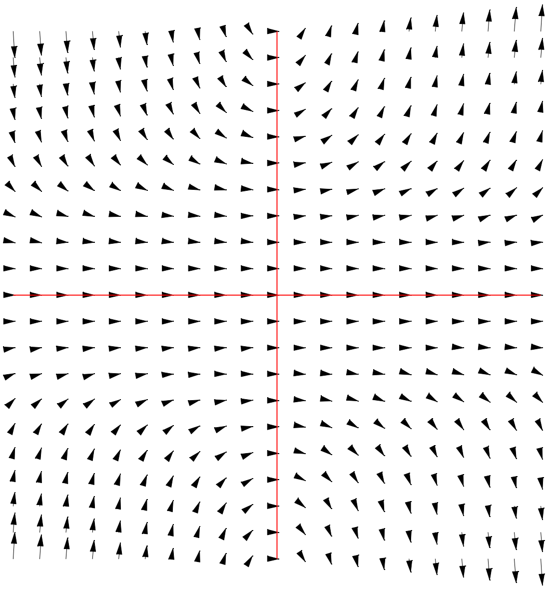
Lösungen

1

a

```
<<Graphics`PlotField`

g1=PlotVectorField[{1, x y^3},{x,-2,2,0.2},{y,-2,2,0.2},Epilog→
{Hue[1],Line[{{-2,0},{2,0}}],Line[{{0,-2},{0,2}}]},AspectRatio→Automatic];
```



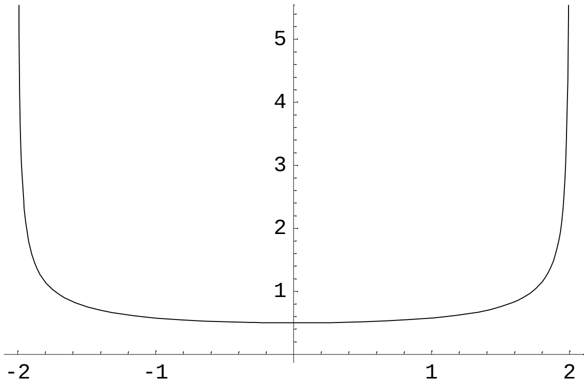
b

```
DSolve[y'[x]==x y[x]^3,y[x],x ]
```

$$\left\{\left\{y[x] \to -\frac{1}{\sqrt{-x^2 - 2\,C[1]}}\right\}, \left\{y[x] \to \frac{1}{\sqrt{-x^2 - 2\,C[1]}}\right\}\right\}$$

c

```
g3=Plot[1/Sqrt[-x^2+4],{x,-2,2}];  (* Eine Lösungskurve *)
```



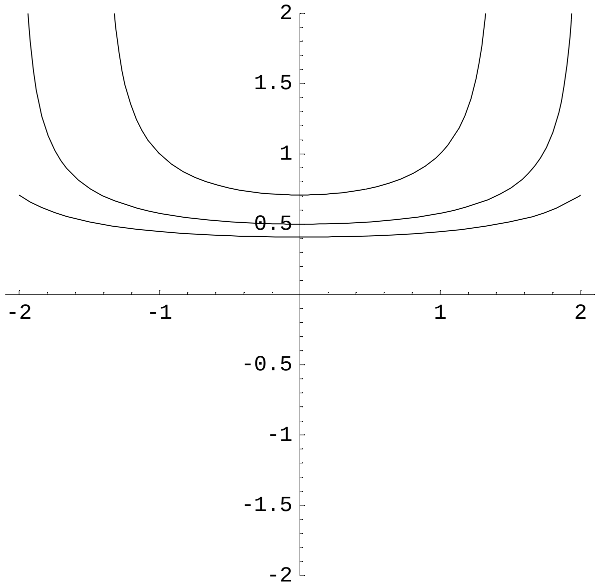
```
g3=ParametricPlot[Evaluate[Table[{x,1/Sqrt[-x^2-2
c]},{c,-3,-1}]],{x,-2,2},AspectRatio→Automatic,PlotRange->{-2,2}];
```

ParametricPlot::pptr : $\{x, \frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = −2.. Mehr…

ParametricPlot::pptr : $\{x, \frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = −1.83773. Mehr…

ParametricPlot::pptr : $\{x, \frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = −1.66076. Mehr…

General::stop : Further output of ParametricPlot::pptr will be suppressed during this calculation. Mehr…



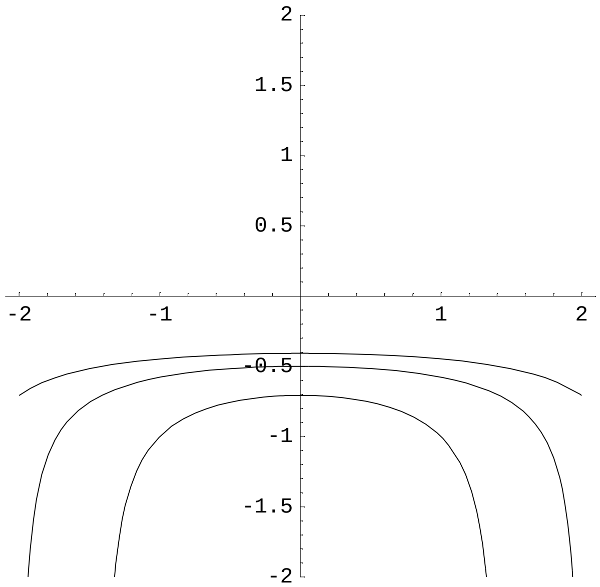
```
g4=ParametricPlot[Evaluate[Table[{x,-1/Sqrt[-x^2-2
c]},{c,-3,-1}]],{x,-2,2},AspectRatio→Automatic,PlotRange->{-2,2}];
```

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = -2.. Mehr…

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = -1.83773. Mehr…

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at x = -1.66076. Mehr…

General::stop : Further output of ParametricPlot::pptr will be suppressed during this calculation. Mehr…



d

```
DSolve[{y'[x] == x y[x]^3, y[1] == 1}, y[x], x ]
```

DSolve::bvnul : For some branches of the general
solution, the given boundary conditions lead to an empty solution. Mehr…

$\{\{y[x] \to \frac{1}{\sqrt{2-x^2}}\}\}$

```
DSolve[{y'[x]==x y[x]^3,y[1]==1},y,x ]
```

DSolve::bvnul : For some branches of the general
solution, the given boundary conditions lead to an empty solution. Mehr…

$\{\{y \to \text{Function}[\{x\}, \frac{1}{\sqrt{2-x^2}}]\}\}$

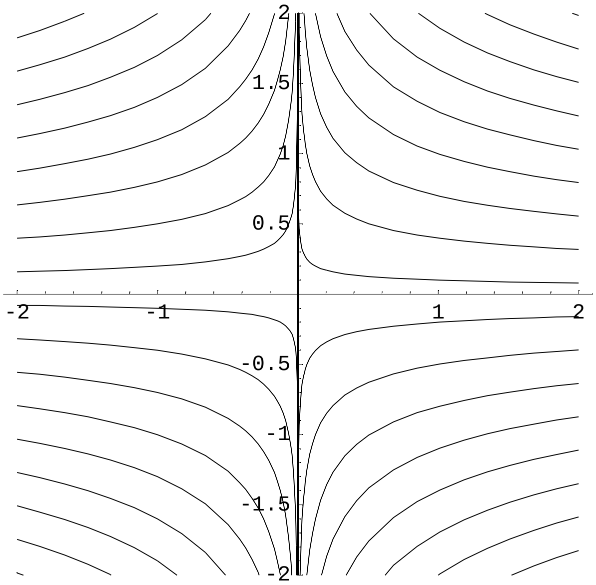
e

```
u1[x_, c_] := Re[PowerExpand[(c / x) ^ (1 / 3)]];
Print[u1[x, c]];
u[x_, c_] := Abs[(c / x)] ^ (1 / 3) Sign[x c];
u[x, c]
```

$\mathrm{Re}\left[\frac{c^{1/3}}{x^{1/3}}\right]$

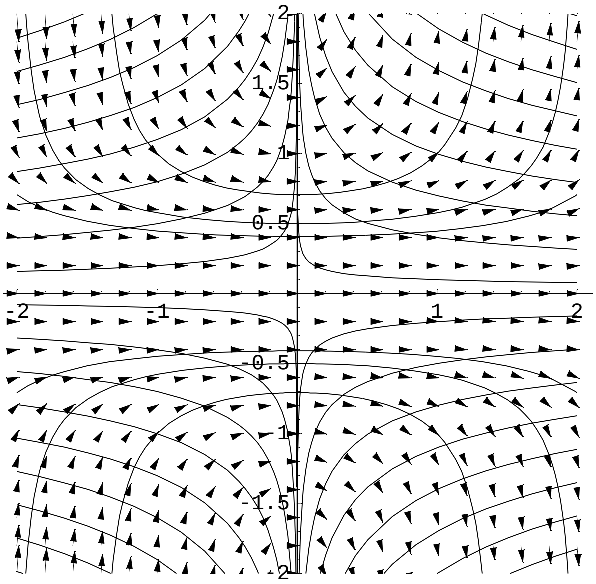
$\mathrm{Abs}\left[\frac{c}{x}\right]^{1/3}\mathrm{Sign}[c\,x]$

```
g2 = Plot[Evaluate[Table[u[x, c^3], {c, -5, 5, 0.3}]],
   {x, -2, 2}, PlotRange → {-2, 2}, AspectRatio → Automatic];
```



f

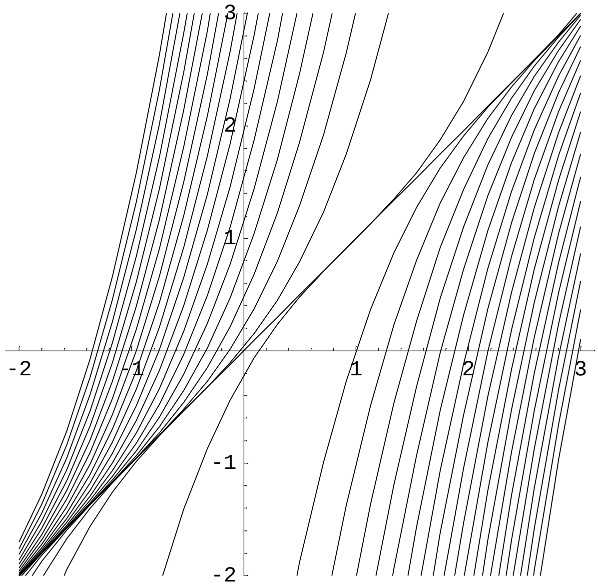
```
Show[g2,g1,g3,g4,AspectRatio→Automatic];
```



2

Wo existiert eine eindeutige Lösung der Differentialgleichung?

```
solution=Evaluate[Table[NDSolve[{y'[x]-1==2 Re[ (y[x]-x)^2]^(1/3),y[1.]==
a},y,{x,-3,3}],{a,-20,20}]];
solution[[1]];
tab=Table[
Re[y[x]]/.solution[[k]],{k,1,Length[solution]}
];
Plot[Evaluate[Union[{{x}},tab]],{x,-2,3},PlotRange->{-2,3},AspectRatio->Automatic];
```



```
Remove["Global`*"]
```

```
f[x_,y_]:=1+(2 (y-x)^2)^(1/3);
(Normal[Series[f[x,y],{y,x0,1}]])//Expand
```

$$1+\frac{2\,2^{1/3}\,x\,x0}{3\,(x^2-2\,x\,x0+x0^2)^{2/3}}-\frac{2\,2^{1/3}\,x0^2}{3\,(x^2-2\,x\,x0+x0^2)^{2/3}}+2^{1/3}\,(x^2-2\,x\,x0+x0^2)^{1/3}-\frac{2\,2^{1/3}\,x\,y}{3\,(x^2-2\,x\,x0+x0^2)^{2/3}}+\frac{2\,2^{1/3}\,x0\,y}{3\,(x^2-2\,x\,x0+x0^2)^{2/3}}$$

```
(Normal[Series[f[x,y],{y,x0,1}]]/.x0->x)//Expand
```

Power::infy : Infinite expression $\frac{1}{0^{2/3}}$ encountered. Mehr…

Power::infy : Infinite expression $\frac{1}{0^{2/3}}$ encountered. Mehr…

∞::indet : Indeterminate expression ComplexInfinity + ComplexInfinity encountered. Mehr…

Indeterminate

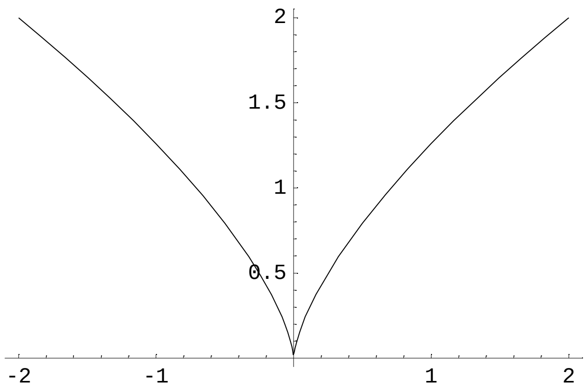
```
D[f[x,y],y]
```

$$\frac{2\,2^{1/3}\,(-x+y)}{3\,((-x+y)^2)^{2/3}}$$

Entwickelt man die Funktion auf der rechten Seite in eine Potenzreihe mit der Variablen y und dem Zentrum y0=x, dass ein Pol entsteht. Die Ableitung nach y (Steigung Richtung y) hat bei y = x einen Pol. Bei y = x kann daher eine Differenz zweier Funktionswerte |f(x,y1)-f(x,y2)| (welche weben dem Pol nicht definiert sind) nicht kleiner als L*|y1-y2| sein

Beispiel x = 0

```
Plot[f[0,y]-f[0,0],{y,-2,2}];
```



Für x ungleich y und kleine Differenzen |y1-y2| ist Lipschitz erfüllbar.

3

a

Lösung

```
DSolve[y'[x]==x y[x],y[x],x ]
```

$\left\{\left\{y[x] \to e^{\frac{x^2}{2}} C[1]\right\}\right\}$

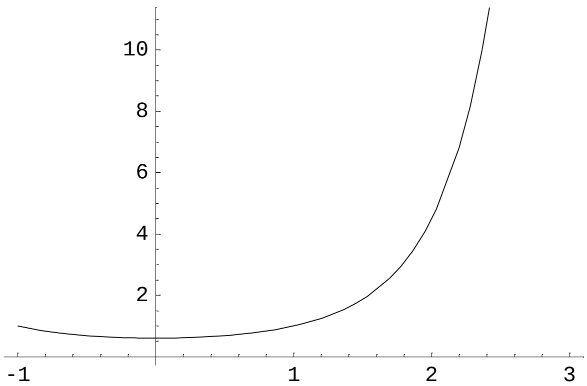
```
DSolve[{y'[x] == x y[x], y[1] == 1}, y[x], x ]
```

$\left\{\left\{y[x] \to e^{-\frac{1}{2}+\frac{x^2}{2}}\right\}\right\}$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

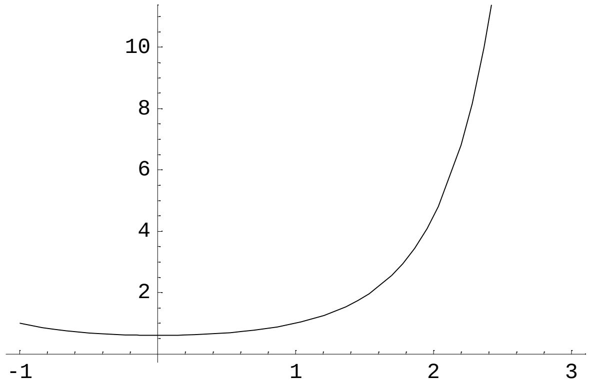
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==x y[x],y[1]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-1,3}];
```

$e^{\frac{1}{2}(-1+x^2)}$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == x y[x], y[1] == 1}, y, {x, -1, 3}, WorkingPrecision → 24];
Plot[y[x] /. solution, {x, -1, 3}];
```



b

Lösung

```
DSolve[y'[x]==Sin[x]/Cos[y[x]],y[x],x ]
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

```
{{y[x] → ArcSin[C[1] - Cos[x]]}}
```

```
DSolve[{y'[x] == Sin[x] / Cos[y[x]], y[0] == 1}, y[x], x ]
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

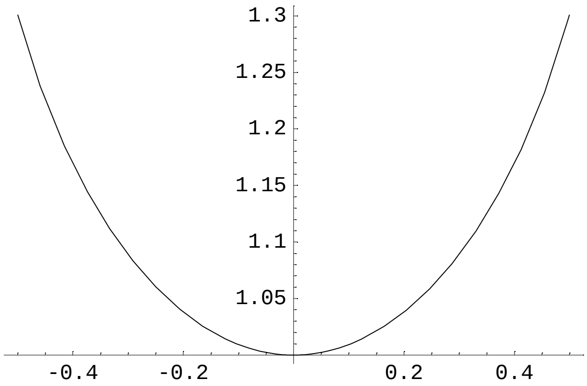
```
{{y[x] → ArcSin[1 - Cos[x] + Sin[1]]}}
```

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==Sin[x]/Cos[y[x]],y[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-0.5,0.5}];
```

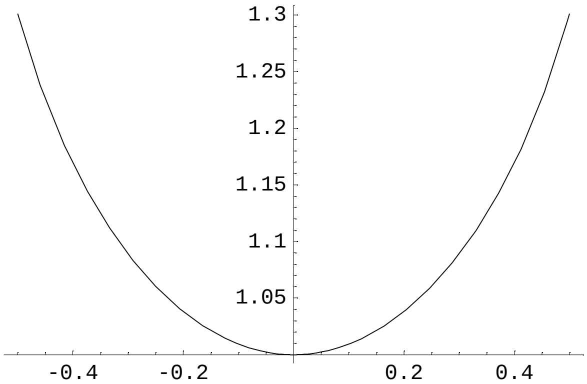
```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

```
ArcSin[1 - Cos[x] + Sin[1]]
```



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == Sin[x] / Cos[y[x]], y[0] == 1}, y, {x, -0.5, 0.5},
  WorkingPrecision → 24]; Plot[y[x] /. solution, {x, -0.5, 0.5}];
```



3

a

Lösung

```
Remove["Global`*"];
DSolve[y'[x]==x y[x],y[x],x ]
```

$\left\{\left\{y[x] \to e^{\frac{x^2}{2}} C[1]\right\}\right\}$

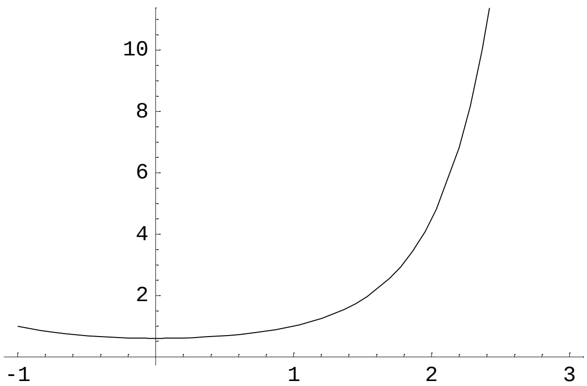
```
DSolve[{y'[x] == x y[x], y[1] == 1}, y[x], x ]
```

$\left\{\left\{y[x] \to e^{-\frac{1}{2}+\frac{x^2}{2}}\right\}\right\}$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

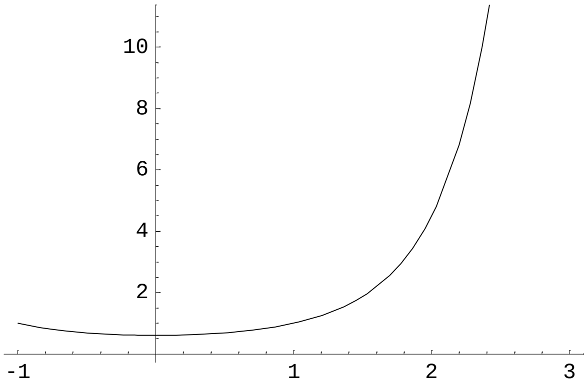
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==x y[x],y[1]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-1,3}];
```

$e^{\frac{1}{2}(-1+x^2)}$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == x y[x], y[1] == 1}, y, {x, -1, 3}, WorkingPrecision → 24];
Plot[y[x] /. solution, {x, -1, 3}];
```



x

Lösung

```
Remove["Global`*"];
DSolve[y'[x]==x^4/y[x],y[x],x ]
```

$$\left\{\left\{y[x] \to -\sqrt{\frac{2}{5}}\,\sqrt{x^5 + 5\,C[1]}\right\}, \left\{y[x] \to \sqrt{\frac{2}{5}}\,\sqrt{x^5 + 5\,C[1]}\right\}\right\}$$

```
DSolve[{y'[x] == x^4 / y[x], y[1] == 2}, y[x], x ]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr…

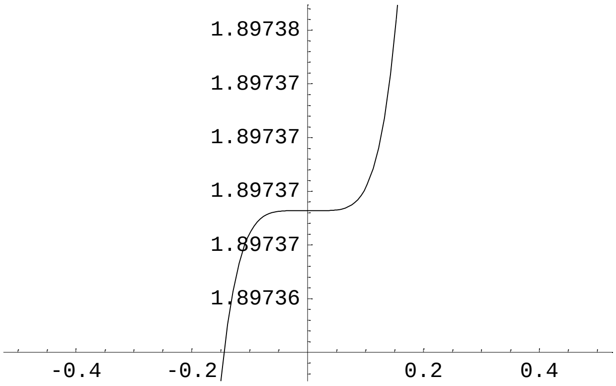
$$\left\{\left\{y[x] \to \sqrt{\frac{2}{5}}\,\sqrt{9 + x^5}\right\}\right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==x^4/y[x],y[1]==2},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-0.5,0.5}];
```

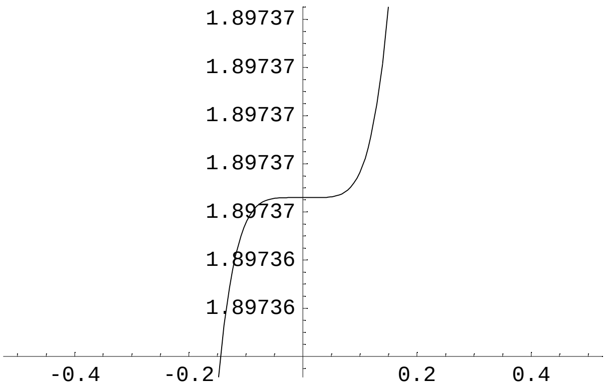
```
DSolve::bvnul : For some branches of the general
   solution, the given boundary conditions lead to an empty solution. Mehr…
```

$$\sqrt{\frac{2}{5}}\ \sqrt{9+x^5}$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution =
 NDSolve[{y'[x] == x^4 / y[x], y[1] == 2}, y, {x, -0.5, 0.5}, WorkingPrecision → 24];
Plot[y[x] /. solution, {x, -0.5, 0.5}];
```



4

a

Lösung

```
DSolve[y'[x]== 4x + 7 y[x]+3,y[x],x ]
```

$$\left\{\left\{y[x] \to -\frac{25}{49} - \frac{4x}{7} + e^{7x} C[1]\right\}\right\}$$

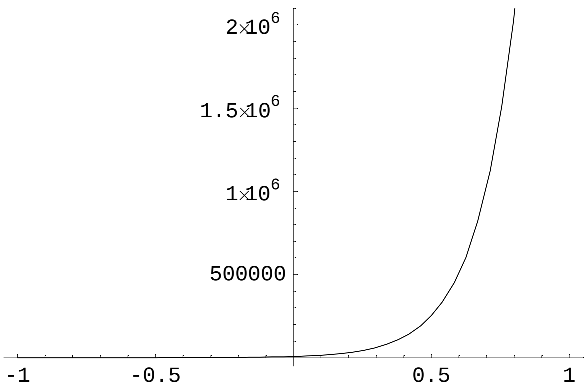
```
DSolve[{y'[x] == 4 x + 7 y[x] + 3, y[-1] == 7}, y[x], x ]
```

$$\left\{\left\{y[x] \to \frac{1}{49} \left(-25 + 340\, e^{7+7x} - 28x\right)\right\}\right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

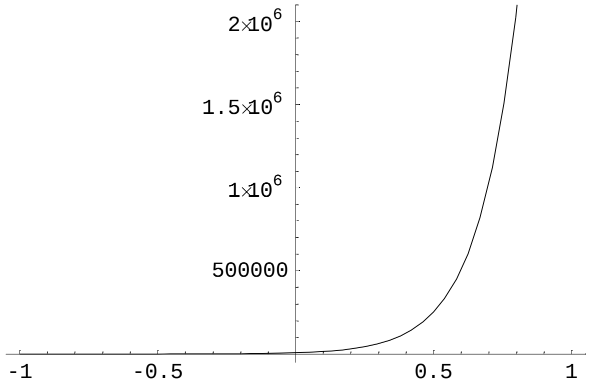
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==4x+7 y[x]+3,y[-1]==7},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-1,1}];
```

$$\frac{1}{49} \left(-25 + 340\, e^{7+7x} - 28x\right)$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == 4 x + 7 y[x] + 3, y[-1] == 7}, y,
  {x, -1, 3}, WorkingPrecision -> 24]; Plot[y[x] /. solution, {x, -1, 1}];
```



b

Lösung

```
DSolve[y'[x]== (y[x]+1)/x+3,y[x],x ]
```

$\{\{y[x] \to x\,C[1] + x\left(-\frac{1}{x} + 3\,\mathrm{Log}[x]\right)\}\}$

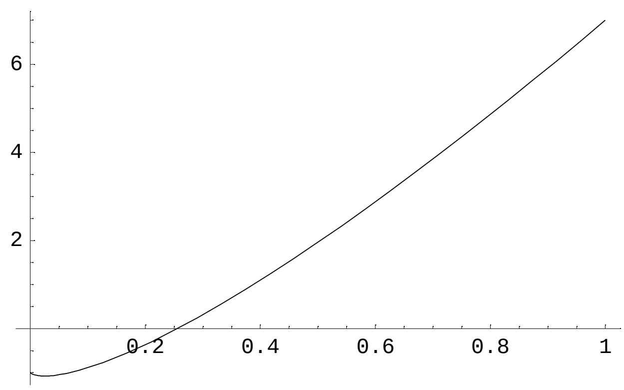
```
DSolve[{y'[x] == (y[x] + 1) / x + 3, y[1] == 7}, y[x], x ]
```

$\{\{y[x] \to -1 + 8\,x + 3\,x\,\mathrm{Log}[x]\}\}$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

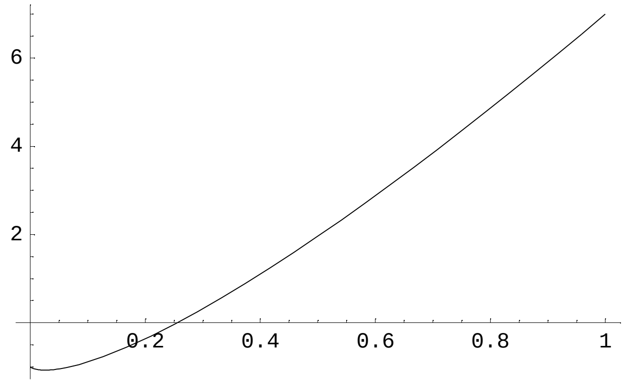
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==(y[x]+1)/x+3,y[1]==7},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,0,1}];
```

$-1 + 8\,x + 3\,x\,\mathrm{Log}[x]$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == (y[x] + 1) / x + 3, y[1] == 7}, y,
  {x, -1, 3}, WorkingPrecision → 24]; Plot[y[x] /. solution, {x, 0, 1}];
```



C

Lösung

```
D[x+Sin[x y+x],x] dx + D[x+Sin[x y+x],y] dy
```

dy x Cos[x + x y] + dx (1 + (1 + y) Cos[x + x y])

```
D[x+Sin[x y+x],x] dx + D[x+Sin[x y+x],y] dy //InputForm
```

dy*x*Cos[x + x*y] + dx*(1 + (1 + y)*Cos[x + x*y])

```
(1 + (1 + y)*Cos[x + x*y])/(x*Cos[x + x*y])
```

$$\frac{(1 + (1 + y)\,\text{Cos}[x + x\,y])\ \text{Sec}[x + x\,y]}{x}$$

```
DSolve[y'[x]==-(1 + (1 + y[x])*Cos[x + x*y[x]])/(x*Cos[x + x*y[x]]), y[x], x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

$$\left\{\left\{y[x] \to \frac{-x - \text{ArcSin}[x - C[1]]}{x}\right\}\right\}$$

```
DSolve[
 {y'[x] == -(1 + (1 + y[x]) * Cos[x + x * y[x]]) / (x * Cos[x + x * y[x]]), y[1 / 2] == 1}, y[x], x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

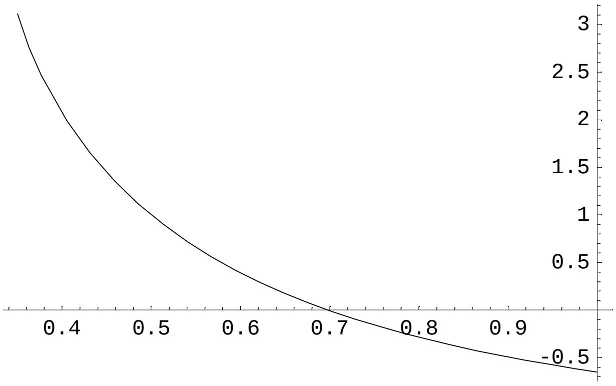
$$\left\{\left\{y[x] \to \frac{-x - \text{ArcSin}[x + \frac{1}{2}\,(-1 - 2\,\text{Sin}[1])]}{x}\right\}\right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==-(1+(1+y[x])*Cos[x+x*y[x]])/(x*Cos[x+x*y[x]]),y[1/2]==
1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,0.35,1}];
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

$$-1+\frac{\mathrm{ArcSin}[\frac{1}{2}-x+\mathrm{Sin}[1]]}{x}$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution =
 NDSolve[{y'[x] == -(1 + (1 + y[x]) * Cos[x + x * y[x]]) / (x * Cos[x + x * y[x]]), y[1 / 2] == 1},
  y, {x, -1, 3}, WorkingPrecision → 24]; Plot[y[x] /. solution, {x, 35, 1}];
```

```
NDSolve::ndsz : At x == 0.341470984834437209602096357892962342 62`24.,
   step size is effectively zero; singularity or stiff system suspected. Mehr…

NDSolve::ndsz : At x == 2.341470984658452687643663425491678234 26`24.,
   step size is effectively zero; singularity or stiff system suspected. Mehr…

InterpolatingFunction::dmval : Input value {2.37928} lies outside the
   range of data in the interpolating function. Extrapolation will be used. Mehr…

InterpolatingFunction::dprec : The precision of input value
   {2.37928} and/or the interpolation grid is insufficient to compute the value. Mehr…

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 2.379277713479137`. Mehr…

InterpolatingFunction::dmval : Input value {3.8835} lies outside the
   range of data in the interpolating function. Extrapolation will be used. Mehr…

InterpolatingFunction::dprec : The precision of input value
   {3.8835} and/or the interpolation grid is insufficient to compute the value. Mehr…

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 3.8834991952187057`. Mehr…

InterpolatingFunction::dmval : Input value {2.35643} lies outside the
   range of data in the interpolating function. Extrapolation will be used. Mehr…

General::stop :
 Further output of InterpolatingFunction::dmval will be suppressed during this calculation. Mehr…

InterpolatingFunction::dprec : The precision of input value
   {2.35643} and/or the interpolation grid is insufficient to compute the value. Mehr…

General::stop :
 Further output of InterpolatingFunction::dprec will be suppressed during this calculation. Mehr…

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 2.356434054419371`. Mehr…

General::stop : Further output of Plot::plnr will be suppressed during this calculation. Mehr…
```