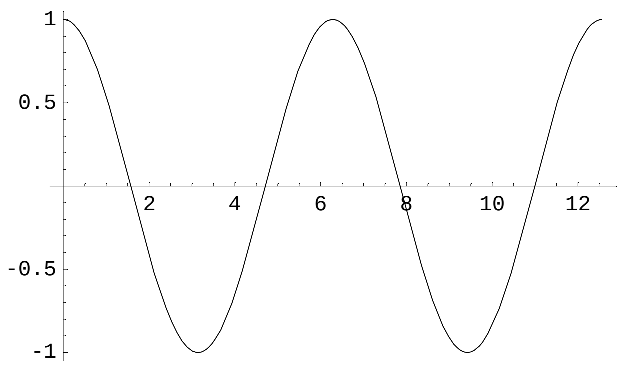
Lösungen

1 Schwingungen

a

```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=0; f=1; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

```
m = 1  d = 0  f = 1  funktion = 0

==>  y[x] = Cos[x]
```

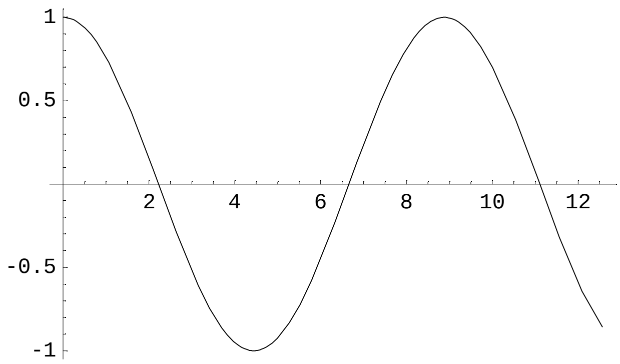


b

```
Remove["Global`*"];
(* Neues f = 1/2 * altes f  *)
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=0; f=1/2; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 1 d = 0 f = $\frac{1}{2}$ funktion = 0

==> y[x] = Cos$\left[\frac{x}{\sqrt{2}}\right]$

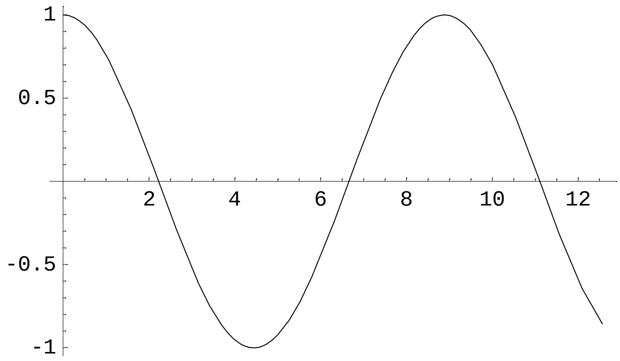


c

```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=2; d=0; f=1; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 2 d = 0 f = 1 funktion = 0

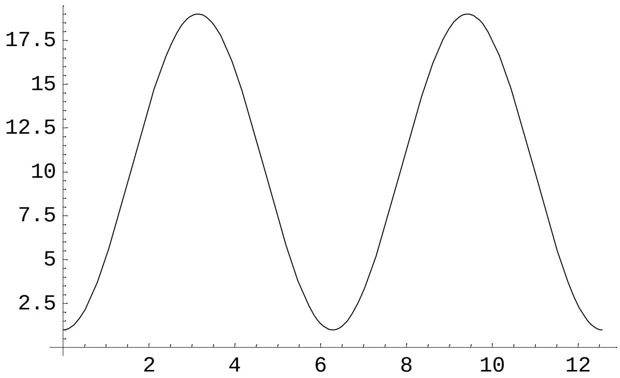
==> y[x] = $\text{Cos}\left[\frac{x}{\sqrt{2}}\right]$



d

```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=0; f=1; funktion=10;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

```
m = 1  d = 0  f = 1  funktion = 10

==>  y[x] = 10 - 9 Cos[x]
```

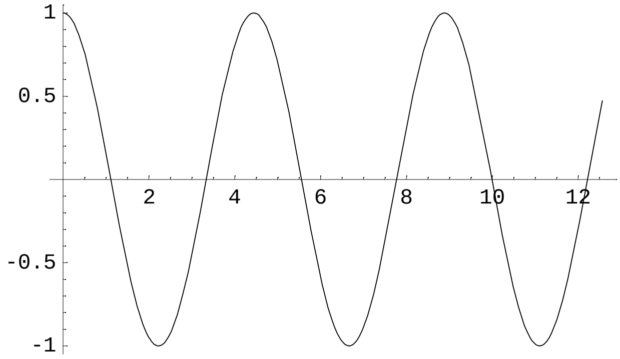


e

```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] - f*(-y[x])== funktion,
y[0]==1, y'[0]==0};
m=1; d=0; f=1; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 1 d = 0 f = 1 funktion = 0

==> y[x] = Cos[$\sqrt{2}$ x]



f

```
Remove["Global`*"];

links1 = 1*y1''[x] +  1*y1[x] - 1*(y2[x]-y1[x]);
links2 = 1*y2''[x] +  1*y2[x] - 1*(y1[x]-y2[x]);
{links1 + links2 == 0+0, links1 - links2 == 0+0}
```

{y1[x] + y2[x] + y1′′[x] + y2′′[x] == 0, 3 y1[x] − 3 y2[x] + y1′′[x] − y2′′[x] == 0}

Sei r[x_] := y1[x] + y2[x], s[x_] := y1[x] - y2[x]

```
links1 + links2
```

y1[x] + y2[x] + y1′′[x] + y2′′[x]

```
u1 = links1 + links2 /. {y1[x]+y2[x] -> r[x]} /.{y1''[x]+y2''[x] -> r''[x]}
```

r[x] + r′′[x]

```
links1 - links2
```

3 y1[x] − 3 y2[x] + y1′′[x] − y2′′[x]

```
u2 = links1 - links2
```

3 y1[x] − 3 y2[x] + y1′′[x] − y2′′[x]

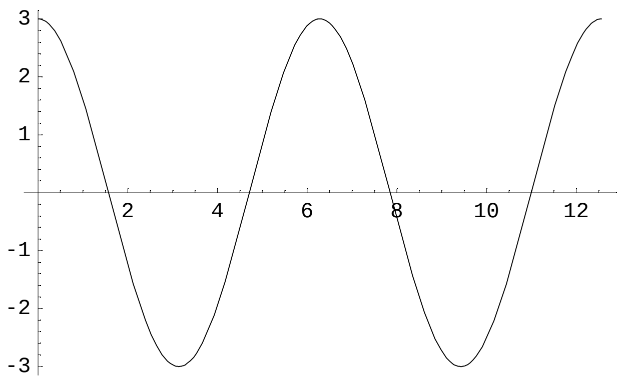
```
u2 = links1 - links2 /. {3 y1[x]- 3 y2[x] -> 3 s[x]} /.{y1''[x]-y2''[x] -> s''[x]}
```

```
3 s[x] + s''[x]
```

Das ergibt die Differentialgleichungen:
r[x] + r''[x] = 0, r[x] = y1[x] + y2[x], r[0] = 6, r'[0] = 0 und
s[x] + s''[x] = 0, s[x] = y1[x] - y2[x], s[0] = -4, s'[0] = 0
Diese Gleichungen sind unabhängig lösbar.

```
Remove[m,d,f,funktion,r,s,x];
gleichung[m_,d_,f_,funktion_]:={m*r''[x]+ d*r'[x]+ f*r[x] == funktion, r[0]==3,
r'[0]==0};
m=1; d=0; f=1; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],r,x];
r = r/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  r[x] = ",r[x]//Simplify];
Plot[r[x],{x,0,4Pi}];
```

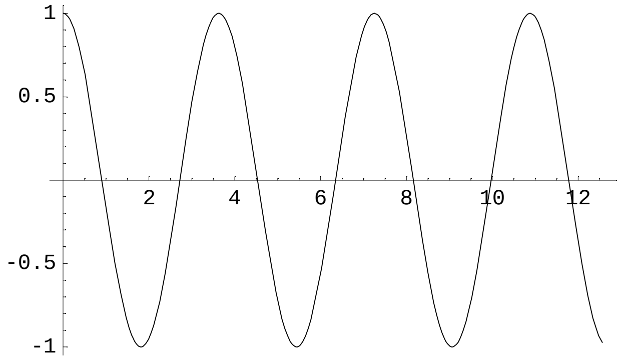
```
m = 1  d = 0  f = 1  funktion = 0

==>  r[x] = 3 Cos[x]
```



```
Remove[m,d,f,funktion,s,x];
gleichung[m_,d_,f_,funktion_]:={m*s''[x]+ d*s'[x]+ f*s[x] == funktion, s[0]==1,
s'[0]==0};
m=1; d=0; f=3; funktion=0;
solv =
DSolve[gleichung[m,d,f,funktion],s,x];
s = s/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  s[x] = ",s[x]//Simplify];
Plot[s[x],{x,0,4Pi}];
```

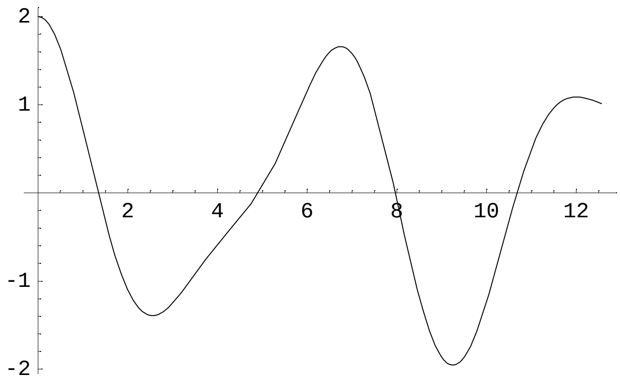
m = 1 d = 0 f = 3 funktion = 0

==> s[x] = $\mathrm{Cos}[\sqrt{3}\,x]$

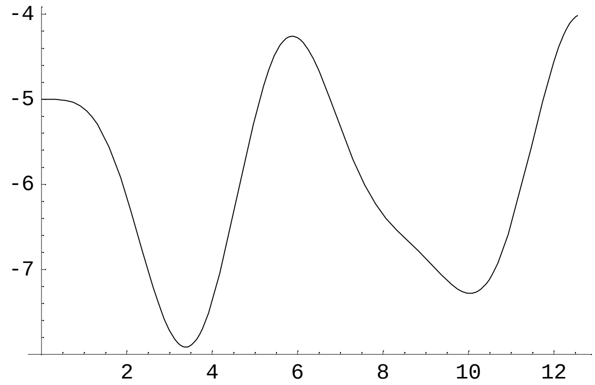


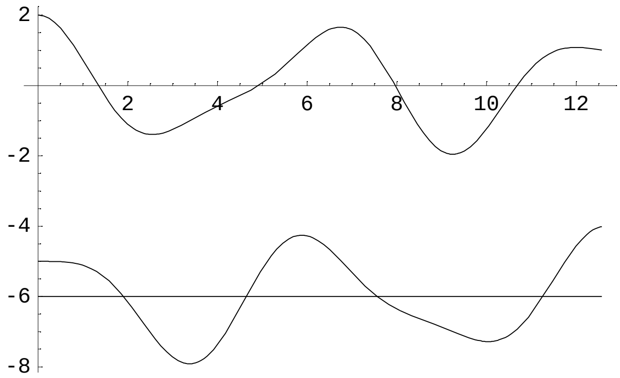
```
y1[x_]:= (r[x]+s[x])/2;
y2[x_]:= (r[x]-s[x])/2 -6; (* Koordinatenverschiebung -6 *)
Print["==>  y1[x] = ",y1[x]//Simplify];
Plot[y1[x],{x,0,4Pi}];
Print["==>  y2[x] = ",y2[x]-6//Simplify];
Plot[y2[x],{x,0,4Pi}];
Plot[{y1[x],y2[x],-6},{x,0,4Pi}];
```

==> y1[x] = $\frac{1}{2}\,(3\,\mathrm{Cos}[x] + \mathrm{Cos}[\sqrt{3}\,x])$

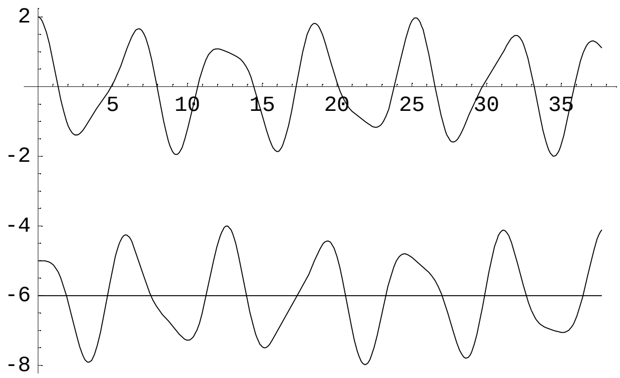


==> y2[x] = $\frac{1}{2}\,(-24 + 3\,\mathrm{Cos}[x] - \mathrm{Cos}[\sqrt{3}\,x])$





Plot[{y1[x],y2[x],-6},{x,0,12Pi}];

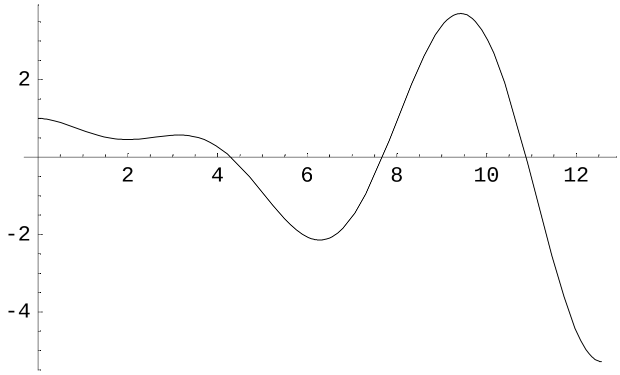


g

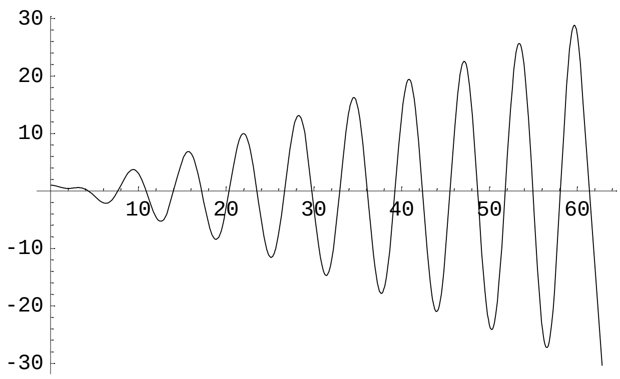
```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=0; f=1; funktion=Sin[x];
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 1 d = 0 f = 1 funktion = Sin[x]

==> y[x] = $\frac{1}{2}$ (−(−2 + x) Cos[x] + Sin[x])




```
Plot[y[x],{x,0,20Pi}];
```

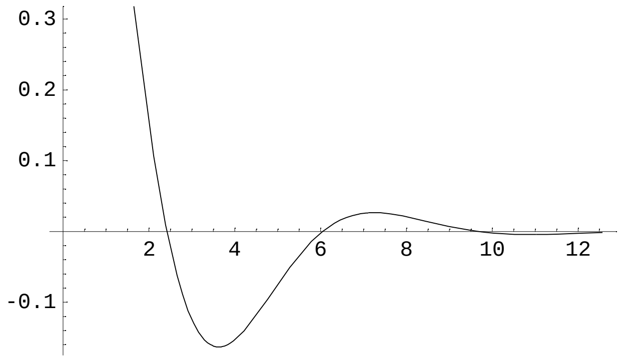


h

```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=1; f=1; funktion= 0;
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 1 d = 1 f = 1 funktion = 0

==> y[x] = $\frac{1}{3} e^{-x/2} \left(3 \mathrm{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + \sqrt{3}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right)$

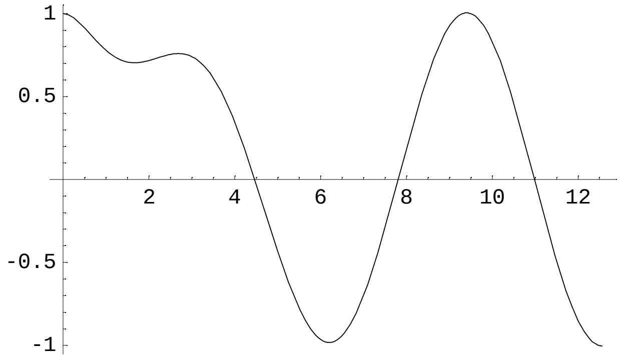


i

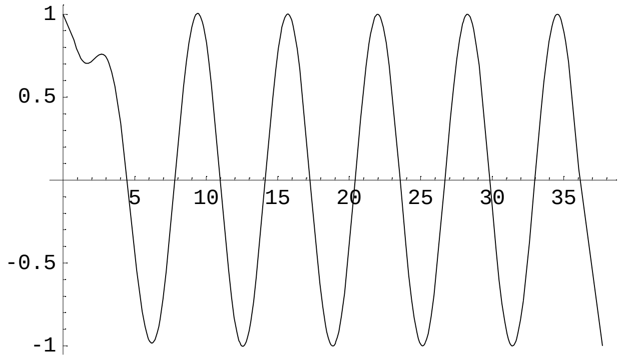
```
Remove["Global`*"];
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==1,
y'[0]==0};
m=1; d=1; f=1; funktion= Sin[x];
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

m = 1 d = 1 f = 1 funktion = Sin[x]

==> $y[x] = -\mathrm{Cos}[x] + \frac{2}{3} e^{-x/2} \left(3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + \sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right)$




```
Plot[y[x],{x,0,12Pi}];
```



j

```
Remove["Global`*"];
f1 = 3; f2 = 4;
geichungen= {F == f x, F == a x1, F == b x2, x == x1+x2};
solv = Solve[geichungen, {f,x1,x2,x,F}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…

$\left\{\{x1 \to 0,\ x2 \to 0,\ F \to 0,\ x \to 0\},\ \left\{f \to \frac{a\,b}{a+b},\ x1 \to \frac{b\,x}{a+b},\ x2 \to \frac{a\,x}{a+b},\ F \to \frac{a\,b\,x}{a+b}\right\}\right\}$

```
solv[[2]][[1]]
```

$$f \to \frac{a\,b}{a+b}$$

```
solv1=solv[[2]][[1]] /. {a->3, b->4}
```

$$f \to \frac{12}{7}$$

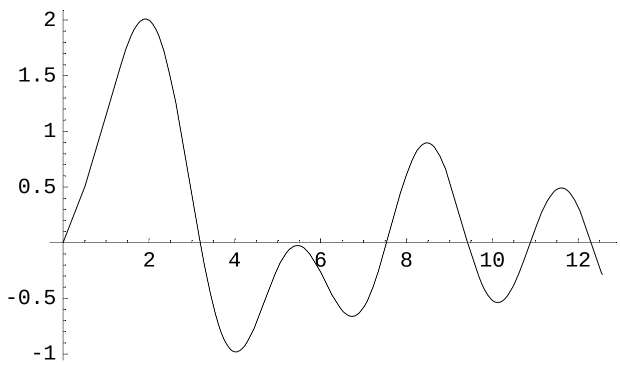
```
f = f /. solv1
```

$$\frac{12}{7}$$

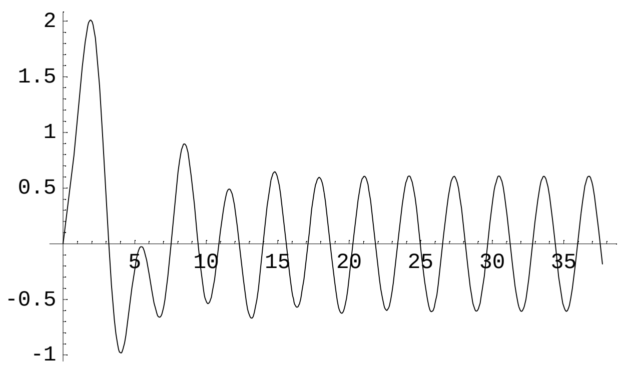
```
gleichung[m_,d_,f_,funktion_]:={m*y''[x]+ d*y'[x]+ f*y[x] == funktion, y[0]==0,
y'[0]==1};
m=2; d=1; f=f; funktion= 4 Sin[2 x];
solv =
DSolve[gleichung[m,d,f,funktion],y,x];
y = y/.solv[[1]];
Print["m = ",m,"  d = ",d,"  f = ",f,"  funktion = ",funktion];
Print["==>  y[x] = ",y[x]//Simplify];
Plot[y[x],{x,0,4Pi}];
```

$m = 2 \quad d = 1 \quad f = \frac{12}{7} \quad \text{funktion} = 4\,\text{Sin}[2\,x]$

$$\text{==>}\quad y[x] = \frac{1}{47437} \left(2\left(-4361\,\text{Cos}[2\,x] + 4361\,e^{-x/4}\,\text{Cos}\left[\frac{1}{4}\sqrt{\frac{89}{7}}\,x\right] - 13706\,\text{Sin}[2\,x] + 2347\sqrt{623}\,e^{-x/4}\,\text{Sin}\left[\frac{1}{4}\sqrt{\frac{89}{7}}\,x\right]\right)\right)$$




```
Plot[y[x],{x,0,12Pi}];
```



2 Linienintegrale

a

i

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
v[t]:= {t^2+1,2 t^2,t^3};
D[v[t],t]
```

$\{2\,t,\ 4\,t,\ 3\,t^2\}$

```
integrand[t_]:= F[x,y,z].Evaluate[D[v[t],t]] /.{x->v[t][[1]], y->v[t][[2]],
z->v[t][[3]]};
integrand[t]
```

$20\,t^4 + 30\,t^2\,(1+t^2) + 12\,t^3\,(1+t^2)$

```
integrand[t]/.t->u
```

$20\,u^4 + 30\,u^2\,(1+u^2) + 12\,u^3\,(1+u^2)$

```
Integrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

576

ii

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
absF[x_,y_,z_]:= Sqrt[F[x,y,z].F[x,y,z]];
absF[x,y,z]
```

$\sqrt{100\,x^2 + 9\,x^2\,y^2 + 25\,z^2}$

```
v[t] := {t^2 + 1, 2 t^2, t^3};
Sqrt[D[v[t], t].D[v[t], t]]
```

$\sqrt{20\,t^2 + 9\,t^4}$

```
integrand[t_]:= absF[x,y,z]* Evaluate[Sqrt[D[v[t],t].D[v[t],t]]] /.{x->v[t][[1]],
y->v[t][[2]], z->v[t][[3]]};
integrand[t]
```

$\sqrt{20\,t^2 + 9\,t^4}\ \sqrt{25\,t^6 + 100\,(1+t^2)^2 + 36\,t^4\,(1+t^2)^2}$

```
integrand[t]/.t->u
```

$\sqrt{20\,u^2 + 9\,u^4}\ \sqrt{25\,u^6 + 100\,(1+u^2)^2 + 36\,u^4\,(1+u^2)^2}$

```
Integrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

$$\int_0^2 \sqrt{20\,u^2 + 9\,u^4}\,\sqrt{25\,u^6 + 100\,(1+u^2)^2 + 36\,u^4\,(1+u^2)^2}\,du$$

```
NIntegrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

758.233

b

i

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
v[t]:= {t^3,2 t^2,t^2+1};
D[v[t],t]
```

$\{3\,t^2,\ 4\,t,\ 2\,t\}$

```
integrand[t_]:= F[x,y,z].Evaluate[D[v[t],t]] /.{x->v[t][[1]], y->v[t][[2]],
z->v[t][[3]]};
integrand[t]
```

$20\,t^4 + 18\,t^7 + 20\,t\,(1+t^2)$

```
integrand[t]/.t->u
```

$20\,u^4 + 18\,u^7 + 20\,u\,(1+u^2)$

```
Integrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

824

ii

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
absF[x_,y_,z_]:= Sqrt[F[x,y,z].F[x,y,z]];
absF[x,y,z]
```

$\sqrt{100\,x^2 + 9\,x^2\,y^2 + 25\,z^2}$

$\{2\,t,\ 4\,t,\ 3\,t^2\}$

```
v[t] := {t^3, 2 t^2, t^2 + 1};
Sqrt[D[v[t], t].D[v[t], t]]
```

$\sqrt{20\,t^2 + 9\,t^4}$

```
integrand[t_]:= absF[x,y,z]* Evaluate[Sqrt[D[v[t],t].D[v[t],t]]] /.{x->v[t][[1]],
y->v[t][[2]], z->v[t][[3]]};
integrand[t]
```

$\sqrt{20\,t^2 + 9\,t^4}\,\sqrt{100\,t^6 + 36\,t^{10} + 25\,(1+t^2)^2}$

```
integrand[t]/.t->u
```

$\sqrt{20\,u^2+9\,u^4}\,\sqrt{100\,u^6+36\,u^{10}+25\,(1+u^2)^2}$

```
Integrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

$\int_0^2 \sqrt{20\,u^2+9\,u^4}\,\sqrt{100\,u^6+36\,u^{10}+25\,(1+u^2)^2}\,du$

```
NIntegrate[Evaluate[integrand[t]/.t->u],{u,0,2}]
```

913.579

c

i

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
v[t]:={Cos[t],Sin[2 t],1+Sin[3 t]};
D[v[t],t]
```

{-Sin[t], 2 Cos[2 t], 3 Cos[3 t]}

```
integrand[t_]:= F[x,y,z].Evaluate[D[v[t],t]] /.{x->v[t][[1]], y->v[t][[2]],
z->v[t][[3]]};
integrand[t]
```

30 Cos[t] Cos[3 t] - 3 Cos[t] Sin[t] Sin[2 t] + 10 Cos[2 t] (1 + Sin[3 t])

```
integrand[t]/.t->u
```

30 Cos[u] Cos[3 u] - 3 Cos[u] Sin[u] Sin[2 u] + 10 Cos[2 u] (1 + Sin[3 u])

```
(* Integrate[Evaluate[integrand[t]/.t->u],{u,0,2Pi}] *)
```

```
NIntegrate[Evaluate[integrand[t]/.t->u],{u,0,2Pi}]
```

-4.71239

ii

```
Remove["Global`*"];
F[x_,y_,z_]:= {3 x y, 5 z, 10 x};
absF[x_,y_,z_]:= Sqrt[F[x,y,z].F[x,y,z]];
absF[x,y,z]
```

$\sqrt{100\,x^2+9\,x^2\,y^2+25\,z^2}$

$\{2\,t,\ 4\,t,\ 3\,t^2\}$

```
v[t] := {Cos[t], Sin[2 t], 1 + Sin[3 t]};
Sqrt[D[v[t], t].D[v[t], t]]
```

$\sqrt{4\,\mathrm{Cos}[2\,t]^2+9\,\mathrm{Cos}[3\,t]^2+\mathrm{Sin}[t]^2}$

```
integrand[t_]:= absF[x,y,z]* Evaluate[Sqrt[D[v[t],t].D[v[t],t]]] /.{x->v[t][[1]],
y->v[t][[2]], z->v[t][[3]]};
integrand[t]
```

$$\sqrt{4\,\mathrm{Cos}[2\,t]^2+9\,\mathrm{Cos}[3\,t]^2+\mathrm{Sin}[t]^2}\;\sqrt{100\,\mathrm{Cos}[t]^2+9\,\mathrm{Cos}[t]^2\,\mathrm{Sin}[2\,t]^2+25\,(1+\mathrm{Sin}[3\,t])^2}$$

```
integrand[t]/.t->u
```

$$\sqrt{4\,\mathrm{Cos}[2\,u]^2+9\,\mathrm{Cos}[3\,u]^2+\mathrm{Sin}[u]^2}\;\sqrt{100\,\mathrm{Cos}[u]^2+9\,\mathrm{Cos}[u]^2\,\mathrm{Sin}[2\,u]^2+25\,(1+\mathrm{Sin}[3\,u])^2}$$

```
(* Integrate[Evaluate[integrand[t]/.t->u],{u,0,2}] *)
```

```
NIntegrate[Evaluate[integrand[t]/.t->u],{u,0,2 Pi}]
```

138.451

3 Allgemeine Lösungen wichtiger Differentialgleichungen mit der Maschine

a) y'(x) + y(x) = f(x)

```
Remove["Global`*"];
solv1 = DSolve[ y'[x] + α y[x] == f[x], y, x];
u[x_] := y /. solv1[[1]];
a = (u[x][x] // InputForm)[[1]][[2]][[2]][[2]][[1]];
v[x_] := u[x][x] /. a → t;
Print["y(x) = ", v[x]];
Print["     = ", v[x] // Simplify];
v[w] // Simplify
```

$$y(x) \;=\; e^{-x\alpha}\,C[1]+e^{-x\alpha}\int_1^x e^{t\alpha}\,f[t]\,dt$$

$$=\; e^{-x\alpha}\left(C[1]+\int_1^x e^{t\alpha}\,f[t]\,dt\right)$$

$$e^{-w\alpha}\left(C[1]+\int_1^w e^{t\alpha}\,f[t]\,dt\right)$$

b) y''(x) + y'(x) + y(x) = f(x)

```
Remove["Global`*"];
solv2 = DSolve[y''[x] + α y'[x] + β y[x] == f[x], y, x][[1]];
u[x_] := y /. solv2[[1]];
a = u[x][[2]][[3]][[2]][[2]][[1]];
b = u[x][[2]][[4]][[2]][[2]][[1]];
v[x_] := u[x][x] /. {a → t, b → t};
Print["y(x) = ", v[x]];
Print["     = ", v[x] // Simplify];
v[w] // Simplify
```

$$y(x) = e^{\frac{1}{2} x \left(-\alpha-\sqrt{\alpha^2-4\beta}\right)} C[1] + e^{\frac{1}{2} x \left(-\alpha+\sqrt{\alpha^2-4\beta}\right)} C[2] + e^{\frac{1}{2} x \left(-\alpha+\sqrt{\alpha^2-4\beta}\right)} \int_1^x \frac{e^{t\alpha+\frac{1}{2} t \left(-\alpha-\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt + e^{\frac{1}{2} x \left(-\alpha-\sqrt{\alpha^2-4\beta}\right)} \int_1^x -\frac{e^{t\alpha+\frac{1}{2} t \left(-\alpha+\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt$$

$$= e^{-\frac{1}{2} x \left(\alpha+\sqrt{\alpha^2-4\beta}\right)} \left(C[1] + e^{x\sqrt{\alpha^2-4\beta}} C[2] + e^{x\sqrt{\alpha^2-4\beta}} \int_1^x \frac{e^{\frac{1}{2} t \left(\alpha-\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt + \int_1^x -\frac{e^{\frac{1}{2} t \left(\alpha+\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt\right)$$

$$e^{-\frac{1}{2} w \left(\alpha+\sqrt{\alpha^2-4\beta}\right)} \left(C[1] + e^{w\sqrt{\alpha^2-4\beta}} C[2] + e^{w\sqrt{\alpha^2-4\beta}} \int_1^w \frac{e^{\frac{1}{2} t \left(\alpha-\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt + \int_1^w -\frac{e^{\frac{1}{2} t \left(\alpha+\sqrt{\alpha^2-4\beta}\right)} f[t]}{\sqrt{\alpha^2-4\beta}} dt\right)$$

c) Wahl von Parametern = 2, = -1, C[1] = 1, C[2] = 1, f(x) = cos(x), Plot

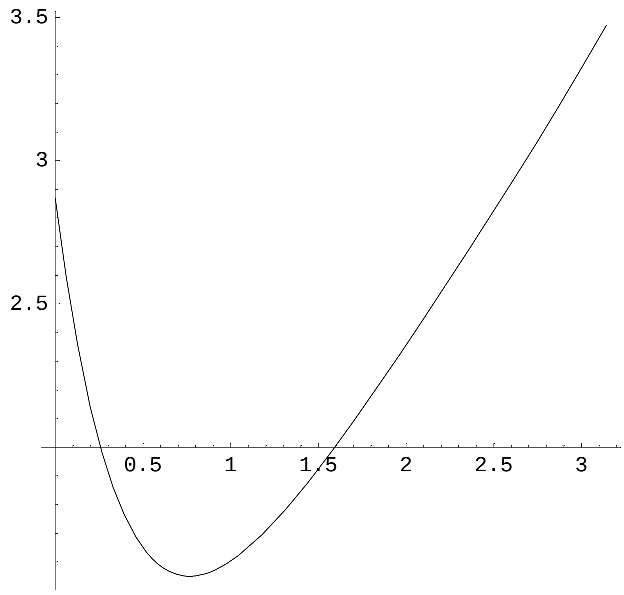
Maschinenlösung

```
Remove["Global`*"];
solv2 = DSolve[y''[x] + α y'[x] + β y[x] == f[x], y, x][[1]];
u[x_] := y /. solv2[[1]];
a = u[x][[2]][[3]][[2]][[2]][[1]];
b = u[x][[2]][[4]][[2]][[2]][[1]];
v[x_] := u[x][x] /. {a → t, b → t};
s[u_] := v[x] /. {x → u, α → 2, β → -1, C[1] → 1, C[2] → 1, f[t] → Cos[t]};
Print["y(x) = ", s[x] // N // Simplify];
Print["y(0) = ", s[0] // N];
Print["y'(0) = ", (D[s[m], m] /. m → 0) // N];
Plot[s[m], {m, 0, Pi}, AspectRatio → 1];
```

y(x) = $2.24228\, e^{-2.41421 x} + 0.876817\, e^{0.414214 x} - 0.25 \operatorname{Cos}[x] + 0.25 \operatorname{Sin}[x]$

y(0) = 2.86909

y'(0) = -4.80015

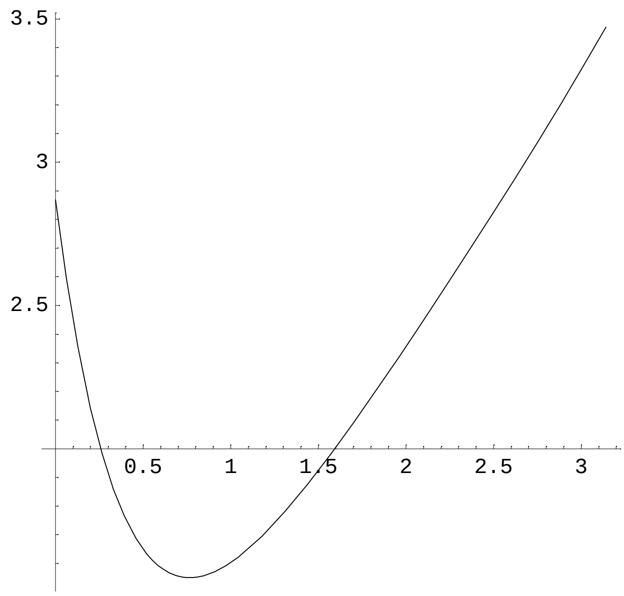


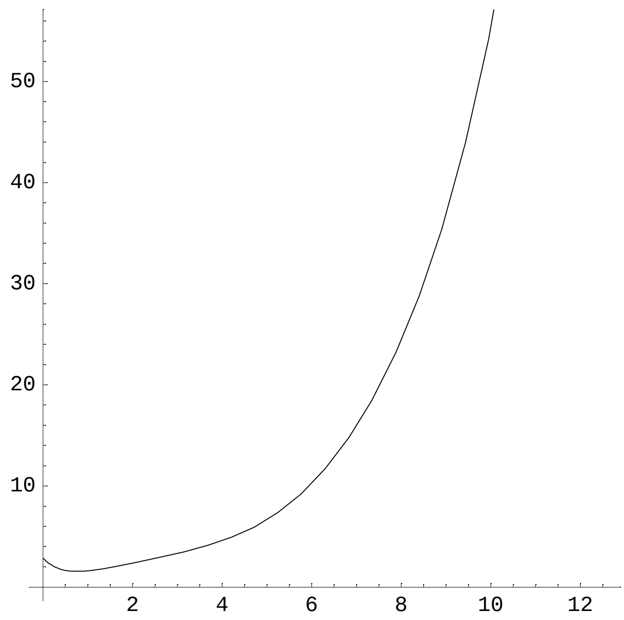
Direkter Plot der exakten Lösung mit Ausgabe der Funktion

```
Remove["Global`*"];
solv = Flatten[DSolve[{y''[x]+2 y'[x]-1 y[x]==
Cos[x],y[0]==2.86909,y'[0]==-4.80015},y,x]];
y = y/.solv;
Print["y(x) = ",Simplify[y[x]]];
Print["y(0) = ", y[0], " y'(0) = ",(D[y[x],x] /. x->0) //N];
Plot[y[x],{x,0,Pi},AspectRatio→1];
Plot[y[x],{x,0,4 Pi},AspectRatio→1];
```

y(x) = $0.876812\, e^{(-1+\sqrt{2})x} + 2.24228\, e^{-(1+\sqrt{2})x} - 0.25 \operatorname{Cos}[x] + 0.25 \operatorname{Sin}[x]$

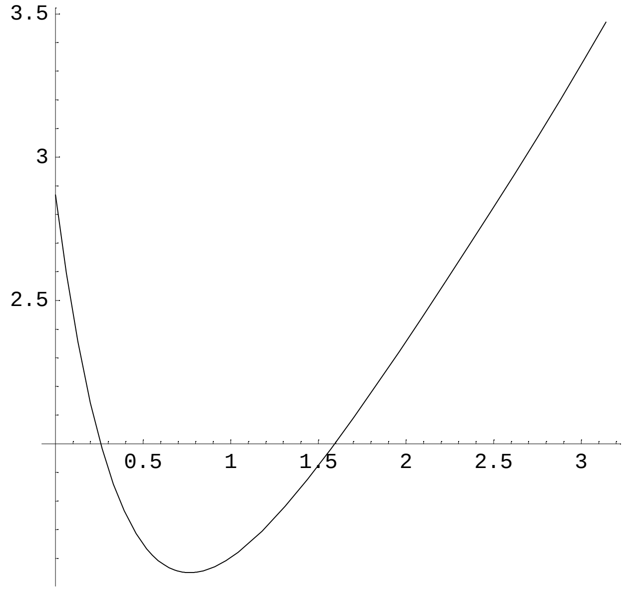
y(0) = 2.86909 y'(0) = -4.80015

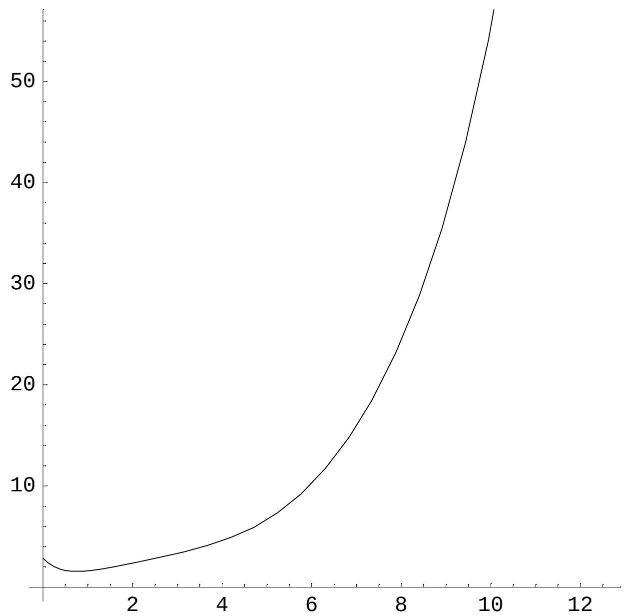




Direkter Plot der numerischen Lösung

```
Remove["Global`*"];
solution=NDSolve[{y''[x]+2 y'[x]-1 y[x]==
Cos[x],y[0]==2.86909,y'[0]==-4.80015},y,{x,0,4 Pi}];
Plot[y[x]/. solution,{x,0,Pi},AspectRatio→1];
Plot[y[x]/. solution,{x,0,4 Pi},AspectRatio→1];
```


3.5
3
2.5
0.5
1
1.5
2
2.5
3


50
40
30
20
10
2
4
6
8
10
12

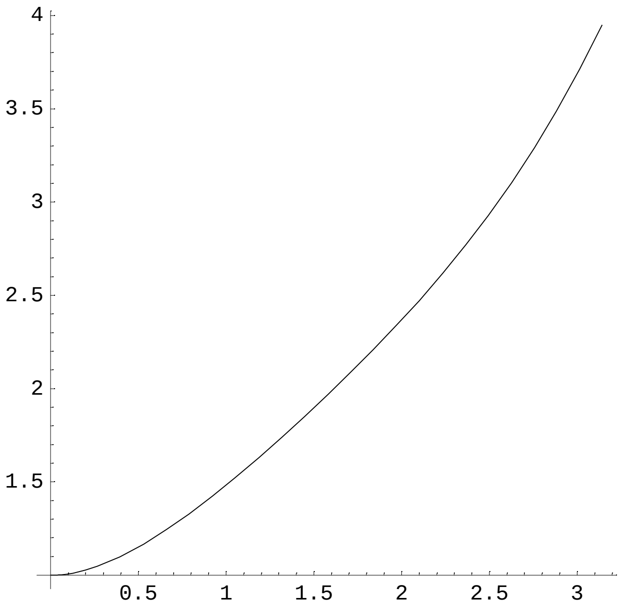
d) Direkter Plot der exakten Lösung einer anderen D'Gl.

Maschinenlösung

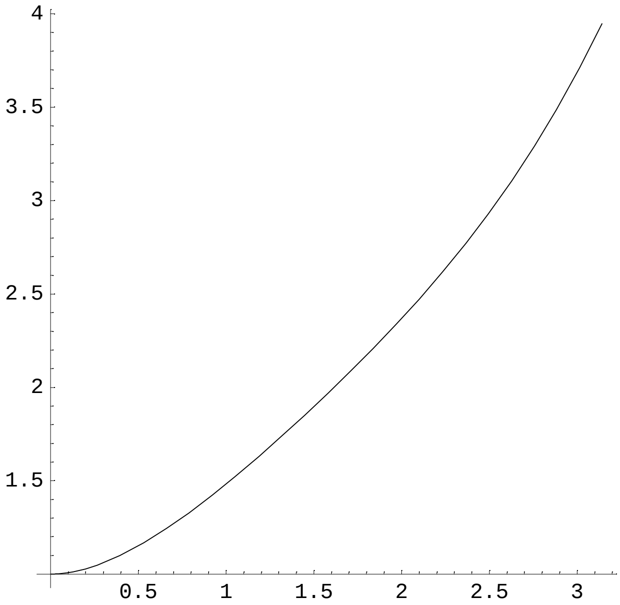
```
Remove["Global`*"];
solv = Flatten[DSolve[{y''[x] + y'[x] - y[x] ==Cos[x+1], y[0]==1, y'[0]==0},y,x]];
y = y/.solv;
Print["y(x) = ",Simplify[y[x]]];
Print["Numerisch y(x) = ",Simplify[y[x]]//N];
Plot[y[x],{x,0,Pi},AspectRatio→1];
```

$$y(x) = -\frac{1}{10} e^{-\frac{1}{2}(1+\sqrt{5})x} \Big(-5+\sqrt{5}-5 e^{\sqrt{5}x}-\sqrt{5}\, e^{\sqrt{5}x}-2\,\mathrm{Cos}[1]-2 e^{\sqrt{5}x}\mathrm{Cos}[1]+4 e^{\frac{1}{2}(1+\sqrt{5})x}\mathrm{Cos}[1+x]+\mathrm{Sin}[1]-\sqrt{5}\,\mathrm{Sin}[1]+e^{\sqrt{5}x}\mathrm{Sin}[1]+\sqrt{5}\, e^{\sqrt{5}x}\mathrm{Sin}[1]-2 e^{\frac{1}{2}(1+\sqrt{5})x}\mathrm{Sin}[1+x]\Big)$$

$$\text{Numerisch } y(x) = -0.1\; 2.71828^{-1.61803\,x}\left(-4.88465-5.59362\; 2.71828^{2.23607\,x}+4.\; 2.71828^{1.61803\,x}\,\mathrm{Cos}[1.+x]-2.\; 2.71828^{1.61803\,x}\,\mathrm{Sin}[1.+x]\right)$$



```
p1=Plot[y[x],{x,0,Pi},AspectRatio→1];
```



Handlösung

```
solv=Solve[λ^2+λ-1==0,{λ}][[1]];
loes1=λ /. solv;
loes1//TeXForm
```

\frac{1}{2} \left(-1-\sqrt{5}\right)

```
solv=Solve[λ^2+λ-1==0,{λ}][[2]];
loes2=λ /. solv;
loes2//TeXForm
```

\frac{1}{2} \left(-1+\sqrt{5}\right)

```
Remove[x,y,c1,c2]
solv = Flatten[DSolve[y''[x] + y'[x] - y[x]==0,y,x]];
y = y/.solv;
y[x]
```

$e^{\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right) x} C[1] + e^{\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right) x} C[2]$

```
Cos[x+1]//TrigExpand
```

Cos[1] Cos[x] − Sin[1] Sin[x]

```
solv3=Flatten[Simplify[Solve[{-2 r+s==Cos[1], -r-2s==-Sin[1]},{r,s}]]]
```

$\left\{r \to \frac{1}{5} (-2 \text{Cos}[1] + \text{Sin}[1]),\ s \to \frac{1}{5} (\text{Cos}[1] + 2 \text{Sin}[1])\right\}$

```
r=r /.solv3[[1]]; s=s /.solv3[[2]]
```

$\frac{1}{5} (\text{Cos}[1] + 2 \text{Sin}[1])$

```
Remove[y];
y[x_,c1_,c2_]:= c1 E^(1/2(-1-Sqrt[5])x)+c2 E^(1/2(-1+Sqrt[5])x)+ r  Cos[x]+ s
Sin[x];
y[x,c1,c2]//Simplify
```

$$\frac{1}{5}\left(5\, e^{-\frac{1}{2}\left(1+\sqrt{5}\right)x}\left(c1+c2\, e^{\sqrt{5}\,x}\right)-2\,\mathrm{Cos}[1+x]+\mathrm{Sin}[1+x]\right)$$

```
y[x,c1,c2]
```

$$c1\, e^{\frac{1}{2}\left(-1-\sqrt{5}\right)x}+c2\, e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}+\frac{1}{5}\,\mathrm{Cos}[x]\,(-2\,\mathrm{Cos}[1]+\mathrm{Sin}[1])+\frac{1}{5}\,(\mathrm{Cos}[1]+2\,\mathrm{Sin}[1])\,\mathrm{Sin}[x]$$

```
D[y[x,c1,c2],x]//Simplify
```

$$\frac{1}{10}\left(2\,\mathrm{Cos}[1+x]+e^{-\frac{1}{2}\left(1+\sqrt{5}\right)x}\left(-5\left(\left(1+\sqrt{5}\right)c1-\left(-1+\sqrt{5}\right)c2\, e^{\sqrt{5}\,x}\right)+4\, e^{\frac{1}{2}\left(1+\sqrt{5}\right)x}\,\mathrm{Sin}[1+x]\right)\right)$$

```
y[0,c1,c2]
```

$$c1+c2+\frac{1}{5}\,(-2\,\mathrm{Cos}[1]+\mathrm{Sin}[1])$$

```
(Simplify[D[y[x,c1,c2],x]]/.x->0)
```

$$\frac{1}{10}\left(-5\left(\left(1+\sqrt{5}\right)c1-\left(-1+\sqrt{5}\right)c2\right)+2\,\mathrm{Cos}[1]+4\,\mathrm{Sin}[1]\right)$$

```
solv=Flatten[Solve[Evaluate[{y[0,c1,c2]==1, (D[y[x,c1,c2],x]/.x->0)==0}],{c1,c2} ]]
```

$$\left\{c1\to\frac{1}{10}\left(5-\sqrt{5}+2\,\mathrm{Cos}[1]-\mathrm{Sin}[1]+\sqrt{5}\,\mathrm{Sin}[1]\right),\; c2\to\frac{5+5\sqrt{5}+2\sqrt{5}\,\mathrm{Cos}[1]-5\,\mathrm{Sin}[1]-\sqrt{5}\,\mathrm{Sin}[1]}{10\sqrt{5}}\right\}$$

```
c1=c1/.solv[[1]]//Simplify
```

$$\frac{1}{10}\left(5-\sqrt{5}+2\,\mathrm{Cos}[1]+\left(-1+\sqrt{5}\right)\mathrm{Sin}[1]\right)$$

```
c2=c2/.solv[[2]]//Simplify
```

$$\frac{1}{10}\left(5+\sqrt{5}+2\,\mathrm{Cos}[1]-\left(1+\sqrt{5}\right)\mathrm{Sin}[1]\right)$$

```
y[x,c1,c2]
```

$$\frac{1}{5}\,\mathrm{Cos}[x]\,(-2\,\mathrm{Cos}[1]+\mathrm{Sin}[1])+\frac{1}{10}\, e^{\frac{1}{2}\left(-1-\sqrt{5}\right)x}\left(5-\sqrt{5}+2\,\mathrm{Cos}[1]+\left(-1+\sqrt{5}\right)\mathrm{Sin}[1]\right)+\frac{1}{10}\, e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}\left(5+\sqrt{5}+2\,\mathrm{Cos}[1]-\left(1+\sqrt{5}\right)\mathrm{Sin}[1]\right)+\frac{1}{5}\,(\mathrm{Cos}[1]+2\,\mathrm{Sin}[1])\,\mathrm{Sin}[x]$$

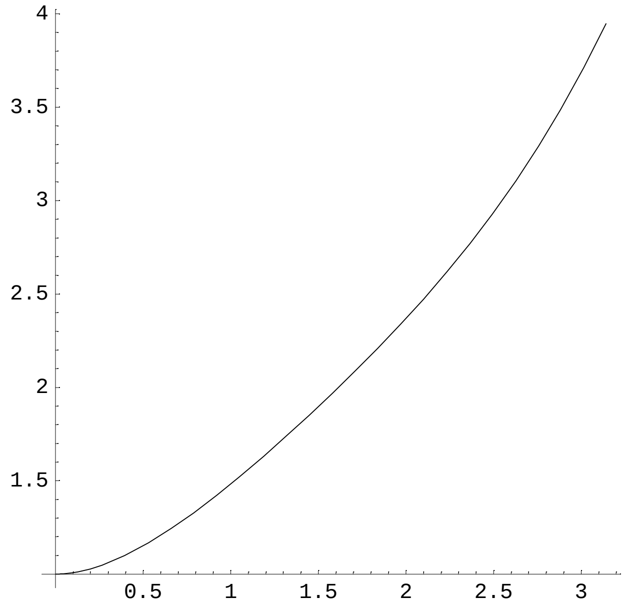
```
y[x,c1,c2]//Simplify
```

$$\frac{1}{10}\left(2\,\mathrm{Cos}[x]\,(-2\,\mathrm{Cos}[1]+\mathrm{Sin}[1])+e^{-\frac{1}{2}\left(1+\sqrt{5}\right)x}\left(5-\sqrt{5}+2\,\mathrm{Cos}[1]+\left(-1+\sqrt{5}\right)\mathrm{Sin}[1]\right)+e^{\frac{1}{2}\left(-1+\sqrt{5}\right)x}\left(5+\sqrt{5}+2\,\mathrm{Cos}[1]-\left(1+\sqrt{5}\right)\mathrm{Sin}[1]\right)+2\,(\mathrm{Cos}[1]+2\,\mathrm{Sin}[1])\,\mathrm{Sin}[x]\right)$$

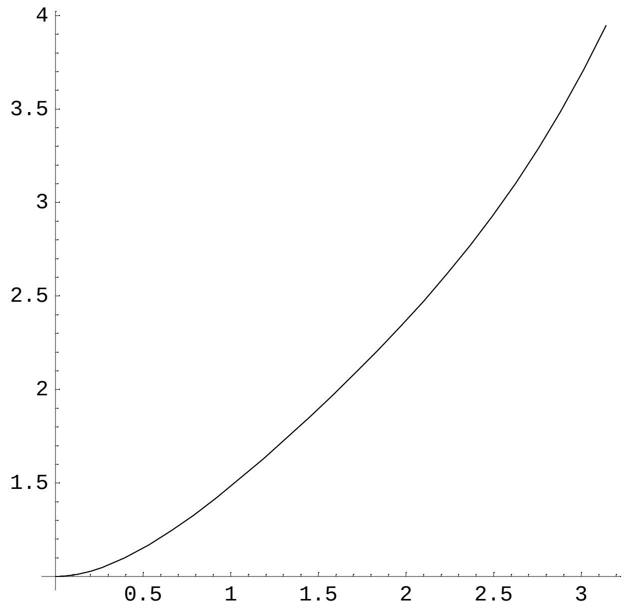
```
y[x,c1,c2]//N//Simplify
```

$$0.488465\, e^{-1.61803\,x}+0.559362\, e^{0.618034\,x}-0.0478267\,\mathrm{Cos}[x]+0.444649\,\mathrm{Sin}[x]$$

```
p2 = Plot[y[x,c1,c2],{x,0,Pi},AspectRatio→1];
```




```
Show[p1,p2];
```

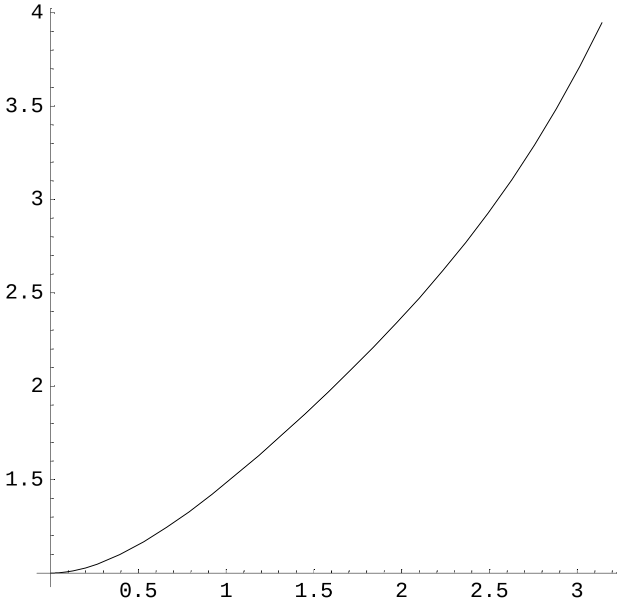


```
Remove[x,y];
solv = Flatten[DSolve[{y''[x] + y'[x] - y[x] ==Cos[x+1], y[0]==1, y'[0]==0},y,x]];
y = y/.solv;
Print["y(x) = ",Simplify[y[x]]];
Print["Numerisch y(x) = ",Simplify[y[x]]//N];
Plot[y[x],{x,0,Pi},AspectRatio→1];
```

y(x) =

$$-\frac{1}{10}\, e^{-\frac{1}{2}(1+\sqrt{5})x}\left(-5+\sqrt{5}-5\,e^{\sqrt{5}x}-\sqrt{5}\,e^{\sqrt{5}x}-2\,\mathrm{Cos}[1]-2\,e^{\sqrt{5}x}\,\mathrm{Cos}[1]+4\,e^{\frac{1}{2}(1+\sqrt{5})x}\,\mathrm{Cos}[1+x]+\mathrm{Sin}[1]-\sqrt{5}\,\mathrm{Sin}[1]+e^{\sqrt{5}x}\,\mathrm{Sin}[1]+\sqrt{5}\,e^{\sqrt{5}x}\,\mathrm{Sin}[1]-2\,e^{\frac{1}{2}(1+\sqrt{5})x}\,\mathrm{Sin}[1+x]\right)$$

Numerisch y(x) = $-0.1\; 2.71828^{-1.61803\,x}\,(-4.88465-5.59362\; 2.71828^{2.23607\,x}+4.\; 2.71828^{1.61803\,x}\,\mathrm{Cos}[1.+x]-2.\; 2.71828^{1.61803\,x}\,\mathrm{Sin}[1.+x])$




```
y[x]//Simplify//N
```

$-0.1\; 2.71828^{-1.61803\,x}\,(-4.88465-5.59362\; 2.71828^{2.23607\,x}+4.\; 2.71828^{1.61803\,x}\,\mathrm{Cos}[1.+x]-2.\; 2.71828^{1.61803\,x}\,\mathrm{Sin}[1.+x])$