Lösungen

1

```
Remove["Global`*"]

Dreh[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; Dreh[φ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

```
P1 = {2, 1}; P2 = {3, 2}; P3 = {1, 3};
φ = 71 Degree;
Q1 = Dreh[φ].P1 // N
```

```
{-0.294382, 2.21661}
```

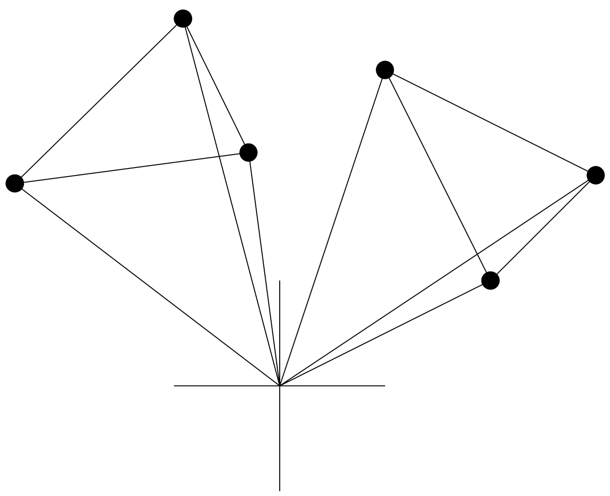
```
Q2 = Dreh[φ].P2 // N
```

```
{-0.914333, 3.48769}
```

```
Q3 = Dreh[φ].P3 // N
```

```
{-2.51099, 1.92222}
```

```
o = {0, 0};
Show[Graphics[
   {
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}],
    Line[{P1, P2, P3, P1}], Line[{Q1, Q2, Q3, Q1}],
    Line[{o, P1}], Line[{o, P2}], Line[{o, P3}],
    Line[{o, Q1}], Line[{o, Q2}], Line[{o, Q3}],
    PointSize[0.03], Point[P1],
    Point[P2], Point[P3], Point[Q1], Point[Q2], Point[Q3]
   }
  ], AspectRatio → Automatic];
```



2

```
α[x_] := ArcTan[x[[2]] / x[[1]]]
x1 = {4, 1}; α[x1]
```

$\text{ArcTan}\left[\frac{1}{4}\right]$

```
% // N
```

0.244979

```
S[0] = {{1, 0}, {0, -1}}; S[0] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

```
S[α_] := Dreh[α].S[0].Dreh[-α];  S[α] // Simplify // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[2\alpha] & \text{Sin}[2\alpha] \\ \text{Sin}[2\alpha] & -\text{Cos}[2\alpha] \end{pmatrix}$$

```
S1 = S[α[x1]].P1
```

$\left\{\frac{38}{17}, \frac{1}{17}\right\}$

```
S1 // N
```

{2.23529, 0.0588235}

```
S2 = S[α[x1]].P2
```

$\left\{\frac{61}{17}, -\frac{6}{17}\right\}$

```
S2 // N
```

{3.58824, -0.352941}

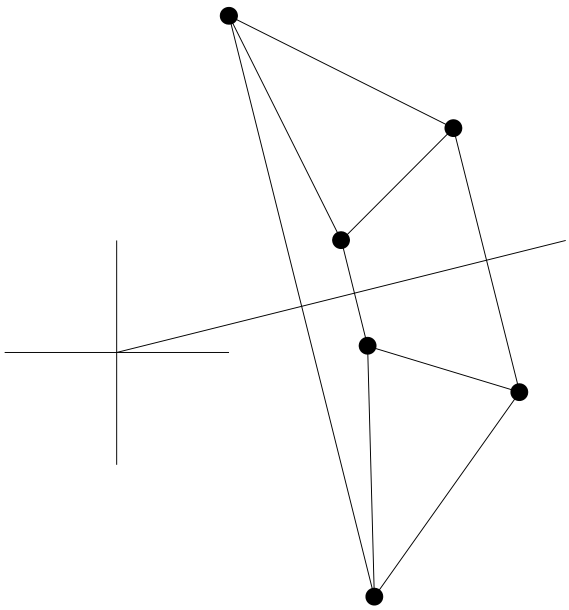
```
S3 = S[α[x1]].P3
```

$\left\{\frac{39}{17}, -\frac{37}{17}\right\}$

```
S2 // N
```

{3.58824, -0.352941}

```
Show[Graphics[
   {
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}],
    Line[{P1, P2, P3, P1}], Line[{S1, S2, S3, S1}],
    Line[{o, x1}],
    Line[{P1, S1}], Line[{P2, S2}], Line[{P3, S3}],
    PointSize[0.03], Point[P1],
    Point[P2], Point[P3], Point[S1], Point[S2], Point[S3]
   }
  ], AspectRatio → Automatic];
```



3

```
Eigensystem[S[α[x1]]]
```

$\left\{\{-1, 1\}, \left\{\left\{-\frac{1}{4}, 1\right\}, \{4, 1\}\right\}\right\}$

{4, 1} ist x1 gestreckt (Richtung der Spiegelungsgerade). $\left\{ \frac{1}{4}, 1\right\}$ ist darauf senkrecht. Der Eigenwert 1 bewirkt die Spiegelung, der Eigenwert 1 das Fixhalten der Abbildung senkrecht zur Speigelungsachse.

4

```
Remove["Global`*"]
```

```
vec0 = {0, 0, 0};
a = {3, 1, 2}; b = {-1, 2, -2}; u = {2, 3, -1}; M = Transpose[{a, b, u}] ; Det[M]
```

−7

a, b, u sind linear unabhängig, da die Determinante nicht 0 ist.

Proj.Transpose[{a,b,u}]=Transpose[{a,b,0}] > Proj = Transpose[{a,b,0}].Inverse[M]

```
M // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{pmatrix}$$

```
Transpose[{a, b, vec0}] // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & 0 \\ 1 & 2 & 0 \\ 2 & -2 & 0 \end{pmatrix}$$

```
vec0 = {0, 0, 0};
 Proj = Transpose[{a, b, vec0}].Inverse[M]; Proj // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{7} & \frac{8}{7} & 2 \\ -\frac{18}{7} & \frac{19}{7} & 3 \\ \frac{6}{7} & -\frac{4}{7} & 0 \end{pmatrix}$$

```
% // N // MatrixForm
```

$$\begin{pmatrix} -0.714286 & 1.14286 & 2. \\ -2.57143 & 2.71429 & 3. \\ 0.857143 & -0.571429 & 0. \end{pmatrix}$$

```
T1 = {0, 2, 3}; T2 = {1, 1, 0}; T3 = {2, 0, 2};
```

```
N1 = Proj.T1
```

$\left\{\frac{58}{7}, \frac{101}{7}, -\frac{8}{7}\right\}$

```
% // N
```

{8.28571, 14.4286, −1.14286}

```
N2 = Proj.T2
```

$\left\{\frac{3}{7}, \frac{1}{7}, \frac{2}{7}\right\}$

```
% // N
```

{0.428571, 0.142857, 0.285714}

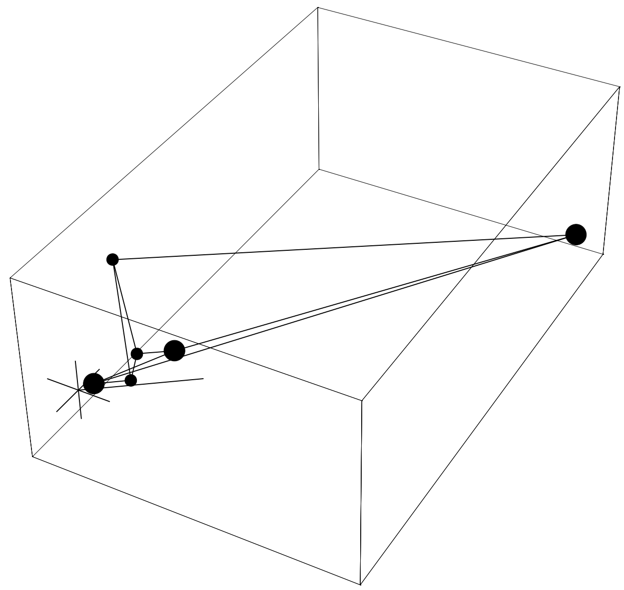
```
N3 = Proj.T3
```

$\left\{\frac{18}{7}, \frac{6}{7}, \frac{12}{7}\right\}$

```
% // N
```

{2.57143, 0.857143, 1.71429}

```
o = {0, 0, 0};
Show[Graphics3D[
   {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}], Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{N1, N2, N3, N1}], Line[{T1, T2, T3, T1}],
    Line[{o, u}],
    Line[{T1, N1}], Line[{T2, N2}], Line[{T3, N3}],
    PointSize[0.02], Point[T1], Point[T2],
    Point[T3], PointSize[0.035], Point[N1], Point[N2], Point[N3]
   }
  ], AspectRatio → Automatic, PlotRange → {{-1, 9}, {-1, 15}, {-2, 4}} ];
```



5

```
M1 = (T1 + N1) / 2
```

$\left\{\frac{29}{7}, \frac{115}{14}, \frac{13}{14}\right\}$

```
N[%]
```

{4.14286, 8.21429, 0.928571}

```
M2 = (T2 + N2) / 2
```

$\left\{\frac{5}{7}, \frac{4}{7}, \frac{1}{7}\right\}$

```
N[%]
```

{0.714286, 0.571429, 0.142857}

```
M3 = (T3 + N3) / 2
```

$\left\{\frac{16}{7}, \frac{3}{7}, \frac{13}{7}\right\}$

```
N[%]
```

{2.28571, 0.428571, 1.85714}

```
Matr = Transpose[{M1, M2, M3}].Inverse[Transpose[{T1, T2, T3}]]; Matr // MatrixForm
```

$$\begin{pmatrix} \frac{1}{7} & \frac{4}{7} & 1 \\ -\frac{9}{7} & \frac{13}{7} & \frac{3}{2} \\ \frac{3}{7} & -\frac{2}{7} & \frac{1}{2} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.142857 & 0.571429 & 1. \\ -1.28571 & 1.85714 & 1.5 \\ 0.428571 & -0.285714 & 0.5 \end{pmatrix}$$

```
Eigensystem[Matr] // N
```

{{1., 1., 0.5}, {{1.16667, 0., 1.}, {0.666667, 1., 0.}, {-2., -3., 1.}}}

a={3,1,2}; b={-1,2,-2}; u={2,3,-1};
Auf den Ersten Blick fällt auf, dass unter den Eigenvektoren -u und daher u (gestreckt) vorkommt. Der dazugehörige Eigenwert ist 0.5, während die andern beiden 1 sind (keine Streckung in diese Richtungen).

```
Matr.T1
```

$\left\{\frac{29}{7}, \frac{115}{14}, \frac{13}{14}\right\}$

Richtig

```
Matr.T2
```

$\left\{\frac{5}{7}, \frac{4}{7}, \frac{1}{7}\right\}$

Richtig

```
Matr.T3
```

$\left\{\frac{16}{7}, \frac{3}{7}, \frac{13}{7}\right\}$

Richtig

6

```
Remove["Global`*"]
```

```
a = {3, 1, 2};
b1 = a / Norm[a]
```

$\left\{\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \sqrt{\frac{2}{7}}\right\}$

```
{1, y1, 0}.a == 0
```

$3 + y1 == 0$

```
Solve[{1, y1, 0}.a == 0, {y1}]
```

$\{\{y1 \to -3\}\}$

```
b2work = {1, -3, 0}
```

$\{1, -3, 0\}$

```
b2 = b2work / Norm[b2work]
```

$\left\{\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}, 0\right\}$

```
b3 = Cross[b1, b2]
```

$\left\{\frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, -\sqrt{\frac{5}{7}}\right\}$

```
U = Transpose[{b1, b2, b3}]; U // MatrixForm
```

$$\begin{pmatrix} \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{35}} \\ \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{35}} \\ \sqrt{\frac{2}{7}} & 0 & -\sqrt{\frac{5}{7}} \end{pmatrix}$$

```
Dreh0[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}};
Dreh0[φ] // MatrixForm
```

```
(1  0       0      )
(0  Cos[φ]  -Sin[φ])
(0  Sin[φ]  Cos[φ] )
```

```
DrehAchse[φ_] := U.Dreh0[φ].Inverse[U]
```

```
Dreh = DrehAchse[56 Degree] // N; Dreh // MatrixForm
```

```
(0.842569    -0.348681  0.410487 )
(0.537598    0.590679   -0.601736)
(-0.0326523  0.727681   0.685138 )
```

```
T1 = {0, 2, 3}; T2 = {1, 1, 0}; T3 = {2, 0, 2};
```

```
R1 = Dreh.T1
```

```
{0.5341, -0.623851, 3.51078}
```

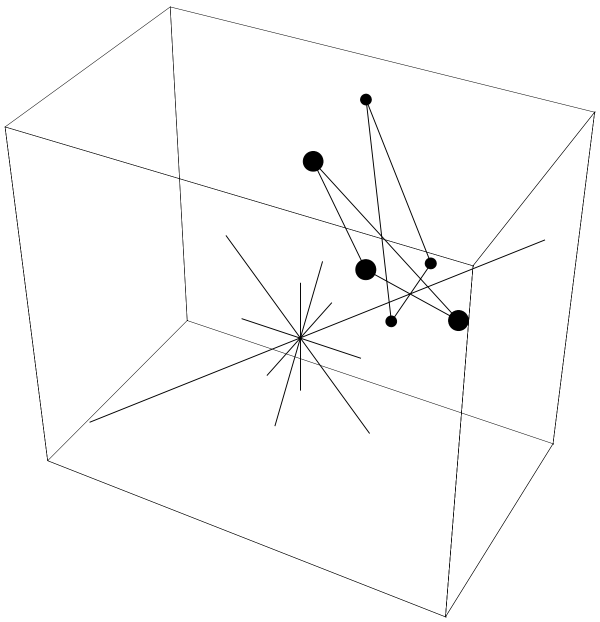
```
R2 = Dreh.T2
```

```
{0.493888, 1.12828, 0.695029}
```

```
R3 = Dreh.T3
```

```
{2.50611, -0.128277, 1.30497}
```

```
o = {0, 0, 0};
Show[Graphics3D[
   {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}], Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{-4 b1, 4 b1}], Line[{-2 b2, 2 b2}], Line[{-2 b3, 2 b3}],
    Line[{R1, R2, R3, R1}], Line[{T1, T2, T3, T1}],
    PointSize[0.02], Point[T1], Point[T2],
    Point[T3], PointSize[0.035], Point[R1], Point[R2], Point[R3]
   }
  ], AspectRatio → Automatic ];
```



7

a Berechnung der Punkte

```
Remove["Global`*"]
```

```
a = {3, 1, 2};
b = {-1, 3, 0};
c = Cross[a, b]
```

```
{-6, -2, 10}
```

```
B = Transpose[{a, b, c}];  B // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & -6 \\ 1 & 3 & -2 \\ 2 & 0 & 10 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
A = B.Dλ.Inverse[B]; A // MatrixForm
```

$$\begin{pmatrix} \frac{17}{35} & -\frac{6}{35} & \frac{6}{7} \\ -\frac{6}{35} & \frac{33}{35} & \frac{2}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix}$$

```
T1 = {0, 2, 3}; T2 = {1, 1, 0}; T3 = {2, 0, 2};
```

```
S1 = A.T1
```

$$\left\{\frac{78}{35}, \frac{96}{35}, -\frac{5}{7}\right\}$$

```
S2 = A.T2
```

$$\left\{\frac{11}{35}, \frac{27}{35}, \frac{8}{7}\right\}$$

```
S3 = A.T3
```

$$\left\{\frac{94}{35}, \frac{8}{35}, \frac{6}{7}\right\}$$

```
M1 = (T1 + S1) / 2
```

$$\left\{\frac{39}{35}, \frac{83}{35}, \frac{8}{7}\right\}$$

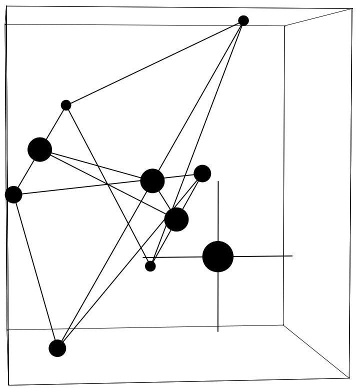
```
M2 = (T2 + S2) / 2
```

$$\left\{\frac{23}{35}, \frac{31}{35}, \frac{4}{7}\right\}$$

```
M3 = (T3 + S3) / 2
```

$$\left\{\frac{82}{35}, \frac{4}{35}, \frac{10}{7}\right\}$$

```
o = {0, 0, 0};
Show[Graphics3D[
   {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}], Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{S1, S2, S3, S1}], Line[{T1, T2, T3, T1}], Line[{M1, M2, M3, M1}],
    Line[{S1, T1}], Line[{S2, T2}], Line[{S3, T3}],
    PointSize[0.03], Point[T1], Point[T2], Point[T3],
    PointSize[0.05], Point[S1], Point[S2], Point[S3], PointSize[0.07],
    Point[M1], Point[M2], Point[M3], PointSize[0.09], Point[o]
   }
  ], AspectRatio → Automatic, ViewPoint -> {0.478, 4.341, 0.259} ];
```



b Kontrolle: Liegen die Punkte M_k in der Ebene ?

```
Solve[λ a + μ b + ν c == M1, {λ, μ, ν}]
```

$\left\{\left\{\lambda \to \frac{4}{7}, \mu \to \frac{3}{5}, \nu \to 0\right\}\right\}$

```
Solve[λ a + μ b + ν c == M2, {λ, μ, ν}]
```

$\left\{\left\{\lambda \to \frac{2}{7}, \mu \to \frac{1}{5}, \nu \to 0\right\}\right\}$

```
Solve[λ a + μ b + ν c == M3, {λ, μ, ν}]
```

$\left\{\left\{\lambda \to \frac{5}{7}, \mu \to -\frac{1}{5}, \nu \to 0\right\}\right\}$

In Ordnung, denn der Koeffizient für die Richtung c ist immer 0.