

# Lösungen

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1

a

```
Integrate[x^2 Cos[x], x] + c
```

```
c + 2 x Cos[x] + (-2 + x^2) Sin[x]
```

b

```
Integrate[Sin[2x] Cos[x/2], x] + c
```

```
c - 1/3 Cos[3x/2] - 1/5 Cos[5x/2]
```

c

```
Ln[x_] := Log[x]
```

```
Integrate[Ln[2x], x] + c
```

```
c - x + x Log[2 x]
```

```
Integrate[Ln[2x], {x, 1, 2}]
```

```
-1 + Log[8]
```

```
N[%]
```

```
1.07944
```

d

```
Integrate[x^3 E^x, x] + c
```

```
c + e^x (-6 + 6 x - 3 x^2 + x^3)
```

```
Integrate[x^3 E^x, {x, 0, 1}]
```

```
6 - 2 e
```

```
N[%]
```

```
0.563436
```

**e**`Integrate[Cos[x]^2, x] + c`

$$c + \frac{x}{2} + \frac{1}{4} \sin[2x]$$

`Integrate[Cos[x]^2, {x, 0, 2Pi}]` $\pi$ `N[%]`

3.14159

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**2****a**`Integrate[x Sqrt[x^2-4], x] + c`

$$c + \frac{1}{3} (-4 + x^2)^{3/2}$$

**b**`Integrate[Cos[x]^5 Sin[x], x] + c`

$$c - \frac{5}{64} \cos[2x] - \frac{1}{32} \cos[4x] - \frac{1}{192} \cos[6x]$$

**c**`Integrate[x Sqrt[1+x^2], x] + c`

$$c + \frac{1}{3} (1 + x^2)^{3/2}$$

`Integrate[x Sqrt[1+x^2], {x, 0, 1}]`

$$\frac{1}{3} (-1 + 2\sqrt{2})$$

`N[%]`

0.609476

**d**`Integrate[Sqrt[4-x^2], x] + c`

$$c + \frac{1}{2} x \sqrt{4 - x^2} + 2 \operatorname{ArcSin}\left[\frac{x}{2}\right]$$

**e**`Integrate[1/Sqrt[4+x^2], x] + c`

$$c + \operatorname{ArcSinh}\left[\frac{x}{2}\right]$$

`Integrate[1/Sqrt[4+x^2], {x,0, 1}]`

$$\operatorname{ArcSinh}\left[\frac{1}{2}\right]$$

`N[%]`

0.481212

**3**`Integrate[Cos[a x] Sin[a x], {x,0,b}]^2`

$$\frac{\sin[ab]^4}{4 a^2}$$

`Integrate[Cos[a x]^2, {x,0,b}]`

$$\frac{2 a b + \sin[2 a b]}{4 a}$$

`Integrate[Sin[a x]^2, {x,0,b}]`

$$\frac{b}{2} - \frac{\sin[2 a b]}{4 a}$$

`Integrate[Cos[a x]^2, {x,0,b}]*Integrate[Sin[a x]^2, {x,0,b}] //Expand`

$$\frac{b^2}{4} - \frac{\sin[2 a b]^2}{16 a^2}$$

**4****a**`Integrate[x^2/(x^3-7), x] + c`

$$c + \frac{1}{3} \operatorname{Log}[-7 + x^3]$$

**b**

`Integrate[E^{2x}/(E^{2x}+7),x] + c`

$$\left\{c + \frac{1}{2} \operatorname{Log}[7 + e^{2x}]\right\}$$

**c**

`Integrate[(2 Cos[2x] + 4x + E^{-x})/(Sin[2x] + 2x^2 - E^{-x} + 4),x] + c`

$$c - x + \operatorname{Log}[-1 + 2 e^x (2 + x^2) + e^x \operatorname{Sin}[2x]]$$

**d**

`Integrate[Cot[x],x] + c //Expand`

$$c + \operatorname{Log}[\operatorname{Sin}[x]]$$

**5****a**

`Apart[1/(x(x-1))]`

$$\frac{1}{-1+x} - \frac{1}{x}$$

`Integrate[1/(x(x-1)),x] + c`

$$c + \operatorname{Log}[-1+x] - \operatorname{Log}[x]$$

**b**

`Apart[(x^4-16 x-3)/((x^2+1)(x-1)^2 (x+2))]`

$$-\frac{3}{(-1+x)^2} + \frac{2}{-1+x} + \frac{1}{2+x} + \frac{3-2x}{1+x^2}$$

`Integrate[(x^4-16 x-3)/((x^2+1)(x-1)^2 (x+2)),x] + c`

$$c + \frac{3}{-1+x} + 3 \operatorname{ArcTan}[x] + 2 \operatorname{Log}[-1+x] + \operatorname{Log}[2+x] - \operatorname{Log}[1+x^2]$$

**c**

$$\text{Apart}[(x^4-16x-3)/((x^2+1)(x-1)^2)]$$

$$1 - \frac{9}{(-1+x)^2} + \frac{3}{-1+x} + \frac{8-x}{1+x^2}$$

$$\text{Integrate}[(x^4-16x-3)/((x^2+1)(x-1)^2),x] + c$$

$$c + \frac{9}{-1+x} + x + 8 \text{ArcTan}[x] + 3 \text{Log}[-1+x] - \frac{1}{2} \text{Log}[1+x^2]$$

**d**

$$\text{Apart}[(x^2-6x-6)/(x^2(x^2+3x+3))]$$

$$-\frac{2}{x^2} + \frac{3}{3+3x+x^2}$$

$$\text{Integrate}[(x^2-6x-6)/(x^2(x^2+3x+3)),x] + c$$

$$c + \frac{2}{x} + 2\sqrt{3} \text{ArcTan}\left[\frac{3+2x}{\sqrt{3}}\right]$$

**e**

$$\text{Apart}[(90x^3-76x^2-25x+19)/((2x-1)(3x+1)(5x-3))]$$

$$3 + \frac{1}{-1+2x} + \frac{2}{1+3x} - \frac{7}{-3+5x}$$

$$\text{Integrate}[(90x^3-76x^2-25x+19)/((2x-1)(3x+1)(5x-3)),x] + c$$

$$c + 3x + \frac{1}{2} \text{Log}[-1+2x] + \frac{2}{3} \text{Log}[1+3x] - \frac{7}{5} \text{Log}[-3+5x]$$