Lösungen

1

Lösung: Siehe Skript Analysis von R. Wirz
Ausgabe d/f Seite 263
Ausgabe d Seite 234

2

```
Remove["Global`*"]

f[x_,y_]:=1-x^2-2(y+1)^2;
f[x,y]
```

$$1 - x^2 - 2\,(1 + y)^2$$

```
f[{x_,y_}]:=f[x,y];
p={0,2};
```

a

```
grad[f_,x_,y_]:={D[f,x],D[f,y]};
grad[f[x,y],x,y]
```

$$\{-2\,x,\ -4\,(1 + y)\}$$

```
Solve[Evaluate[grad[f[x,y],x,y]==0],{x,y}]
```

$$\{\{x \to 0,\ y \to -1\}\}$$

b

```
e30={Cos[30 Degree],Sin[30 Degree]}
```

$$\left\{\frac{\sqrt{3}}{2},\ \frac{1}{2}\right\}$$

```
grad[f[x,y],x,y].e30
```

$$-\sqrt{3}\,x - 2\,(1 + y)$$

```
grad[f[x,y],x,y].e30 /. {x->0,y->2}
```

$$-6$$

c

```
ArcTan[-6] // N
```

```
-1.40565
```

```
ArcTan[-6]/Degree // N
```

```
-80.5377
```

d

```
Norm[grad[f[x,y],x,y]]
```

$$\sqrt{4\,\text{Abs}[x]^2+16\,\text{Abs}[1+y]^2}$$

```
Norm[grad[f[x,y],x,y]] /. {x->0,y->2}
```

```
12
```

```
ArcTan[12] // N
```

```
1.48766
```

```
ArcTan[12]/Degree // N
```

```
85.2364
```

e

```
gradP = grad[f[x,y],x,y] /. {x->0,y->2}
```

```
{0, -12}
```

```
Φ[x_, y_] := f[p] + gradP[[1]] (x - p[[1]]) + gradP[[2]] (y - p[[2]]);
Φ[x, y]
```

```
-17 - 12 (-2 + y)
```

```
Solve[{Φ[x,y]==0,y==0},{x}]
```

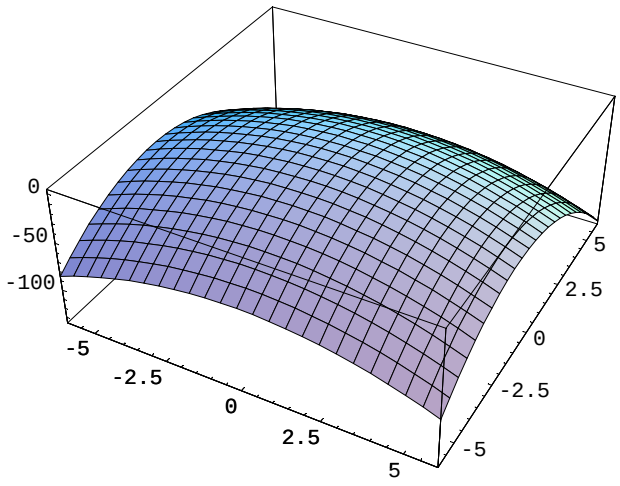
```
{}
```

```
%// N
```

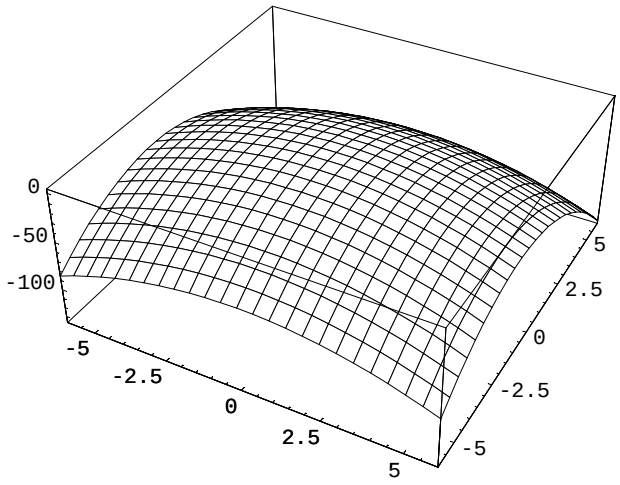
```
{}
```

f

```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6}];
```



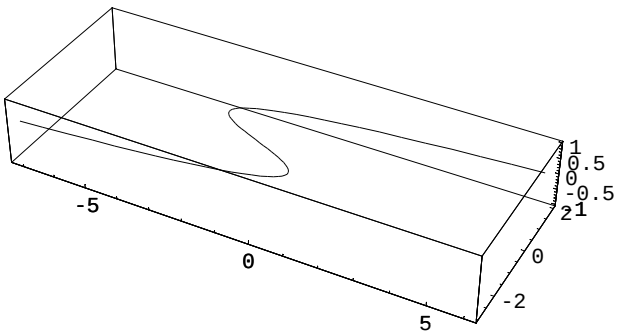

```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6},Shading -> False];
```



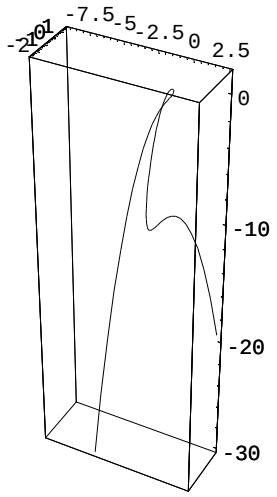

```
v[t_]:= {Expand[t(t-1)(t+2)-2],t};  v[t]
```

$\{-2 - 2\,t + t^2 + t^3,\ t\}$

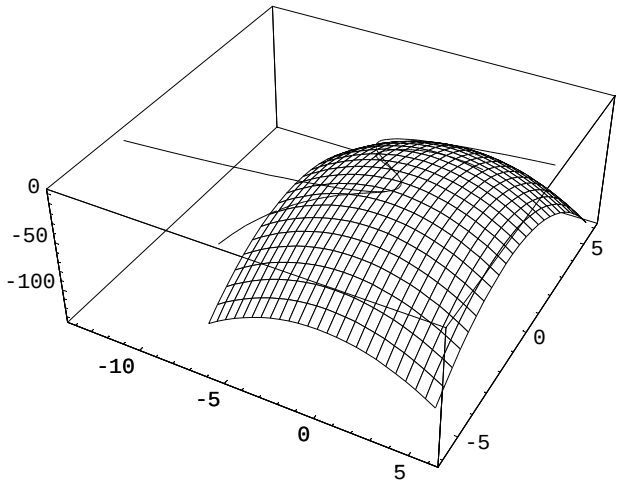
```
pl2=ParametricPlot3D[{v[t][[1]],v[t][[2]],0},{t,-3,2}];
```



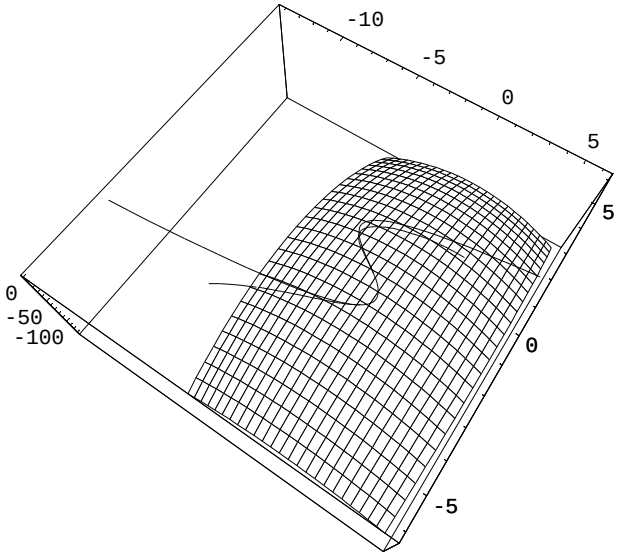
```
pl3=ParametricPlot3D[{v[t][[1]],v[t][[2]],f[v[t]]},{t,-2.7,1.7}];
```



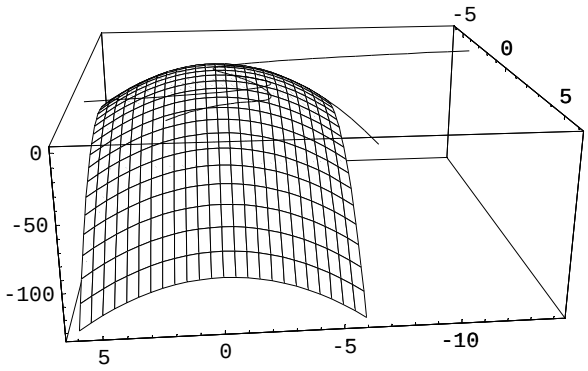

```
Show[pl1,pl3,pl2];
```



```
Show[pl1,pl3,pl2,ViewPoint->{0.424, -0.630, 1.583}];
```




```
Show[pl1,pl3,pl2,ViewPoint->{0.210, 2.200, 0.760}];
```



g

```
v[t_]:= {Expand[t(t-1)(t+2)-2],t};  v[t]
```

$\{-2 - 2\,t + t^2 + t^3,\ t\}$

```
g[x_,y_]:= Evaluate[v[t][[1]] - x  /.t->y];
g[x,y] ==0
```

$-2 - x - 2\,y + y^2 + y^3 == 0$

```
grad[f[x,y],x,y]== λ grad[g[x,y],x,y]
```

$\{-2\,x,\ -4\,(1+y)\} == \{-\lambda,\ (-2+2\,y+3\,y^2)\,\lambda\}$

```
NSolve[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y], g[x,y]==0}, {λ,x,y}]
```

$\{\{\lambda \to -4.92498,\ x \to -2.46249,\ y \to 0.790255\},\ \{\lambda \to -1.66997,\ x \to -0.834987,\ y \to 1.27807\},$
$\{\lambda \to 0.778958 - 0.50778\,i,\ x \to 0.389479 - 0.25389\,i,\ y \to -1.3675 - 0.33188\,i\},$
$\{\lambda \to 0.778958 + 0.50778\,i,\ x \to 0.389479 + 0.25389\,i,\ y \to -1.3675 + 0.33188\,i\},$
$\{\lambda \to 0.,\ x \to 0.,\ y \to -1.\}\}$

f[v[t]]

$1 - 2\,(1+t)^2 - (-2-2\,t+t^2+t^3)^2$

solv1 = Solve[Evaluate[D[f[v[t]],t]==0],{t}] //N // Flatten

$\{t \to -1.,\ t \to -1.3675 - 0.33188\,i,\ t \to -1.3675 + 0.33188\,i,\ t \to 0.790255,\ t \to 1.27807\}$

v[t] /. solv1[[1]]

$\{0.,\ -1.\}$

f[v[t]] /. solv1[[1]]

$1.$

v[t] /. solv1[[4]]

$\{-2.46249,\ 0.790255\}$

f[v[t]] /. solv1[[4]]

-11.4739

v[t] /. solv1[[5]]

$\{-0.834987,\ 1.27807\}$

f[v[t]] /. solv1[[5]]

-10.0764

==> Punkt {0,-1,1}

h

D[f[v[t]],t]

$-4\,(1+t) - 2\,(-2+2\,t+3\,t^2)\,(-2-2\,t+t^2+t^3)$

D[f[v[t]],t] // Expand

$-12-4\,t+24\,t^2+12\,t^3-10\,t^4-6\,t^5$

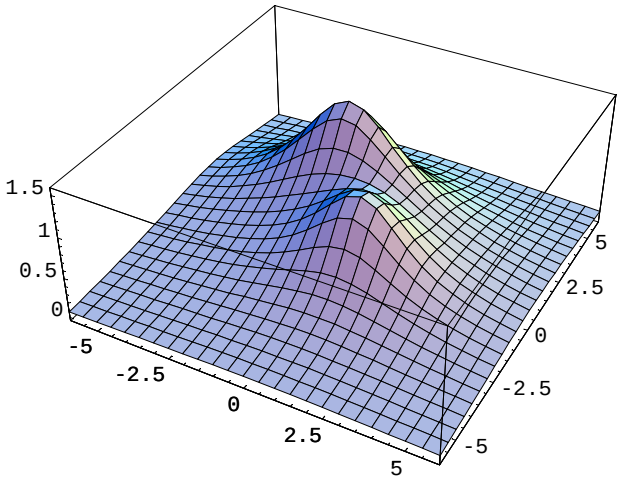
3

Remove["Global`*"]

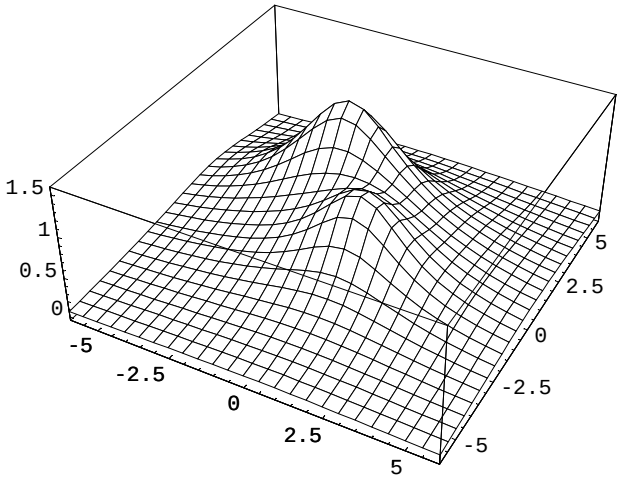
f[x_,y_]:=10/(3 (x-1)^2+5(y+1)^2+10)+ 14/(2 (x+1)^2+5(y-2)^2+10)

a

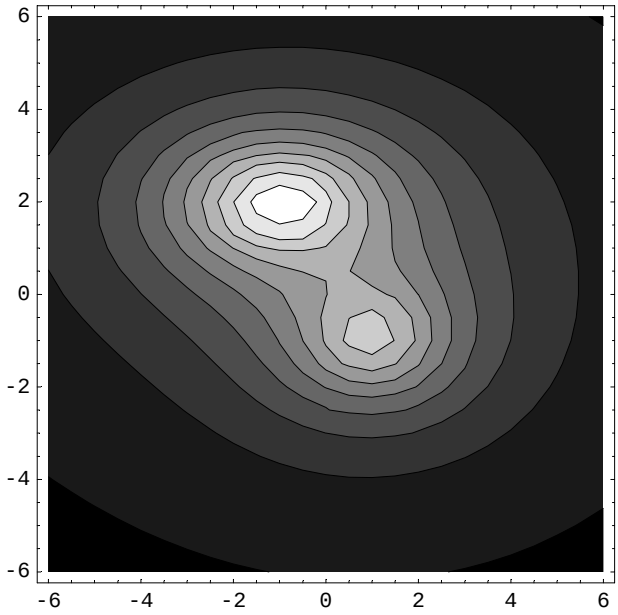
```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6}];
```




```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6},Shading -> False];
```



```
ContourPlot[f[x,y],{x,-6,6},{y,-6,6}];
```



b

```
grad[f_,x_,y_]:={D[f,x],D[f,y]};
grad[f[x,y],x,y]
```

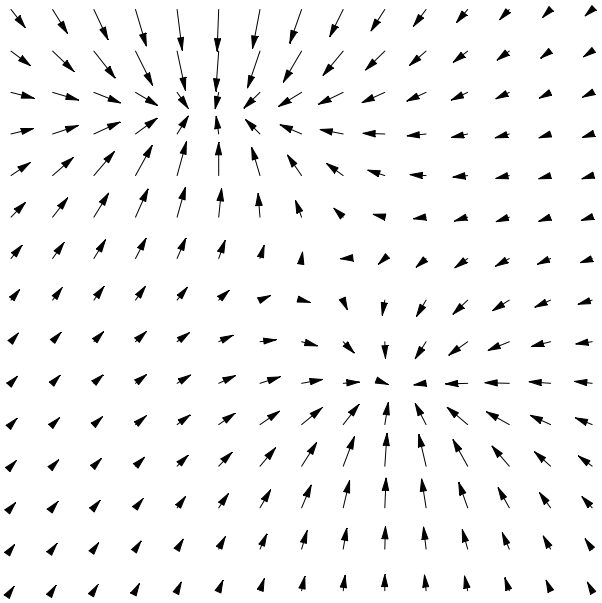
$$\left\{-\frac{56\,(1+x)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{60\,(-1+x)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2},\right.$$
$$\left.-\frac{140\,(-2+y)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{100\,(1+y)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right\}$$

```
grad[f[x,y],x,y] // Simplify
```

$$\left\{-\frac{56\,(1+x)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{60\,(-1+x)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2},\right.$$
$$\left.20\left(-\frac{7\,(-2+y)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{5\,(1+y)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right)\right\}$$

```
<<Graphics`PlotField`
```

```
PlotGradientField[f[x,y],
            {x, -3, 3}, {y, -3, 3}];
```



```
NSolve[Evaluate[grad[f[x,y],x,y]==0],{x,y}]
```

```
{{x → 3.66929 - 11.2219 i, y → 7.62486 + 3.16354 i},
 {x → 3.66929 + 11.2219 i, y → 7.62486 - 3.16354 i},
 {x → 5.7046 - 10.9257 i, y → 8.21161 + 3.28133 i},
 {x → 5.7046 + 10.9257 i, y → 8.21161 - 3.28133 i},
 {x → 1.01289 - 1.67828 i, y → 0.329888 + 3.22855 i},
 {x → 1.01289 + 1.67828 i, y → 0.329888 - 3.22855 i}, {x → -0.950244, y → 1.94983},
 {x → 0.947227, y → -0.882807}, {x → 0.293963, y → 0.350251}}
```

C

```
e30={Cos[30 Degree],Sin[30 Degree]}
```

$$\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}\right\}$$

```
grad[f[x,y],x,y].e30
```

$$\frac{1}{2}\sqrt{3}\left(-\frac{56\,(1+x)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{60\,(-1+x)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right)+\frac{1}{2}\left(-\frac{140\,(-2+y)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{100\,(1+y)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right)$$

```
%//N
```

$$0.866025\left(-\frac{56.\,(1.+x)}{(10.+2.\,(1.+x)^2+5.\,(-2.+y)^2)^2}-\frac{60.\,(-1.+x)}{(10.+3.\,(-1.+x)^2+5.\,(1.+y)^2)^2}\right)+0.5\left(-\frac{140.\,(-2.+y)}{(10.+2.\,(1.+x)^2+5.\,(-2.+y)^2)^2}-\frac{100.\,(1.+y)}{(10.+3.\,(-1.+x)^2+5.\,(1.+y)^2)^2}\right)$$

```
grad[f[x,y],x,y].e30 /. {x->0,y->2}
```

$$-\frac{75}{1682}-\frac{5617}{10092\sqrt{3}}$$

```
%//N
```

$$-0.365931$$

d

i

```
f[{x_,y_}]:=f[x,y];
p={0,2};

gradP = grad[f[x,y],x,y] /. {x->0,y->2}
```

$$\left\{-\frac{5617}{15138},\ -\frac{75}{841}\right\}$$

```
(gradP = grad[f[x,y],x,y] /. {x->0,y->2} ) //N
```

$$\{-0.371053,\ -0.0891795\}$$

```
Φ[x_, y_] := f[p] + gradP[[1]] (x - p[[1]]) + gradP[[2]] (y - p[[2]]);
Φ[x, y]
```

$$\frac{233}{174}-\frac{5617\,x}{15138}-\frac{75}{841}\,(-2+y)$$

```
%//N //Simplify
```

$$1.51744-0.371053\,x-0.0891795\,y$$

```
Solve[{Φ[x,y]==z,x==0,y==0},{z}]
```

$$\left\{\left\{z\to\frac{7657}{5046}\right\}\right\}$$

```
%// N
```

$$\{\{z\to 1.51744\}\}$$

ii

```
Norm[grad[f[x,y],x,y]]
```

$$\sqrt{\left(\text{Abs}\left[-\frac{56\,(1+x)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{60\,(-1+x)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right]^2+\text{Abs}\left[-\frac{140\,(-2+y)}{(10+2\,(1+x)^2+5\,(-2+y)^2)^2}-\frac{100\,(1+y)}{(10+3\,(-1+x)^2+5\,(1+y)^2)^2}\right]^2\right)}$$

```
Norm[grad[f[x,y],x,y]] /. {x->0,y->2}
```

$$\frac{\sqrt{33373189}}{15138}$$

```
%//N
```

```
0.381619
```

```
ArcTan[%] // N
```

```
0.364561
```

```
ArcTan[%%]/Degree // N
```

```
20.8878
```

iii

```
g[x_,y_]:= y-x^2
```

```
solv2= NSolve[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y], g[x,y]==0}, {λ,x,y}]
```

```
{{λ → 0.000167092 - 0.00649497 i, x → -1.33529 + 1.36828 i, y → -0.0891819 - 3.65412 i},
 {λ → 0.000167092 + 0.00649497 i, x → -1.33529 - 1.36828 i, y → -0.0891819 + 3.65412 i},
 {λ → 0.0823729, x → -1.37488, y → 1.89028},
 {λ → -0.0128902 - 0.00987242 i, x → 1.20431 - 1.29879 i, y → -0.236493 - 3.12829 i},
 {λ → -0.0128902 + 0.00987242 i, x → 1.20431 + 1.29879 i, y → -0.236493 + 3.12829 i},
 {λ → -0.0830247 + 0.273961 i, x → -0.170884 + 1.16901 i, y → -1.33739 - 0.399531 i},
 {λ → -0.0830247 - 0.273961 i, x → -0.170884 - 1.16901 i, y → -1.33739 + 0.399531 i},
 {λ → 0.130264, x → -0.441818, y → 0.195204},
 {λ → 0.118471 - 0.0209213 i, x → 1.05079 - 0.244279 i, y → 1.04449 - 0.513372 i},
 {λ → 0.118471 + 0.0209213 i, x → 1.05079 + 0.244279 i, y → 1.04449 + 0.513372 i},
 {λ → -0.0934765, x → 0.318855, y → 0.101668}}
```

```
Table[{k,solv2[[k]]},{k,1,Length[solv2]}]//MatrixForm
```

$$\begin{pmatrix} 1 & \{\lambda \to 0.000167092 - 0.00649497\,i,\ x \to -1.33529 + 1.36828\,i,\ y \to -0.0891819 - 3.65412\,i\} \\ 2 & \{\lambda \to 0.000167092 + 0.00649497\,i,\ x \to -1.33529 - 1.36828\,i,\ y \to -0.0891819 + 3.65412\,i\} \\ 3 & \{\lambda \to 0.0823729,\ x \to -1.37488,\ y \to 1.89028\} \\ 4 & \{\lambda \to -0.0128902 - 0.00987242\,i,\ x \to 1.20431 - 1.29879\,i,\ y \to -0.236493 - 3.12829\,i\} \\ 5 & \{\lambda \to -0.0128902 + 0.00987242\,i,\ x \to 1.20431 + 1.29879\,i,\ y \to -0.236493 + 3.12829\,i\} \\ 6 & \{\lambda \to -0.0830247 + 0.273961\,i,\ x \to -0.170884 + 1.16901\,i,\ y \to -1.33739 - 0.399531\,i\} \\ 7 & \{\lambda \to -0.0830247 - 0.273961\,i,\ x \to -0.170884 - 1.16901\,i,\ y \to -1.33739 + 0.399531\,i\} \\ 8 & \{\lambda \to 0.130264,\ x \to -0.441818,\ y \to 0.195204\} \\ 9 & \{\lambda \to 0.118471 - 0.0209213\,i,\ x \to 1.05079 - 0.244279\,i,\ y \to 1.04449 - 0.513372\,i\} \\ 10 & \{\lambda \to 0.118471 + 0.0209213\,i,\ x \to 1.05079 + 0.244279\,i,\ y \to 1.04449 + 0.513372\,i\} \\ 11 & \{\lambda \to -0.0934765,\ x \to 0.318855,\ y \to 0.101668\} \end{pmatrix}$$

```
{solv2[[3]],solv2[[8]],solv2[[11]]}
```

```
{{λ → 0.0823729, x → -1.37488, y → 1.89028}, {λ → 0.130264, x → -0.441818, y → 0.195204},
 {λ → -0.0934765, x → 0.318855, y → 0.101668}}
```

```
f[x,y]/.solv2[[3]]
```

```
1.49939
```

```
f[x,y]/.solv2[[8]]
```

```
0.947994
```

```
f[x,y]/.solv2[[11]]
```

```
1.01722
```

Punkt { -1.37488, 1.89028, 1.49939}

4

a

Series[E^(-x^2),{x,0,8}]

$$1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} + O[x]^9$$

N[%]

$$1. - (x + 0.)^2 + 0.5\,(x + 0.)^4 - 0.166667\,(x + 0.)^6 + 0.0416667\,(x + 0.)^8 + O[x + 0.]^9$$

b

Evaluate[Normal[Series[E^(-x^2),{x,0,8}]]]

$$1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24}$$

N[%]

$$1. - 1.\,x^2 + 0.5\,x^4 - 0.166667\,x^6 + 0.0416667\,x^8$$

Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,8}]]],{x,-2,2}]

$$\frac{3508}{945}$$

%//N

3.71217

Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,100}]]],{x,-2,2}]//N

1.76416

Integrate[E^(-x^2),{x,-2,2}]//N

1.76416

c

Evaluate[Normal[Series[Cos[x^2]+E^(x^2),{x,0,8}]]]

$$2 + x^2 + \frac{x^6}{6} + \frac{x^8}{12}$$

N[%]

$$2. + x^2 + 0.166667\,x^6 + 0.0833333\,x^8$$

Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,8}]]],{x,-2,2}]

$$\frac{3508}{945}$$

%//N

3.71217

Integrate[E^(-x^2),{x,-2,2}]//N

1.76416

Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,100}]]],{x,-2,2}]//N

1.76416

d

Evaluate[Normal[Series[Sqrt[x],{x,1,6}]]]

$$1+\frac{1}{2}(-1+x)-\frac{1}{8}(-1+x)^2+\frac{1}{16}(-1+x)^3-\frac{5}{128}(-1+x)^4+\frac{7}{256}(-1+x)^5-\frac{21(-1+x)^6}{1024}$$

N[%]

$$1.+0.5(-1.+x)-0.125(-1.+x)^2+0.0625(-1.+x)^3-0.0390625(-1.+x)^4+0.0273438(-1.+x)^5-0.0205078(-1.+x)^6$$

e

r=1

1

d

Evaluate[Normal[Series[Sqrt[x],{x,1,6}]]]

$$1+\frac{1}{2}(-1+x)-\frac{1}{8}(-1+x)^2+\frac{1}{16}(-1+x)^3-\frac{5}{128}(-1+x)^4+\frac{7}{256}(-1+x)^5-\frac{21(-1+x)^6}{1024}$$

N[%]

$$1.+0.5(-1.+x)-0.125(-1.+x)^2+0.0625(-1.+x)^3-0.0390625(-1.+x)^4+0.0273438(-1.+x)^5-0.0205078(-1.+x)^6$$

e

r=1

1

f

Evaluate[Normal[Series[Log[x]-Sin[x],{x,1,6}]]]

$$(-1+x)\,(1-\text{Cos}[1])+(-1+x)^5\left(\frac{1}{5}-\frac{\text{Cos}[1]}{120}\right)+(-1+x)^3\left(\frac{1}{3}+\frac{\text{Cos}[1]}{6}\right)+$$
$$(-1+x)^4\left(-\frac{1}{4}-\frac{\text{Sin}[1]}{24}\right)+(-1+x)^6\left(-\frac{1}{6}+\frac{\text{Sin}[1]}{720}\right)+(-1+x)^2\left(-\frac{1}{2}+\frac{\text{Sin}[1]}{2}\right)-\text{Sin}[1]$$

N[%]

$$-0.841471+0.459698\,(-1.+x)-0.0792645\,(-1.+x)^2+0.423384\,(-1.+x)^3-$$
$$0.285061\,(-1.+x)^4+0.195497\,(-1.+x)^5-0.165498\,(-1.+x)^6$$

g

1-1/(1-1/2)+1/(1-1/3)

$$\frac{1}{2}$$

N[%]

0.5

h

Sum[1/k,{k,1,n}]

HarmonicNumber[n]

Limit[Sum[1/k,{k,1,n}],{n->Infinity}]

{∞}

Limit[Sum[1/k,{k,1,n}]-Log[n],{n->Infinity}]

{EulerGamma}

%//N

{0.577216}

$Aborted