Lösungen

1

a

In[474]:=

```
Remove["Global`*"];
```

In[475]:=

```
f[x_,y_]:= 3*x^2 + 9*x^2*y + 2*x*y^3;
g[x_,y_]:= 1 + 3*x^3 - 2*y + 3*x^2*y^2;
```

In[477]:=

```
y'[x]==-f[x,y[x]]/g[x,y[x]]  (* D'Gl. *)
```

Out[477]=

$$y'[x] == -\frac{3\,x^2 + 9\,x^2\,y[x] + 2\,x\,y[x]^3}{1 + 3\,x^3 - 2\,y[x] + 3\,x^2\,y[x]^2}$$

In[478]:=

```
DSolve[y'[x]==-f[x,y[x]]/g[x,y[x]],y[x],x ]
```

Out[478]=

$$\left\{\left\{y[x] \to \frac{1}{3 x^2} - \left(2^{1/3} \left(-1 + 3 x^2 + 9 x^5\right)\right) \Big/ \left(3 x^2 \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right) + \frac{1}{3\, 2^{1/3} x^2} \left(\left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right)\right\},\right.$$

$$\left\{y[x] \to \frac{1}{3 x^2} + \left(\left(1 + i \sqrt{3}\right) \left(-1 + 3 x^2 + 9 x^5\right)\right) \Big/ \left(3\, 2^{2/3} x^2 \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right) - \frac{1}{6\, 2^{1/3} x^2} \left(\left(1 - i \sqrt{3}\right) \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right)\right\},$$

$$\left.\left\{y[x] \to \frac{1}{3 x^2} + \left(\left(1 - i \sqrt{3}\right) \left(-1 + 3 x^2 + 9 x^5\right)\right) \Big/ \left(3\, 2^{2/3} x^2 \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right) - \frac{1}{6\, 2^{1/3} x^2} \left(\left(1 + i \sqrt{3}\right) \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1] + \sqrt{4 \left(-1 + 3 x^2 + 9 x^5\right)^3 + \left(2 - 9 x^2 - 27 x^5 - 27 x^7 + 27 x^4 C[1]\right)^2}\right)^{1/3}\right)\right\}\right\}$$

In[479]:=

```
solv1=DSolve[y'[x]==-f[x,y[x]]/g[x,y[x]],y[x],x ]//N
```

Out[479]=

$$\{\{y[x] \to \frac{0.333333}{x^2} - (0.419974\,(-1. + 3.\,x^2 + 9.\,x^5)) \Big/ \Big(x^2 \Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big) + \frac{1}{x^2}\Big(0.264567\,\Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big)\},$$

$$\{y[x] \to \frac{0.333333}{x^2} + ((0.209987 + 0.363708\,i)\,(-1. + 3.\,x^2 + 9.\,x^5)) \Big/ \Big(x^2 \Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big) - \frac{1}{x^2}\Big((0.132283 - 0.229122\,i)\,\Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big)\},$$

$$\{y[x] \to \frac{0.333333}{x^2} + ((0.209987 - 0.363708\,i)\,(-1. + 3.\,x^2 + 9.\,x^5)) \Big/ \Big(x^2 \Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big) - \frac{1}{x^2}\Big((0.132283 + 0.229122\,i)\,\Big(2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1] + \sqrt{\left(4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 - 27.\,x^5 - 27.\,x^7 + 27.\,x^4\,C[1])^2\right)}\Big)^{1/3}\Big)\}\}$$

In[480]:=

```
z[x_]:=y[x]/.solv1[[1]]
```

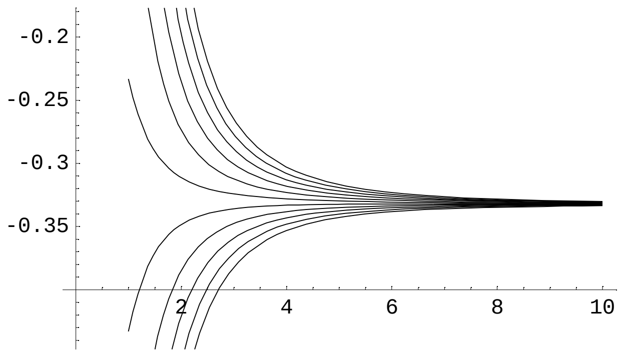
In[481]:=

```
z[x]/.C[1]->c
```

Out[481]=

$$\frac{0.333333}{x^2} - (0.419974\,(-1. + 3.\,x^2 + 9.\,x^5)) \Big/ \Big(x^2 \Big(2. - 9.\,x^2 + 27.\,c\,x^4 - 27.\,x^5 - 27.\,x^7 + \sqrt{4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 + 27.\,c\,x^4 - 27.\,x^5 - 27.\,x^7)^2}\Big)^{1/3}\Big) + \frac{1}{x^2}\Big(0.264567\,\Big(2. - 9.\,x^2 + 27.\,c\,x^4 - 27.\,x^5 - 27.\,x^7 + \sqrt{4.\,(-1. + 3.\,x^2 + 9.\,x^5)^3 + (2. - 9.\,x^2 + 27.\,c\,x^4 - 27.\,x^5 - 27.\,x^7)^2}\Big)^{1/3}\Big)$$

In[482]:=
```
Plot[Evaluate[Table[z[x]/.C[1]->c,{c,-5,5}]],{x,1,10}];
```



In[483]:=
```
(*Gesucht ist nur die reelle Lösung! *)
```

b

In[484]:=
```
Remove["Global`*"];
```

In[485]:=
```
f[x_,y_]:= 3*x^2 + 9*x^2*y + 2*x*y^3;
g[x_,y_]:= 1 + 3*x^3 - 2*y + 3*x^2*y^2;
```

In[487]:=
```
F[x_,y_]:= x^2 y^3 +3x^3 y +x^3 - y^2 +y
```

In[488]:=
```
Dt[F[x, y]]  (* Totales Differential *)
```

Out[488]=

$3 x^2 \mathrm{Dt}[x] + 9 x^2 y \mathrm{Dt}[x] + 2 x y^3 \mathrm{Dt}[x] + \mathrm{Dt}[y] + 3 x^3 \mathrm{Dt}[y] - 2 y \mathrm{Dt}[y] + 3 x^2 y^2 \mathrm{Dt}[y]$

In[489]:=
```
Collect[%,{ Dt[x], Dt[y]}]
```

Out[489]=

$(3 x^2 + 9 x^2 y + 2 x y^3) \mathrm{Dt}[x] + (1 + 3 x^3 - 2 y + 3 x^2 y^2) \mathrm{Dt}[y]$

In[490]:=
```
D[F[x,y],x]==f[x,y]
```

Out[490]=
```
True
```

In[491]:=
```
D[F[x,y],y]==g[x,y]
```

Out[491]=
```
True
```

In[492]:=

```
Solve[F[x,y]==c,{y}]
```

Out[492]=

$$\left\{\left\{y\to\frac{1}{3x^2}-\left(2^{1/3}\left(-1+3x^2+9x^5\right)\right)\Big/\left(3x^2\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)+\frac{1}{3\,2^{1/3}x^2}\left(\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)\right\},\right.$$

$$\left\{y\to\frac{1}{3x^2}+\left(\left(1+i\sqrt{3}\right)\left(-1+3x^2+9x^5\right)\right)\Big/\left(3\,2^{2/3}x^2\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)-\frac{1}{6\,2^{1/3}x^2}\left(\left(1-i\sqrt{3}\right)\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)\right\},$$

$$\left.\left\{y\to\frac{1}{3x^2}+\left(\left(1-i\sqrt{3}\right)\left(-1+3x^2+9x^5\right)\right)\Big/\left(3\,2^{2/3}x^2\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)-\frac{1}{6\,2^{1/3}x^2}\left(\left(1+i\sqrt{3}\right)\left(2-9x^2+27cx^4-27x^5-27x^7+\sqrt{4\left(-1+3x^2+9x^5\right)^3+\left(2-9x^2+27cx^4-27x^5-27x^7\right)^2}\right)^{1/3}\right)\right\}\right\}$$

C

In[493]:=

```
Remove["Global`*"];
```

In[494]:=

```
f[x_,y_]:= 3*x^2 + 9*x^2*y + 2*x*y^3;
g[x_,y_]:= 1 + 3*x^3 - 2*y + 3*x^2*y^2;
```

In[496]:=

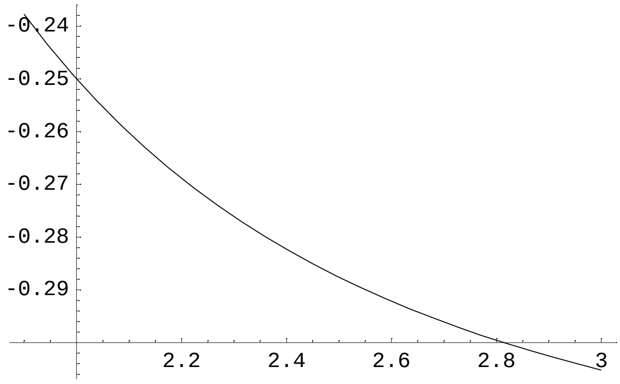
```
Flatten[DSolve[{y'[x]==-f[x,y[x]]/g[x,y[x]],y[2]== -0.25},y[x],x ]];
```

```
DSolve::bvnul : For some branches of the general
   solution, the given boundary conditions lead to an empty solution. Mehr…

DSolve::bvnul : For some branches of the general
   solution, the given boundary conditions lead to an empty solution. Mehr…
```

In[497]:=
```
solution=NDSolve[{y'[x]==-f[x,y[x]]/g[x,y[x]],y[2]== -0.25},y,{x,1.9,3},
    WorkingPrecision→24];Plot[y[x]/.solution,{x,1.9,3}];
```

NDSolve::precw : The precision of the differential equation ($\left\{y'[x] == -\frac{3x^2 + 9x^2 y[x] + 2xy[x]^3}{1 + 3x^3 - 2y[x] + 3x^2 y[\ll 1 \gg]^2}, y[2] == -0.25\right\}$) is less than WorkingPrecision (24.\`). Mehr…



d

In[498]:=
```
Remove["Global`*"];
```

In[499]:=
```
f[x_,y_]:= 3*x^2 + 9*x^2*y + 2*x*y^3;
g[x_,y_]:= 1 + 3*x^3 - 2*y + 3*x^2*y^2;
```

In[501]:=
```
solv2 = DSolve[y'[x]==-(f[x,y[x]]+x)/g[x,y[x]],y[x],x ]
```

Out[501]=

$$\Big\{\Big\{y[x] \to \frac{1}{3x^2} - (-4+12x^2+36x^5) \Big/ \Big(3\,2^{2/3} x^2 \Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1]+\sqrt{\big(4(-4+12x^2+36x^5)^3 + (16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big) + \frac{1}{6\,2^{1/3} x^2}\Big(\Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1] + \sqrt{\big(4(-4+12x^2+36x^5)^3+(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big)\Big\},$$

$$\Big\{y[x] \to \frac{1}{3x^2} + \Big((1+i\sqrt{3})(-4+12x^2+36x^5)\Big) \Big/ \Big(6\,2^{2/3} x^2 \Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1]+\sqrt{\big(4(-4+12x^2+36x^5)^3 + (16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big) - \frac{1}{12\,2^{1/3} x^2}\Big((1-i\sqrt{3})\Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1] + \sqrt{\big(4(-4+12x^2+36x^5)^3+(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big)\Big\},$$

$$\Big\{y[x] \to \frac{1}{3x^2} + \Big((1-i\sqrt{3})(-4+12x^2+36x^5)\Big) \Big/ \Big(6\,2^{2/3} x^2 \Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1]+\sqrt{\big(4(-4+12x^2+36x^5)^3 + (16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big) - \frac{1}{12\,2^{1/3} x^2}\Big((1+i\sqrt{3})\Big(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1] + \sqrt{\big(4(-4+12x^2+36x^5)^3+(16-72x^2-216x^5-108x^6-216x^7+216x^4 C[1])^2\big)}\Big)^{1/3}\Big)\Big\}\Big\}$$

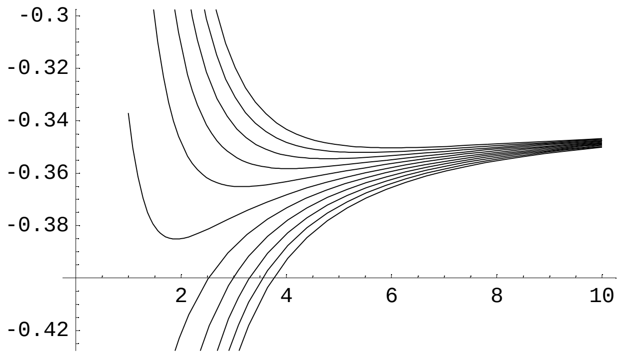
In[502]:=
```
z[x_]:=y[x]/.solv2[[1]]
```

In[503]:=
```
z[x]/.C[1]->c
```

Out[503]=

$$\frac{1}{3x^2} - (-4+12x^2+36x^5) \Big/ \Big(3\,2^{2/3} x^2 \Big(16-72x^2+216\,c\,x^4-216x^5-108x^6-216x^7 + \sqrt{\big(4(-4+12x^2+36x^5)^3+(16-72x^2+216\,c\,x^4-216x^5-108x^6-216x^7)^2\big)}\Big)^{1/3}\Big) + \frac{1}{6\,2^{1/3} x^2}\Big(\Big(16-72x^2+216\,c\,x^4-216x^5-108x^6-216x^7 + \sqrt{4(-4+12x^2+36x^5)^3+(16-72x^2+216\,c\,x^4-216x^5-108x^6-216x^7)^2}\Big)^{1/3}\Big)$$

In[504]:=
```
Plot[Evaluate[Table[z[x]/.C[1]->c,{c,-5,5}]],{x,1,10}];
```



2

a) Plot der exakten Lösung

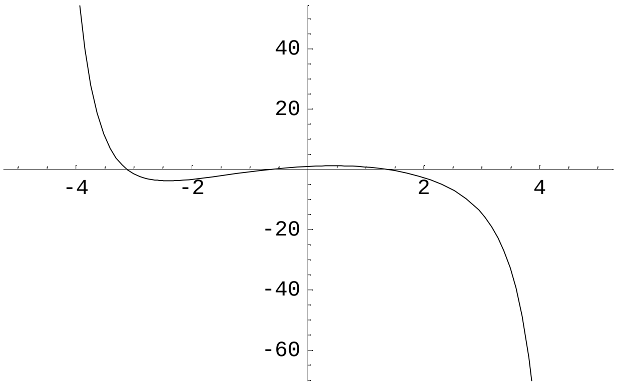
In[505]:=
```
Remove["Global`*"];
DSolve[y''[x]- x y'[x]+ 2 y[x]==0,y,x]
```

Out[506]=

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \left(-1+x^2\right) C[1] + \frac{1}{4} C[2] \left(-2 e^{\frac{x^2}{2}} x - \sqrt{2\pi}\, \text{Erfi}\left[\frac{x}{\sqrt{2}}\right] + \sqrt{2\pi}\, x^2\, \text{Erfi}\left[\frac{x}{\sqrt{2}}\right]\right)\right]\right\}\right\}$$

In[507]:=
```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]- x y'[x]+ 2 y[x]==0, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

$$\frac{1}{4}\left(4 + 2 e^{\frac{x^2}{2}} x - 4 x^2 - \sqrt{2\pi}\,\left(-1+x^2\right) \text{Erfi}\left[\frac{x}{\sqrt{2}}\right]\right)$$



b) Plot der exakten Lösung

In[512]:=

```
Remove["Global`*"];
DSolve[y''[x]+ x y'[x]+ 2 y[x]==0,y,x]
```

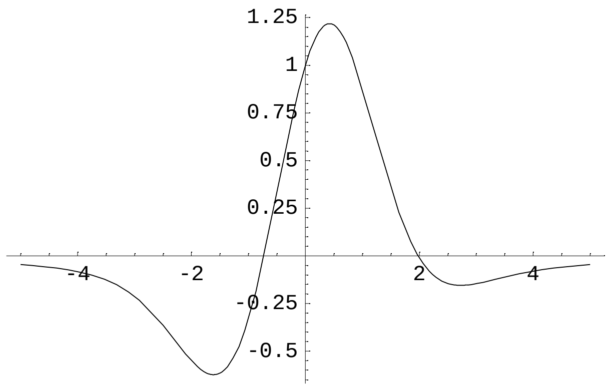
Out[513]=

$$\left\{\left\{y \to \text{Function}\left[\{x\},\ \sqrt{2}\, e^{-\frac{x^2}{2}}\, x\, C[1] - \frac{e^{-\frac{x^2}{2}} \sqrt{-x^2}\, C[2] \left(-2\sqrt{\pi} - \frac{2\sqrt{2}\, e^{\frac{x^2}{2}}}{\sqrt{-x^2}} + 2\sqrt{\pi}\left(1 - \frac{\sqrt{-x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]}{\sqrt{x^2}}\right)\right)}{2\sqrt{2}}\right]\right\}\right\}$$

In[514]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ x y'[x]+ 2 y[x]==0, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

$$1 + e^{-\frac{x^2}{2}}\, x - e^{-\frac{x^2}{2}} \sqrt{\frac{\pi}{2}}\, \sqrt{x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]$$



c) Plot der exakten Lösung

In[519]:=

```
Remove["Global`*"];
DSolve[y''[x]+ x y'[x]+ 2 y[x]==x,y,x]
```

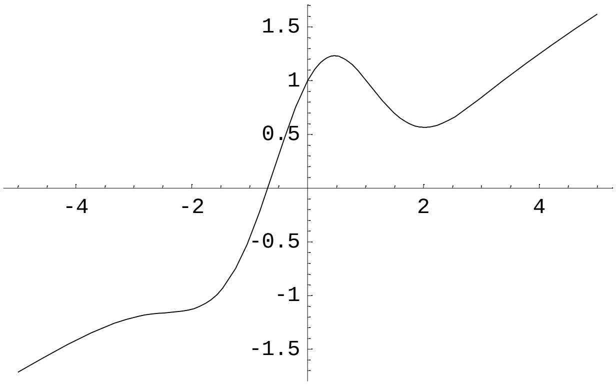
Out[520]=

$$\left\{\left\{y \to \text{Function}\left[\{x\},\ \frac{x}{3} + \sqrt{2}\, e^{-\frac{x^2}{2}}\, x\, C[1] - \frac{e^{-\frac{x^2}{2}} \sqrt{-x^2}\, C[2] \left(-2\sqrt{\pi} - \frac{2\sqrt{2}\, e^{\frac{x^2}{2}}}{\sqrt{-x^2}} + 2\sqrt{\pi}\left(1 - \frac{\sqrt{-x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]}{\sqrt{x^2}}\right)\right)}{2\sqrt{2}}\right]\right\}\right\}$$

In[521]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ x y'[x]+ 2 y[x]==x, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

$$\frac{1}{6}\, e^{-\frac{x^2}{2}} \left(4\, x + 2\, e^{\frac{x^2}{2}}\, (3 + x) - 3\, \sqrt{2\,\pi}\, \sqrt{x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]\right)$$



d) Plot der exakten Lösung

In[526]:=

```
Remove["Global`*"];
DSolve[y''[x]+ x y'[x]+ 2 y[x]==x^2,y,x]
```

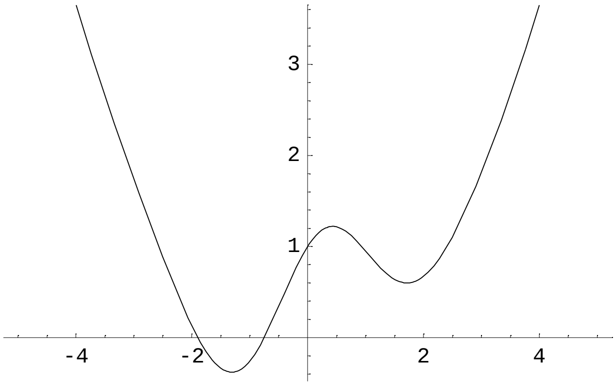
Out[527]=

$$\Big\{\Big\{y \to \text{Function}\Big[\{x\},\ \sqrt{2}\, e^{-\frac{x^2}{2}}\, x\, C[1] + \frac{e^{-\frac{x^2}{2}}\, x \left(2\, e^{\frac{x^2}{2}}\, x\, \sqrt{x^2} - 4\, \sqrt{2\,\pi}\, \sqrt{x^2}\, \text{Erfi}\left[\frac{x}{\sqrt{2}}\right] + 3\, \sqrt{2\,\pi}\, x\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]\right)}{8\, \sqrt{x^2}} - \frac{e^{-\frac{x^2}{2}}\, \sqrt{-x^2}\, C[2] \left(-2\, \sqrt{\pi} - \frac{2\, \sqrt{2}\, e^{\frac{x^2}{2}}}{\sqrt{-x^2}} + 2\, \sqrt{\pi} \left(1 - \frac{\sqrt{-x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]}{\sqrt{x^2}}\right)\right)}{2\, \sqrt{2}}\Big]\Big\}\Big\}$$

In[528]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ x y'[x]+ 2 y[x]==x^2, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

$$\frac{1}{8} e^{-\frac{x^2}{2}} \left(8 e^{\frac{x^2}{2}} + 8 x + 2 e^{\frac{x^2}{2}} x^2 - 4 \sqrt{2\pi}\, x \,\text{Erfi}\left[\frac{x}{\sqrt{2}}\right] - \sqrt{2\pi} \sqrt{x^2}\, \text{Erfi}\left[\frac{\sqrt{x^2}}{\sqrt{2}}\right]\right)$$



e) Plot der exakten Lösung

In[533]:=

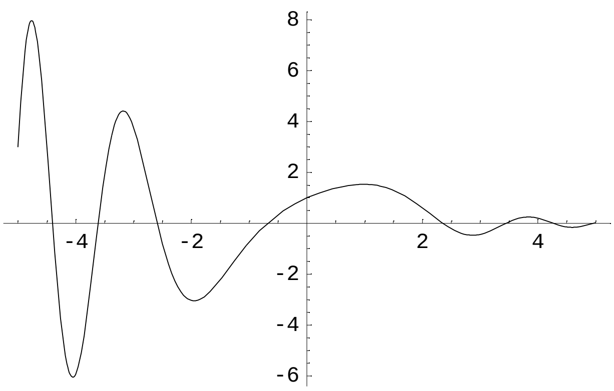
```
Remove["Global`*"];
DSolve[y''[x]+ y'[x]+ x^2 y[x]==0,y,x]
```

Out[534]=

$$\left\{\left\{y \to \text{Function}\left[\{x\}, e^{-\frac{x}{2}-\frac{i x^2}{2}} C[1]\, \text{HermiteH}\left[-\frac{1}{2}+\frac{i}{8}, (-1)^{1/4} x\right] + e^{-\frac{x}{2}-\frac{i x^2}{2}} C[2]\, \text{Hypergeometric1F1}\left[\frac{1}{4}-\frac{i}{16}, \frac{1}{2}, i x^2\right]\right]\right\}\right\}$$

In[535]:=

```
Remove["Global`*"];
solution=NDSolve[{y''[x]+ y'[x]+ x^2  y[x]==0, y[0]==1,y'[0]==1},y,{x,-5,5},
    WorkingPrecision→24];Plot[y[x]/.solution,{x,-5,5}];
```



f) Plot der exakten Lösung

In[537]:=

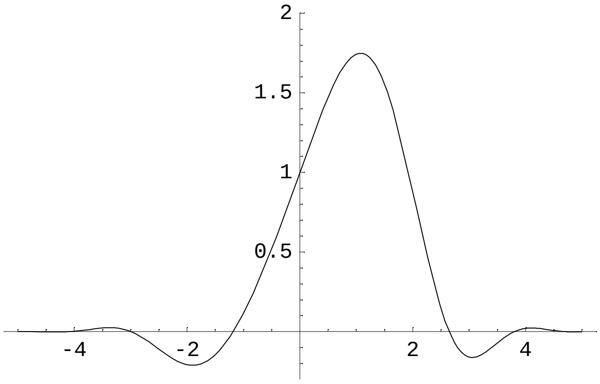
```
Remove["Global`*"];
DSolve[y''[x]+ x y'[x]+ x^2 y[x]==0,y,x]
```

Out[538]=

$$\left\{\left\{y \to \text{Function}\left[\{x\},\ e^{-\frac{1}{4} i \left(-i+\sqrt{3}\right) x^2} C[1]\ \text{HermiteH}\left[\frac{1}{6} i \left(3 i+\sqrt{3}\right),\ \left(\frac{1}{2}+\frac{i}{2}\right) 3^{1/4} x\right] + e^{-\frac{1}{4} i \left(-i+\sqrt{3}\right) x^2} C[2]\ \text{Hypergeometric1F1}\left[-\frac{1}{12} i \left(3 i+\sqrt{3}\right),\ \frac{1}{2},\ \frac{1}{2} i \sqrt{3}\, x^2\right]\right]\right\}\right\}$$

In[539]:=

```
Remove["Global`*"];
solution=NDSolve[{y''[x]+ x y'[x]+ x^2  y[x]==0, y[0]==1,y'[0]==1},y,{x,-5,5},
    WorkingPrecision→24];Plot[y[x]/.solution,{x,-5,5},PlotRange->{-0.3,2}];
```



g) Plot der Lösung

In[541]:=

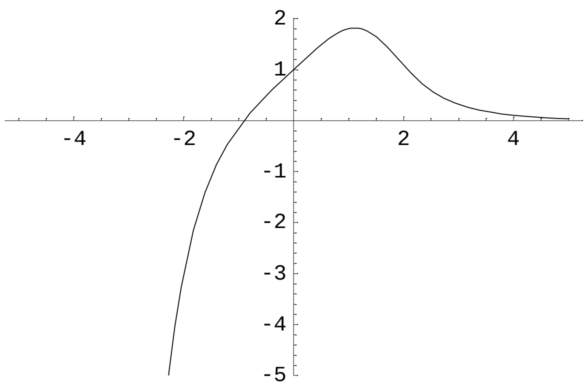
```
Remove["Global`*"];
DSolve[y''[x]+ x^2 y'[x]+ x^2 y[x]==0,y,x]
```

Out[542]=

$$\text{DSolve}[x^2 y[x] + x^2 y'[x] + y''[x] == 0,\ y,\ x]$$

In[543]:=

```
Remove["Global`*"];
solution=NDSolve[{y''[x]+ x^2 y'[x]+ x^2  y[x]==0, y[0]==1,y'[0]==1},y,{x,-5,5},
    WorkingPrecision→24];Plot[y[x]/.solution,{x,-5,5},PlotRange->{-5,2}];
```



h) Plot der Lösung

In[545]:=

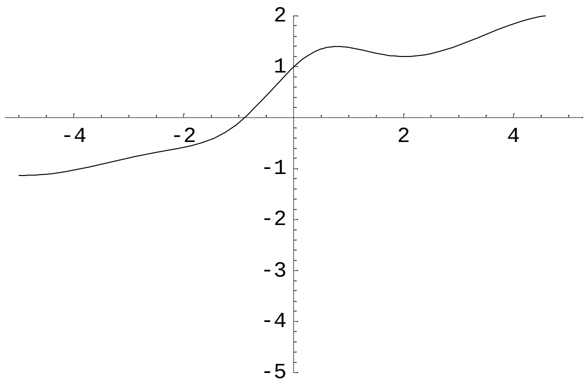
```
Remove["Global`*"];
DSolve[y''[x]+ x y'[x]+ Cos[x] y[x]==0,y,x]
```

Out[546]=

```
DSolve[Cos[x] y[x] + x y′[x] + y″[x] == 0, y, x]
```

In[547]:=

```
Remove["Global`*"];
solution=NDSolve[{y''[x]+ x y'[x]+ Cos[x] y[x]==0, y[0]==1,y'[0]==1},y,{x,-5,5},
    WorkingPrecision→24];Plot[y[x]/.solution,{x,-5,5},PlotRange->{-5,2}];
```



3

a

In[549]:=

```
Remove["Global`*"];
DSolve[y''[x]+ a y'[x]+ b y[x]==0,y,x]
```

Out[550]=

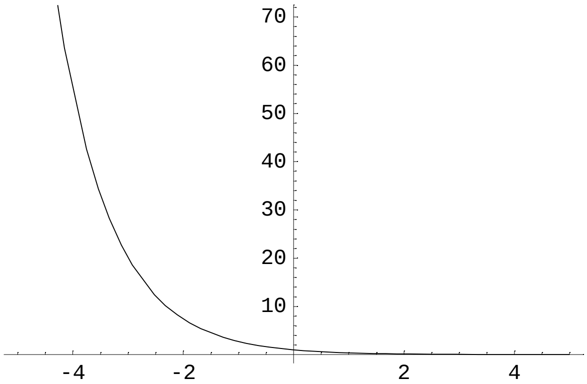
$$\left\{\left\{y \to \text{Function}\left[\{x\},\ e^{\frac{1}{2}\left(-a-\sqrt{a^2-4 b}\right) x} C[1] + e^{\frac{1}{2}\left(-a+\sqrt{a^2-4 b}\right) x} C[2]\right]\right\}\right\}$$

b

In[551]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y'[x]+ y[x]==0, y[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

e^{-x}

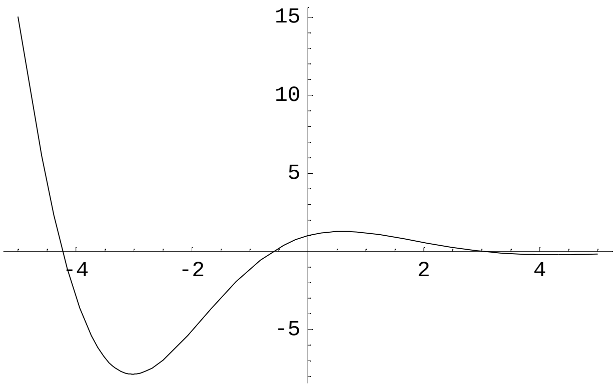


c

In[556]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ y'[x]+ y[x]==0, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,5}];
```

$$e^{-x/2}\left(\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right]+\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right)$$

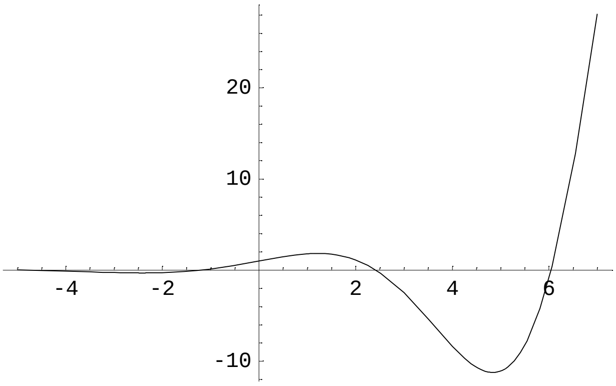


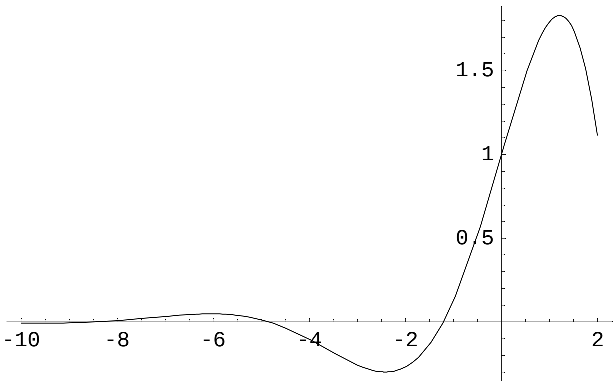
d

In[561]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]- y'[x]+ y[x]==0, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,7}];
Plot[y[x],{x,-10,2}];
```

$$\frac{1}{3} e^{x/2} \left(3 \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + \sqrt{3} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right)$$



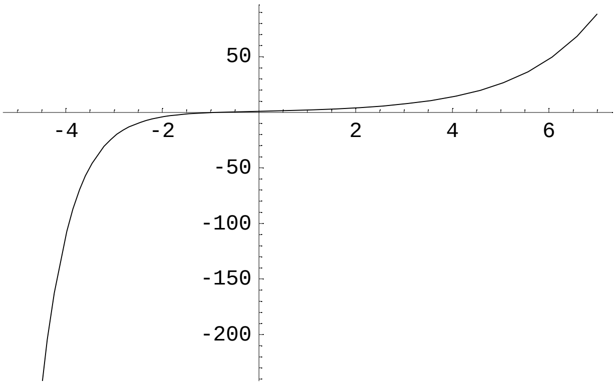


e

In[567]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ y'[x]- y[x]==0, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,7}];
```

$$\frac{1}{10}\, e^{-\frac{1}{2}\left(1+\sqrt{5}\right) x} \left(5-3\sqrt{5}+\left(5+3\sqrt{5}\right) e^{\sqrt{5}\, x}\right)$$

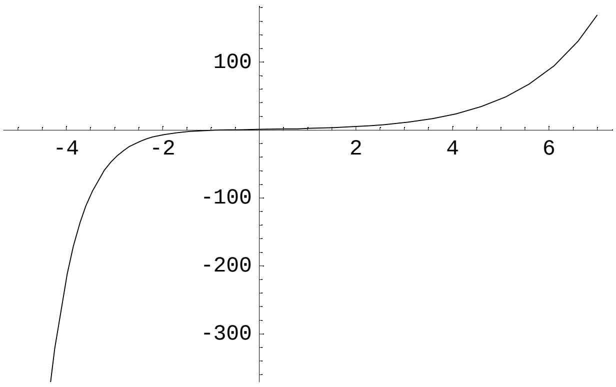


f

In[572]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ y'[x]- y[x]==x, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-5,7}];
```

$$\frac{1}{5}\, e^{-\frac{1}{2}\left(1+\sqrt{5}\right) x} \left(5-3\sqrt{5}+\left(5+3\sqrt{5}\right) e^{\sqrt{5}\, x}-5\, e^{\frac{1}{2}\left(1+\sqrt{5}\right) x}\left(1+x\right)\right)$$

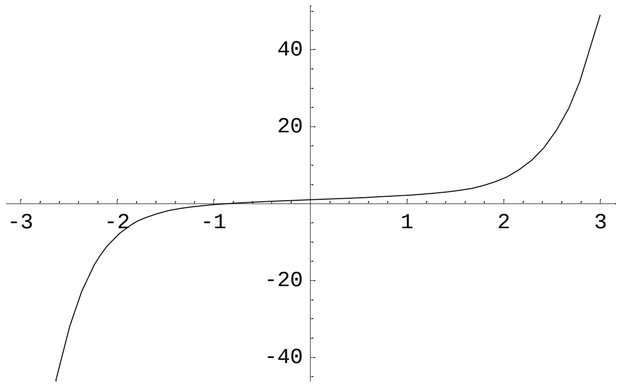


g

In[577]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ y'[x]- y[x]==x^5, y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-3,3}];
```

$$\frac{1}{10} e^{-\frac{1}{2}(1+\sqrt{5})x} \left(4805 - 2163\sqrt{5} + (4805 + 2163\sqrt{5})\, e^{\sqrt{5}\,x} - 10\, e^{\frac{1}{2}(1+\sqrt{5})x} (960 + 600\,x + 180\,x^2 + 40\,x^3 + 5\,x^4 + x^5)\right)$$

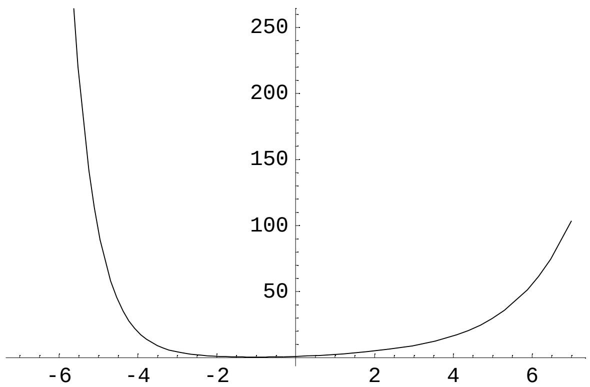


h

In[582]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]+ y'[x]- y[x]==Cos[x], y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-7,7}];
```

$$\frac{1}{10}\left(-4\,\mathrm{Cos}[x] + e^{-\frac{1}{2}(1+\sqrt{5})x}\left(7 - 3\sqrt{5} + (7 + 3\sqrt{5})\, e^{\sqrt{5}\,x} + 2\, e^{\frac{1}{2}(1+\sqrt{5})x}\,\mathrm{Sin}[x]\right)\right)$$

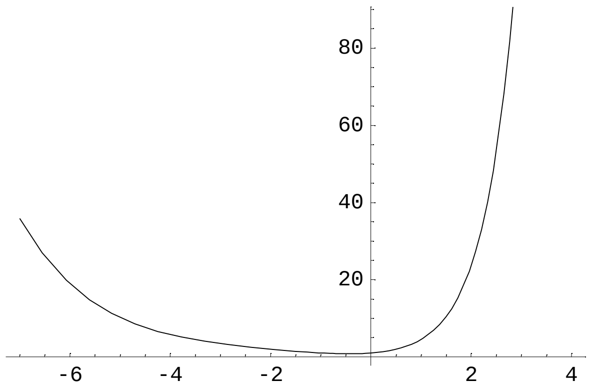


i

In[587]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]- y'[x]- y[x]==Cos[x], y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-7,4}];
```

$$\frac{1}{10}\left(-4\,\mathrm{Cos}[x]+e^{-\frac{1}{2}\left(-1+\sqrt{5}\right)x}\left(7-\sqrt{5}+\left(7+\sqrt{5}\right)e^{\sqrt{5}\,x}-2\,e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}\,\mathrm{Sin}[x]\right)\right)$$

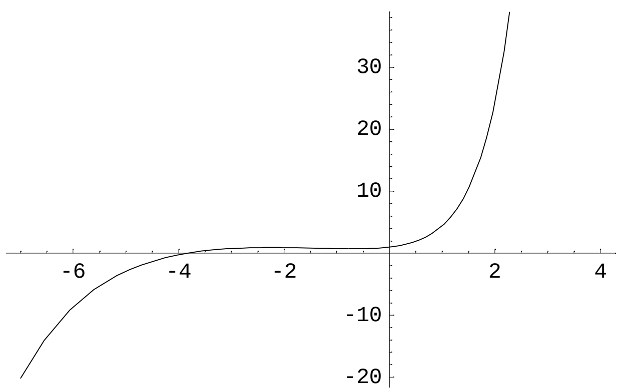


j

In[592]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]- y'[x]- y[x]==x+Cos[x/5+1], y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-7,4}];
```

$$\frac{1}{7010}\left(-6500\,\mathrm{Cos}\left[\frac{5+x}{5}\right]+e^{-\frac{1}{2}\left(-1+\sqrt{5}\right)x}\left(-2804\sqrt{5}-7010\,e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}\left(-1+x\right)+3250\,\mathrm{Cos}[1]+600\sqrt{5}\,\mathrm{Cos}[1]+625\,\mathrm{Sin}[1]+385\sqrt{5}\,\mathrm{Sin}[1]+e^{\sqrt{5}\,x}\left(2804\sqrt{5}-50\left(-65+12\sqrt{5}\right)\mathrm{Cos}[1]+\left(625-385\sqrt{5}\right)\mathrm{Sin}[1]\right)-1250\,e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}\,\mathrm{Sin}\left[\frac{5+x}{5}\right]\right)\right)$$

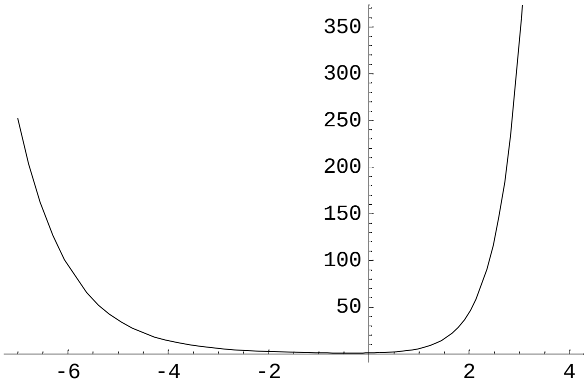


k

In[597]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y''[x]- y'[x]-2 y[x]==x+Cos[x/5+1], y[0]==1,y'[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-7,4}];
```

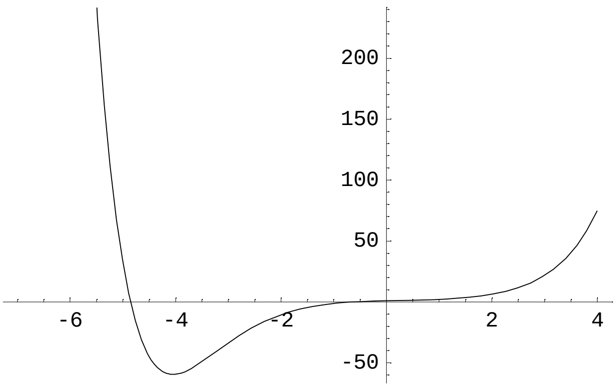
$$\frac{1}{15756}\left(3939+11817\,e^{2x}-7878\,x+5050\,e^{-x}\,\mathrm{Cos}[1]+2600\,e^{2x}\,\mathrm{Cos}[1]-7650\,\mathrm{Cos}\left[\frac{5+x}{5}\right]+1010\,e^{-x}\,\mathrm{Sin}[1]-260\,e^{2x}\,\mathrm{Sin}[1]-750\,\mathrm{Sin}\left[\frac{5+x}{5}\right]\right)$$



In[602]:=

```
Remove["Global`*"];
solv = Flatten[
DSolve[{y'''[x]+y''[x]- y'[x]-2 y[x]==x, y[0]==1,y'[0]==1,y''[0]==-1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-7,4}];
```

```
(ⅇ^(x Root[-2-#1+#1^2+#1^3&,2]) (-14 + 4 Root[-2 - #1 + #1^2 + #1^3 &, 2] + 6 Root[-2 - #1 + #1^2 + #1^3 &, 2]^2 +
      (8 + 3 Root[-2 - #1 + #1^2 + #1^3 &, 1]^2) Root[-2 - #1 + #1^2 + #1^3 &, 3] -
      3 (-4 + Root[-2 - #1 + #1^2 + #1^3 &, 1]) Root[-2 - #1 + #1^2 + #1^3 &, 3]^2) -
   (-1 + 2 x) (5 + 2 Root[-2 - #1 + #1^2 + #1^3 &, 1] Root[-2 - #1 + #1^2 + #1^3 &, 2]^2 +
      2 Root[-2 - #1 + #1^2 + #1^3 &, 1]^2 Root[-2 - #1 + #1^2 + #1^3 &, 3] +
      2 Root[-2 - #1 + #1^2 + #1^3 &, 2] Root[-2 - #1 + #1^2 + #1^3 &, 3]^2) +
   ⅇ^(x Root[-2-#1+#1^2+#1^3&,3]) (29 + (-4 + 3 Root[-2 - #1 + #1^2 + #1^3 &, 1]^2) Root[-2 - #1 + #1^2 + #1^3 &, 3] +
      3 (-2 + Root[-2 - #1 + #1^2 + #1^3 &, 1]) Root[-2 - #1 + #1^2 + #1^3 &, 3]^2 +
      3 Root[-2 - #1 + #1^2 + #1^3 &, 2]^2
        (-4 + 2 Root[-2 - #1 + #1^2 + #1^3 &, 1] + Root[-2 - #1 + #1^2 + #1^3 &, 3]) +
      Root[-2 - #1 + #1^2 + #1^3 &, 2] (-8 + 3 Root[-2 - #1 + #1^2 + #1^3 &, 3]^2)) +
   ⅇ^(x Root[-2-#1+#1^2+#1^3&,1]) (-3 Root[-2 - #1 + #1^2 + #1^3 &, 2]^2 (-2 + Root[-2 - #1 + #1^2 + #1^3 &, 3]) -
      2 Root[-2 - #1 + #1^2 + #1^3 &, 3] (2 + 3 Root[-2 - #1 + #1^2 + #1^3 &, 3]) +
      Root[-2 - #1 + #1^2 + #1^3 &, 2] (4 + 3 Root[-2 - #1 + #1^2 + #1^3 &, 3]^2))) /
 (20 + 8 Root[-2 - #1 + #1^2 + #1^3 &, 1] Root[-2 - #1 + #1^2 + #1^3 &, 2]^2 +
   8 Root[-2 - #1 + #1^2 + #1^3 &, 1]^2 Root[-2 - #1 + #1^2 + #1^3 &, 3] +
   8 Root[-2 - #1 + #1^2 + #1^3 &, 2] Root[-2 - #1 + #1^2 + #1^3 &, 3]^2)
```



In[607]:=

(* Das Programm findet keine klassische Darstellung der Lösung *)