[B.A]
```

$$\left\{\frac{9}{4}, 1\right\}$$

```
Eigensystem[A.B]
```

$$\left\{\left\{\frac{9}{4}, 1\right\}, \left\{\{2, 1\}, \left\{\frac{25}{11}, 1\right\}\right\}\right\}$$

```
Eigensystem[B.A]
```

$$\left\{\left\{\frac{9}{4}, 1\right\}, \left\{\{2, 1\}, \left\{\frac{5}{3}, 1\right\}\right\}\right\}$$

A.B und B.A haben dieselben Eigenwerte, also dieselbe Diagonalmatrix, aber nicht dieselben Eigenvektoren.

i

```
P1 = {-2, -1}; P2 = {-1, -3}; P3 = {2, -2};
```

```
Q1 = A.P1
```

$\left\{3, \frac{3}{2}\right\}$

```
Q2 = A.P2
```

$\left\{\frac{17}{3}, \frac{1}{3}\right\}$

```
Q3 = A.P3
```

$\{2, -2\}$

j

```
Senkr[a_] := {-a[[2]], a[[1]]};
FlaechenProd[a_, b_] := Senkr[a].b
```

```
FlaechenProd[P2 - P1, P3 - P1]
```

7

```
FlaechenProd[Q2 - Q1, Q3 - Q1]
```

$-\frac{21}{2}$

```
FlaechenProd[Q2 - Q1, Q3 - Q1] / FlaechenProd[P2 - P1, P3 - P1]
```

$-\frac{3}{2}$

```
FlaechenProd[Q2 - Q1, Q3 - Q1] / FlaechenProd[P2 - P1, P3 - P1] == λ1 λ2
```

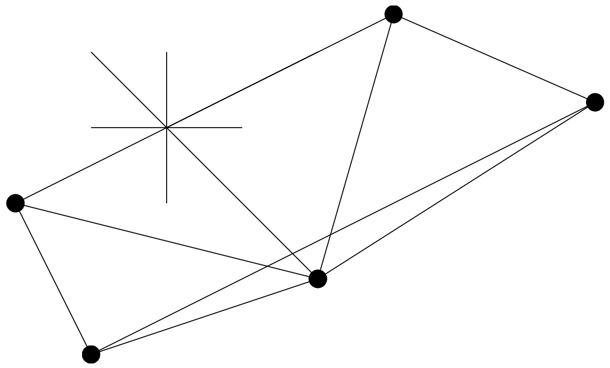
True

```
FlaechenProd[Q2 - Q1, Q3 - Q1] / FlaechenProd[P2 - P1, P3 - P1] == Det[A]
```

True

k

```
o = {0, 0};
Show[Graphics[
   {
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}],
    Line[{o, x1}], Line[{o, x2}],
    Line[{P1, P2, P3, P1}],
    Line[{Q1, Q2, Q3, Q1}],
    Line[{o, P3}], Line[{P1, Q1}], Line[{P2, Q2}],
    PointSize[0.03], Point[P1],
    Point[P2], Point[P3], Point[Q1], Point[Q2], Point[Q3]
   }
  ], AspectRatio → Automatic];
```



2

```
Remove["Global`*"]
```

Die Abbildungen A.B und B.A lassen sich nacheinander konstruieren, was geometrisch interessant ist, weil Fixpunktgeraden auftreten.

```
x1 = {2, 1};  x2 = {-1, 3};
X = Transpose[{x1, x2}]; X // MatrixForm
```

$$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

```
λ1A = 2;   λ2A = 1;  λ1B = 1;   λ2B = -3;
```

```
DλA = {{λ1A, 0}, {0, λ2A}}; DλA // MatrixForm
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

```
DλB = {{λ1B, 0}, {0, λ2B}}; DλB // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$$

```
DλB.DλA // MatrixForm
```

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$$

```
DλA.DλB // MatrixForm
```

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$$

```
A = X.DλA.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} \frac{13}{7} & \frac{2}{7} \\ \frac{3}{7} & \frac{8}{7} \end{pmatrix}$$

```
B = X.DλB.Inverse[X]; B // MatrixForm
```

$$\begin{pmatrix} \frac{3}{7} & \frac{8}{7} \\ \frac{12}{7} & -\frac{17}{7} \end{pmatrix}$$

```
A.B == (X.DλA.Inverse[X]).(X.DλB.Inverse[X])
```

True

```
B.A == (X.DλB.Inverse[X]).(X.DλA.Inverse[X])
```

True

```
A // Eigensystem
```

$$\left\{\{2, 1\}, \left\{\{2, 1\}, \left\{-\frac{1}{3}, 1\right\}\right\}\right\}$$

```
B // Eigensystem
```

$$\left\{\{-3, 1\}, \left\{\left\{-\frac{1}{3}, 1\right\}, \{2, 1\}\right\}\right\}$$

```
A.B // Eigensystem
```

$$\left\{\{-3, 2\}, \left\{\left\{-\frac{1}{3}, 1\right\}, \{2, 1\}\right\}\right\}$$

```
B.A // Eigensystem
```

$$\left\{\{-3, 2\}, \left\{\left\{-\frac{1}{3}, 1\right\}, \{2, 1\}\right\}\right\}$$

```
(X.DλA.Inverse[X]).(X.DλB.Inverse[X]) // MatrixForm
```

$$\begin{pmatrix} \frac{9}{7} & \frac{10}{7} \\ \frac{15}{7} & -\frac{16}{7} \end{pmatrix}$$

```
A.B // MatrixForm
```

$$\begin{pmatrix} \frac{9}{7} & \frac{10}{7} \\ \frac{15}{7} & -\frac{16}{7} \end{pmatrix}$$

```
(X.DλB.Inverse[X]).(X.DλA.Inverse[X]) // MatrixForm
```

$$\begin{pmatrix} \frac{9}{7} & \frac{10}{7} \\ \frac{15}{7} & -\frac{16}{7} \end{pmatrix}$$

```
B.A // MatrixForm
```

$$\begin{pmatrix} \frac{9}{7} & \frac{10}{7} \\ \frac{15}{7} & -\frac{16}{7} \end{pmatrix}$$