

# Lösungen

---

1

a

```
f[x_]:=4x^4+5x^5-6x^6+x^0-2/x^2
```

```
f[x]
```

$$1 - \frac{2}{x^2} + 4x^4 + 5x^5 - 6x^6$$

i

```
Integrate[f[x],x]+c
```

$$c + \frac{2}{x} + x + \frac{4x^5}{5} + \frac{5x^6}{6} - \frac{6x^7}{7}$$

```
N[%]
```

$$c + \frac{2.}{x} + x + 0.8x^5 + 0.833333x^6 - 0.857143x^7$$

ii

```
Integrate[f[x],{x,1,2}]
```

$$-\frac{2209}{70}$$

```
N[%]
```

$$-31.5571$$

iii

```
Integrate[f[x],{x,-1,1}]
```

Integrate::idiv : Integral of  $1 - \frac{2}{x^2} + 4x^4 + 5x^5 - 6x^6$  does not converge on  $\{-1, 1\}$ . Mehr...

$$\int_{-1}^1 \left(1 - \frac{2}{x^2} + 4x^4 + 5x^5 - 6x^6\right) dx$$

iv

**D[Evaluate[Integrate[f[x],{x,0,t}],t]**Integrate::idiv : Integral of  $1 - \frac{2}{x^2} + 4x^4 + 5x^5 - 6x^6$  does not converge on {0, t}. Mehr...

$$1 - \frac{2}{t^2} + 4t^4 + 5t^5 - 6t^6$$

b

**f[x\_]:=1/Sin[x/3-3]^2****f[x]**

$$\text{Csc}\left[3 - \frac{x}{3}\right]^2$$

i

**Integrate[f[x],x]+c**

$$c + 3 \text{Cot}\left[3 - \frac{x}{3}\right]$$

**N[%]**

$$c + 3. \text{Cot}[3. - 0.333333 x]$$

ii

**Integrate[f[x],{x,2,t}]**

$$(-2 + t) \text{If}\left[\text{Re}[t] \leq 9 \mid \mid \text{Im}[t] \neq 0, \frac{3(-\text{Cot}\left[\frac{7}{3}\right] + \text{Cot}\left[3 - \frac{t}{3}\right])}{-2 + t},\right.$$

$$\left. \text{Integrate}\left[\text{Csc}\left[\frac{1}{3}(7 - (-2 + t)x)\right]^2, \{x, 0, 1\}, \text{Assumptions} \rightarrow \text{Im}[t] = 0 \ \&\& \ \text{Re}[t] > 9\right]\right]$$

**Assuming[t ∈ Reals, Integrate[f[x], {x, 2, t}]]**

$$\text{If}\left[2 < t \leq 9, -3 \text{Cot}\left[\frac{7}{3}\right] + 3 \text{Cot}\left[3 - \frac{t}{3}\right],\right.$$

$$\left. \text{Integrate}\left[\text{Csc}\left[3 - \frac{x}{3}\right]^2, \{x, 2, t\}, \text{Assumptions} \rightarrow t \leq 2 \mid \mid t > 9\right]\right]$$

**Integrate[f[x],{x,2,t},GenerateConditions→False]**

$$-3 \text{Cot}\left[\frac{7}{3}\right] + 3 \text{Cot}\left[3 - \frac{t}{3}\right]$$

**D[Evaluate[Integrate[f[x],{x,2,t},GenerateConditions→False]],t]**

$$\text{Csc}\left[3 - \frac{t}{3}\right]^2$$

**Integrate[Evaluate[Evaluate[D[Evaluate[Integrate[f[x],{x,2,t},GenerateConditions→False]],t]],{t,1,u},GenerateConditions→False]**

$$-3 \text{Cot}\left[\frac{8}{3}\right] + 3 \text{Cot}\left[3 - \frac{u}{3}\right]$$

**c****f[x\_]:=1/(x(x+2)(x-2))****f[x]**

$$\frac{1}{(-2+x)x(2+x)}$$

**Apart[f[x]]**

$$\frac{1}{8(-2+x)} - \frac{1}{4x} + \frac{1}{8(2+x)}$$

**i****Integrate[f[x],x]+c**

$$c - \frac{\text{Log}[x]}{4} + \frac{1}{8} \text{Log}[-4+x^2]$$

**Integrate[f[x],{x,3,5}]**

$$\frac{1}{8} \text{Log}\left[\frac{189}{125}\right]$$

**N[%]**

0.0516792

**ii****Integrate[f[x],{x,3,Infinity}]**

$$\frac{1}{8} \text{Log}\left[\frac{9}{5}\right]$$

**N[%]**

0.0734733

**d****f[x\_]:=7 Log[x]-(E^x-E^(-x))/E^(3x)****f[x]**

$$-e^{-3x}(-e^{-x}+e^x)+7\text{Log}[x]$$

**i****Integrate[f[x],{x,1,E}]**

$$\frac{1}{4} \left( 28 + \frac{1}{e^4} - \frac{2}{e^2} - e^{-4e} + 2e^{-2e} \right)$$

**N[%]**

6.93908

**e****f[x\_]:=x Log[x]^2+x****f[x]** $x + x \operatorname{Log}[x]^2$ **i****Integrate[f[x],x]+c** $c + \frac{3x^2}{4} - \frac{1}{2}x^2 \operatorname{Log}[x] + \frac{1}{2}x^2 \operatorname{Log}[x]^2$ **N[%]** $c + 0.75x^2 - 0.5x^2 \operatorname{Log}[x] + 0.5x^2 \operatorname{Log}[x]^2$ **ii****Integrate[f[x],{x,t,2t}]** $\frac{1}{4}t^2(9 + 8 \operatorname{Log}[2]^2 - 4 \operatorname{Log}[4] + 2 \operatorname{Log}[t](-3 + \operatorname{Log}[256] + 3 \operatorname{Log}[t]))$ **N[%]** $0.25t^2(7.29845 + 2. \operatorname{Log}[t](2.54518 + 3. \operatorname{Log}[t]))$ **2****Remove["Global`\*"]****a****f[x]:=Sin[x]; x0=2 Pi; n=8;****Series[f[x],{x,x0,n}]** $(x - 2\pi) - \frac{1}{6}(x - 2\pi)^3 + \frac{1}{120}(x - 2\pi)^5 - \frac{(x - 2\pi)^7}{5040} + O[x - 2\pi]^9$ **f[x]:=Sin[2x];Series[f[x],{x,x0,n}]** $2(x - 2\pi) - \frac{4}{3}(x - 2\pi)^3 + \frac{4}{15}(x - 2\pi)^5 - \frac{8}{315}(x - 2\pi)^7 + O[x - 2\pi]^9$

**N[%]**

$$2. (x - 6.28319) - 1.33333 (x - 6.28319)^3 + 0.266667 (x - 6.28319)^5 - 0.0253968 (x - 6.28319)^7 + O[x - 6.28319]^9$$

**b****f[x\_] := Cos[x^2] + E^(-x^2); x0=0; n=8;****Series[f[x], {x, x0, n}]**

$$2 - x^2 - \frac{x^6}{6} + \frac{x^8}{12} + O[x]^9$$

**N[%]**

$$2. - (x + 0.)^2 - 0.166667 (x + 0.)^6 + 0.0833333 (x + 0.)^8 + O[x + 0.]^9$$

**c****Integrate[Normal[Series[f[x], {x, x0, n}]], {x, -2, 2}]**

$$\frac{1144}{189}$$

**N[%]**

$$6.05291$$

**n=100; Integrate[Normal[Series[f[x], {x, x0, n}]], {x, -2, 2}]**

$$95517222254963855091923543488942132977874204428074748868171863341048024/35546771741509629643290406777212549782492184005430027166664466650390625$$

**N[%]**

$$2.68709$$

**NIntegrate[f[x], {x, -2, 2}]**

$$2.68709$$

**d**

Konvergenzradius von  $e^x$  und von  $\cos(x)$  ist unendlich

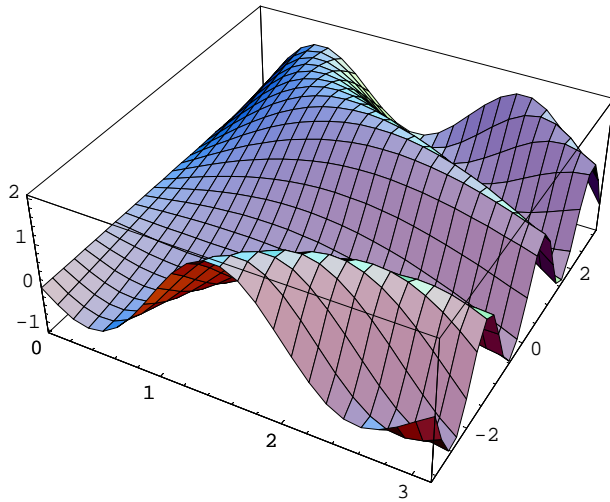
(wegen den Fakultäten im Nenner, folgt aus der Formel für den Konvergenzradius). Wird auch für  $x^2$  u. s. w übernommen.

**3****Remove["Global`\*"]**

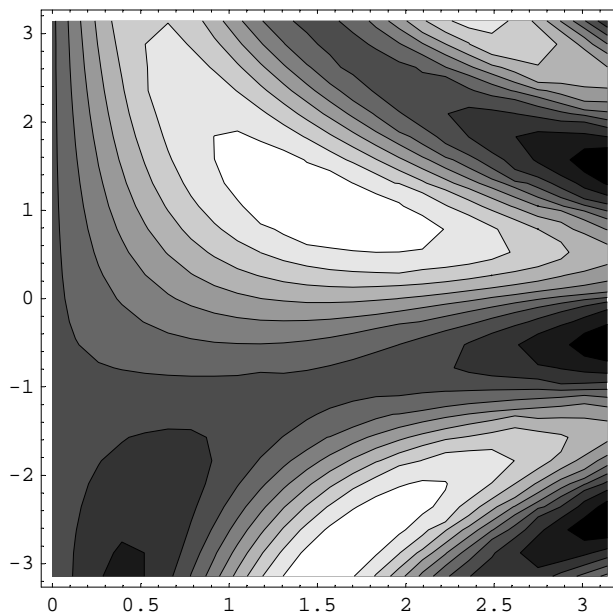
```
f[x_,y_]:=Sin[x y]+Sin[x];
f[{x_,y_}]:=f[x,y];
```

**a**

```
Plot3D[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```



```
ContourPlot[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```



**b**

```
grad[f_,x_,y_]:= {D[f,x],D[f,y]};
grad[f[x,y],x,y]
```

```
{Cos[x] + y Cos[x y], x Cos[x y]}
```

**Solve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]**

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is  $-\text{ArcCos}[\text{Cos}[x]] == 0$ . Mehr...

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is  $\text{ArcCos}[\text{Cos}[xy]] == 0$ . Mehr...

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr...

$\{\{y \rightarrow -1, x \rightarrow 0\}, \{y \rightarrow 1, x \rightarrow -\frac{\pi}{2}\}, \{y \rightarrow 1, x \rightarrow \frac{\pi}{2}\}\}$

**NSolve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]**

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is  $-\text{ArcCos}[\text{Cos}[x]] == 0$ . Mehr...

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is  $\text{ArcCos}[\text{Cos}[xy]] == 0$ . Mehr...

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr...

$\{\{y \rightarrow 1., x \rightarrow -1.5708\}, \{y \rightarrow 1., x \rightarrow 1.5708\}\}$

**FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0}]**

$\{x \rightarrow 0., y \rightarrow -1.\}$

**f[0,-1]**

0

**FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0.5}]**

$\{x \rightarrow 1.5708, y \rightarrow 1.\}$

**f[Pi/2,1]**

2

**FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,3}]**

$\{x \rightarrow 1.5708, y \rightarrow 3.\}$

**f[Pi/2,3]**

0

Rand

**Evaluate[(D[f[x,y],y]==0)/.x->0]**

True

Alles Lösungen

**Evaluate**[(D[f[x,y],y]==0)/.x->Pi]

$$\pi \operatorname{Cos}[\pi y] == 0$$

**Solve**[**Evaluate**[(D[f[x,y],y]==0)/.x->Pi],{y}]

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

$$\left\{ \left\{ y \rightarrow -\frac{1}{2} \right\}, \left\{ y \rightarrow \frac{1}{2} \right\} \right\}$$

**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi]

$$\operatorname{Cos}[x] - \pi \operatorname{Cos}[\pi x] == 0$$

**Solve**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x}]

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way. Mehr...

$$\operatorname{Solve}[\operatorname{Cos}[x] - \pi \operatorname{Cos}[\pi x] == 0, \{x\}]$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,0}]

FindRoot::jsing : Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr...

$$\{x \rightarrow 0.\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,0.1}]

$$\{x \rightarrow 0.405515\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,1}]

$$\{x \rightarrow 5.57793\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,1.1}]

$$\{x \rightarrow 1.50651\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,2}]

$$\{x \rightarrow -1.50651\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->-Pi],{x,2.1}]

$$\{x \rightarrow 3.40044\}$$

**Evaluate**[(D[f[x,y],x]==0)/.y->Pi]

$$\operatorname{Cos}[x] + \pi \operatorname{Cos}[\pi x] == 0$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->Pi],{x,0}]

FindRoot::jsing : Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr...

$$\{x \rightarrow 0.\}$$

**FindRoot**[**Evaluate**[(D[f[x,y],x]==0)/.y->Pi],{x,0.05}]

$$\{x \rightarrow 2.42298\}$$



```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.1}]
```

```
{x -> 1.49203}
```

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,1}]
```

```
{x -> -2.42298}
```

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.5}]
```

```
{x -> 0.58547}
```

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2}]
```

```
{x -> 3.59249}
```

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2.1}]
```

```
{x -> 2.42298}
```

**c**

```
grad[f[x,y],x,y].{1,2} 1/Norm[{1,2}]/.{x->1,y->1}
```

$$\frac{4 \cos[1]}{\sqrt{5}}$$

```
N[%]
```

```
0.966522
```

**d**

```
ArcTan[%]
```

```
0.768376
```

```
%/Degree
```

```
44.0247
```

**e**

```
g[x_,y_]:=y^2-x
```

```
Solve[Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y],g[x,y]==0}],{x,y,λ}]
```

```
Solve::tdep : The equations appear to involve the
variables to be solved for in an essentially non-algebraic way. Mehr...
```

```
Solve[{{Cos[x] + y Cos[x y], x Cos[x y]} == {-λ, 2 y λ}, -x + y^2 == 0}, {x, y, λ}]
```

```
Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y]}/.x->y^2
```

```
{{Cos[y^2] + y Cos[y^3], y^2 Cos[y^3]} == {-λ, 2 y λ}}
```

```
Evaluate[grad[f[x,y],x,y][[2]]== λ grad[g[x,y],x,y][[2]]/.
λ->-grad[f[x,y],x,y][[1]] /.x->y^2
```

$$y^2 \cos[y^3] = 2y (-\cos[y^2] - y \cos[y^3])$$

```
InputForm[%]
```

$$y^2 \cos[y^3] == 2y * (-\cos[y^2] - y \cos[y^3])$$

```
tab1=Table[FindRoot[y^2*Cos[y^3] == 2*y*(-Cos[y^2] - y*Cos[y^3]),{y,k
Pi}],{k,-1,1,0.0125}]/Flatten;tab1=Union[tab1//Chop];
tab1//MatrixForm
```

```
(
y → -3.93852
y → -3.09442
y → -2.9965
y → -2.86363
y → -2.72898
y → -2.59678
y → -2.40392
y → -2.40392
y → -2.40392
y → -2.22876
y → -2.22876
y → -2.00558
y → -1.63135
y → -1.63135
y → -1.11101
y → -1.11101
y → -1.11101
y → -0.634007
y → -0.634007
y → -0.634007
y → 0
y → 1.18504
y → 1.18504
y → 1.18504
y → 1.18504
y → 1.72169
y → 1.96582
y → 1.96582
y → 2.21938
y → 2.21938
y → 2.43266
y → 2.43266
y → 2.57288
y → 2.57288
y → 2.73715
y → 2.87038
y → 2.87038
y → 2.98174
y → 2.98174
y → 2.98174
y → 3.10912
)
```

```

tabl2=Union[Round[100000
Table[{u=y/.tabl1[[k]],u^2},{k,1,Length[tabl1]}]]/100000]/N;
tabl3=Map[Reverse,tabl2] ;
tabl4 = Select[tabl3,#[[1]]<=Pi&]; tabl4//MatrixForm

```

$$\begin{pmatrix} 2.6613 & -1.63135 \\ 1.23435 & -1.11101 \\ 0.40196 & -0.63401 \\ 0. & 0. \\ 1.40431 & 1.18504 \\ 2.9642 & 1.72169 \end{pmatrix}$$

Extrema

$$\begin{pmatrix} 2.6613 & -1.63135 \\ 1.23435 & -1.11101 \\ 0.40196 & -0.63401 \\ 0. & 0. \\ 1.40431 & 1.18504 \\ 2.9642 & 1.72169 \end{pmatrix}$$

Funktionswerte in den Extrema

```
Map[f,tabl4]//MatrixForm
```

$$\begin{pmatrix} 1.39405 \\ -0.0362476 \\ 0.139126 \\ 0. \\ 1.98182 \\ -0.748048 \end{pmatrix}$$