Lösungen / Statistik 1/15

```
Remove["Global`*"]
```

1.

```
<< Graphics`Graphics`

<< Statistics`DescriptiveStatistics`

tb = {{153, 1}, {154, 1}, {155, 2}, {156, 3}, {157, 3},
   {158, 5}, {159, 6}, {160, 4}, {161, 5}, {162, 7}, {163, 5}, {164, 5},
   {165, 6}, {166, 7}, {167, 5}, {168, 5}, {169, 6}, {170, 5}, {171, 6},
   {172, 4}, {173, 3}, {174, 2}, {175, 3}, {176, 1}, {177, 1}, {178, 1}}

{{153, 1}, {154, 1}, {155, 2}, {156, 3}, {157, 3}, {158, 5}, {159, 6}, {160, 4}, {161, 5},
 {162, 7}, {163, 5}, {164, 5}, {165, 6}, {166, 7}, {167, 5}, {168, 5}, {169, 6}, {170, 5},
 {171, 6}, {172, 4}, {173, 3}, {174, 2}, {175, 3}, {176, 1}, {177, 1}, {178, 1}}

s = Sum[tb[[n]][[2]], {n, 1, Length[tb]}]

102

Table[freq[tb[[n]][[1]]] = tb[[n]][[2]], {n, 1, Length[tb]}]

{1, 1, 2, 3, 3, 5, 6, 4, 5, 7, 5, 5, 6, 7, 5, 5, 6, 5, 6, 4, 3, 2, 3, 1, 1, 1}

Table[freq[tb[[n]][[1]]], {n, 1, Length[tb]}]

{1, 1, 2, 3, 3, 5, 6, 4, 5, 7, 5, 5, 6, 7, 5, 5, 6, 5, 6, 4, 3, 2, 3, 1, 1, 1}

freq[157]

3

Table[relfreq[tb[[n]][[1]]] = tb[[n]][[2]] / s, {n, 1, Length[tb]}]
```

$$\left\{\frac{1}{102}, \frac{1}{102}, \frac{1}{51}, \frac{1}{34}, \frac{1}{34}, \frac{5}{102}, \frac{1}{17}, \frac{2}{51}, \frac{5}{102}, \frac{7}{102}, \frac{5}{102}, \frac{5}{102}, \frac{1}{17}, \frac{7}{102}, \frac{5}{102}, \frac{5}{102}, \frac{1}{17}, \frac{5}{102}, \frac{1}{17}, \frac{2}{51}, \frac{1}{34}, \frac{1}{51}, \frac{1}{34}, \frac{1}{102}, \frac{1}{102}, \frac{1}{102}\right\}$$

```
N[%]

{0.00980392, 0.00980392, 0.0196078, 0.0294118, 0.0294118, 0.0490196,
 0.0588235, 0.0392157, 0.0490196, 0.0686275, 0.0490196, 0.0490196, 0.0588235,
 0.0686275, 0.0490196, 0.0490196, 0.0588235, 0.0490196, 0.0588235, 0.0392157,
 0.0294118, 0.0196078, 0.0294118, 0.00980392, 0.00980392, 0.00980392}

μ = Sum[tb[[n]][[1]] * relfreq[tb[[n]][[1]]] , {n, 1, Length[tb]}]
```

$$\frac{2807}{17}$$

```
N[%]

165.118
```

2.

```
Remove[u, v, w]

tab21 = Table[u + v + w, {u, 1, 6}, {v, 1, 6}, {w, 1, 6}] // Flatten

{3, 4, 5, 6, 7, 8, 4, 5, 6, 7, 8, 9, 5, 6, 7, 8, 9, 10, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11,
 12, 8, 9, 10, 11, 12, 13, 4, 5, 6, 7, 8, 9, 5, 6, 7, 8, 9, 10, 6, 7, 8, 9, 10, 11, 7, 8,
 9, 10, 11, 12, 8, 9, 10, 11, 12, 13, 9, 10, 11, 12, 13, 14, 5, 6, 7, 8, 9, 10, 6, 7, 8,
 9, 10, 11, 7, 8, 9, 10, 11, 12, 8, 9, 10, 11, 12, 13, 9, 10, 11, 12, 13, 14, 10, 11,
 12, 13, 14, 15, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11, 12, 8, 9, 10, 11, 12, 13, 9, 10,
 11, 12, 13, 14, 10, 11, 12, 13, 14, 15, 11, 12, 13, 14, 15, 16, 7, 8, 9, 10, 11, 12, 8,
 9, 10, 11, 12, 13, 9, 10, 11, 12, 13, 14, 10, 11, 12, 13, 14, 15, 11, 12, 13, 14, 15,
 16, 12, 13, 14, 15, 16, 17, 8, 9, 10, 11, 12, 13, 9, 10, 11, 12, 13, 14, 10, 11, 12,
 13, 14, 15, 11, 12, 13, 14, 15, 16, 12, 13, 14, 15, 16, 17, 13, 14, 15, 16, 17, 18}

<< Statistics`DataManipulation`

freq = Frequencies[tab21]

{{1, 3}, {3, 4}, {6, 5}, {10, 6}, {15, 7}, {21, 8}, {25, 9}, {27, 10},
 {27, 11}, {25, 12}, {21, 13}, {15, 14}, {10, 15}, {6, 16}, {3, 17}, {1, 18}}

newFreq = Table[{freq[[n]][[2]], freq[[n]][[1]]}, {n, 1, Length[freq]}]

{{3, 1}, {4, 3}, {5, 6}, {6, 10}, {7, 15}, {8, 21}, {9, 25}, {10, 27},
 {11, 27}, {12, 25}, {13, 21}, {14, 15}, {15, 10}, {16, 6}, {17, 3}, {18, 1}}

s = Sum[newFreq[[n]][[2]], {n, 1, Length[newFreq]}]

216

Table[fr[newFreq[[n]][[1]]] = newFreq[[n]][[2]], {n, 1, Length[newFreq]}]

{1, 3, 6, 10, 15, 21, 25, 27, 27, 25, 21, 15, 10, 6, 3, 1}

Table[fr[newFreq[[n]][[1]]], {n, 1, Length[newFreq]}]

{1, 3, 6, 10, 15, 21, 25, 27, 27, 25, 21, 15, 10, 6, 3, 1}

fr[12]

25

Table[relfr[newFreq[[n]][[1]]] = newFreq[[n]][[2]] / s, {n, 1, Length[newFreq]}]
```

$$\left\{\frac{1}{216}, \frac{1}{72}, \frac{1}{36}, \frac{5}{108}, \frac{5}{72}, \frac{7}{72}, \frac{25}{216}, \frac{1}{8}, \frac{1}{8}, \frac{25}{216}, \frac{7}{72}, \frac{5}{72}, \frac{5}{108}, \frac{1}{36}, \frac{1}{72}, \frac{1}{216}\right\}$$

```
N[%]

{0.00462963, 0.0138889, 0.0277778, 0.0462963, 0.0694444, 0.0972222, 0.115741, 0.125,
 0.125, 0.115741, 0.0972222, 0.0694444, 0.0462963, 0.0277778, 0.0138889, 0.00462963}
```

```
μ = Sum[newFreq[[n]][[1]] * relfr[newFreq[[n]][[1]]] , {n, 1, Length[newFreq]}]
```

$\frac{21}{2}$

```
N[%]
```

10.5

```
3 + (18 - 3) / 2
```

$\frac{21}{2}$

3.

```
Remove[a, b, f, F, x]
```

```
f[x_] := a (E^(-x^2) - b); f[x]
```

$a\left(-b + e^{-x^2}\right)$

```
{f[-7], f[7]}
```

$\left\{a\left(-b + \frac{1}{e^{49}}\right), a\left(-b + \frac{1}{e^{49}}\right)\right\}$

```
s = Solve[f[7] == 0, {b}] // Flatten
```

$\left\{b \to \frac{1}{e^{49}}\right\}$

```
f1[x_] := a (E^(-x^2) - b) /. s; f1[x]
```

$a\left(-\frac{1}{e^{49}} + e^{-x^2}\right)$

```
F[x_] := Integrate[f1[t], {t, -7, x}]; F[x]
```

$-\frac{7\,a}{e^{49}} - \frac{a\,x}{e^{49}} + \frac{1}{2}\,a\sqrt{\pi}\,(\text{Erf}[7] + \text{Erf}[x])$

```
F[-7]
```

0

```
F[7]
```

$a\left(-\frac{14}{e^{49}} + \sqrt{\pi}\,\text{Erf}[7]\right)$

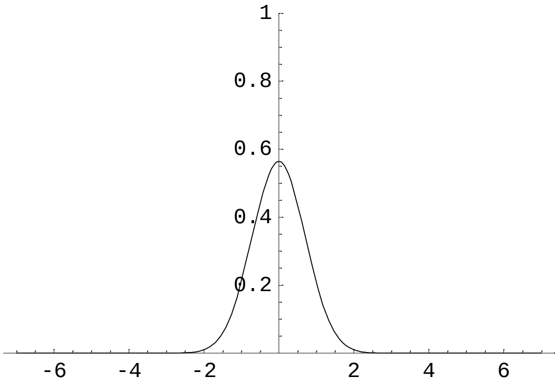
```
ss = Solve[F[7] == 1, {a}] // Flatten
```

$\left\{a \to \frac{e^{49}}{-14 + e^{49}\sqrt{\pi}\,\text{Erf}[7]}\right\}$

```
f2[x_] := f1[x] /. ss; f2[x]
```

$\frac{e^{49}\left(-\frac{1}{e^{49}} + e^{-x^2}\right)}{-14 + e^{49}\sqrt{\pi}\,\text{Erf}[7]}$

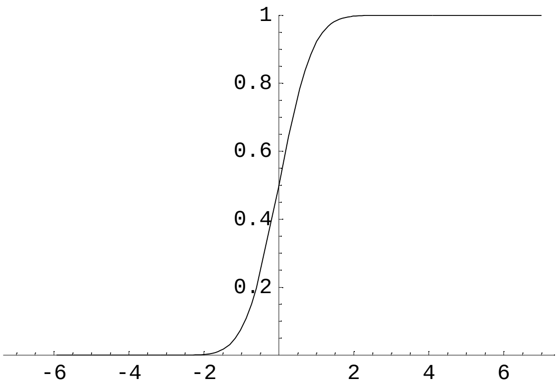
```
Plot[f2[x], {x, -7, 7}, PlotRange → {0, 1}];
```




```
F2[x_] := F[x] /. ss; F2[x]
```

$$-\frac{7}{-14 + e^{49}\sqrt{\pi}\,\text{Erf}[7]} - \frac{x}{-14 + e^{49}\sqrt{\pi}\,\text{Erf}[7]} + \frac{e^{49}\sqrt{\pi}\,(\text{Erf}[7] + \text{Erf}[x])}{2\left(-14 + e^{49}\sqrt{\pi}\,\text{Erf}[7]\right)}$$

```
Plot[F2[x], {x, -7, 7}, PlotRange → {0, 1}];
```



4.

Begriffe siehe angegebene Literatur und Links im Aufgabenblatt.
http://de.wikipedia.org/wiki/Punktsch%C3%A4tzer
http://de.wikipedia.org/wiki/Stichprobe
http://de.wikipedia.org/wiki/Vertrauensintervall
http://de.wikipedia.org/wiki/Konfidenzintervall
http://de.wikipedia.org/wiki/Statistischer_Test
http://de.wikipedia.org/wiki/Vorzeichentest
http://de.wikipedia.org/wiki/Wilcoxon-Vorzeichen-Rang-Test

5.

```
Remove["Global`*"]
```

```
n = 145;
xQuer = 314.0;
σQuadrat = 1000;
σ = Sqrt[1000];
α = 0.01;
```

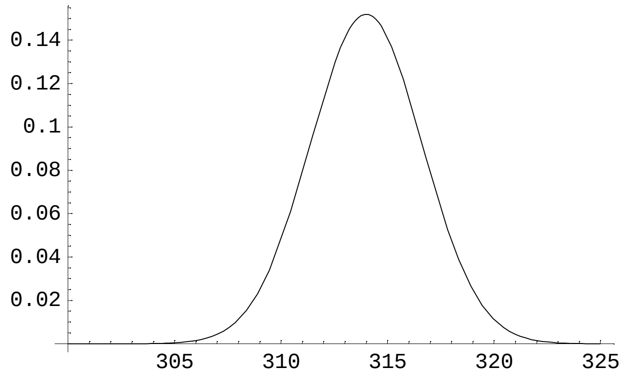
```
σ // N
```

31.6228

```
NV[x_, μ_, σ_, n_] := 1 / (σ / Sqrt[n] Sqrt[2 Pi]) E^(-1 / 2 ((x - μ) Sqrt[n] / σ)^2);
NV[μ, xQuer, σ, n]
```

$$\frac{1}{20} e^{-\frac{29}{400}(-314.+\mu)^2} \sqrt{\frac{29}{\pi}}$$

```
Plot[NV[μ, xQuer, σ, n], {μ, 300, 325}];
```




```
Integrate[NV[μ, xQuer, σ, n], {μ, 280, 340}]
```

1.000000000000

Integriere nur über die halbe Normalverteilung:

```
Integrate[NV[μ, xQuer, σ, n], {μ, xQuer, x}] == (1 - α) / 2
```

$1.25275 \times 10^{-15} + 0.5\,\mathrm{Erf}[-84.5471 + 0.269258\,x] == 0.495$

```
rootOben =
 FindRoot[Evaluate[Integrate[NV[μ, xQuer, σ, n], {μ, xQuer, x}]] == (1 - α) / 2, {x, 320}]
```

{x → 320.764}

```
cOben = x /. rootOben
```

320.764

```
cUnten = xQuer - (cOben - xQuer)
```

307.236

```
Integrate[NV[μ, xQuer, σ, n], {μ, cUnten, cOben}]
```

0.989999999851

```
1 - Integrate[NV[μ, xQuer, σ, n], {μ, cUnten, cOben}]
```

```
0.010000000149
```

Das ist etwa . Lösung: [cUnten, cOben] ist das Konfidenzintervall.

6.

a

Solange nicht mehrmals ohne Zurücklegen gezogen wird, kann man von einer Bernoulli-Verteilung ausgehen.

```
b[k_] := Binomial[20, k] (1 / 2)^k (1 / 2)^(20 - k); b[k]
```

$$\frac{\text{Binomial}[20, k]}{1048576}$$

b

```
Sum[N[b[k]], {k, 5, 15}]
```

```
0.988182
```

c

Mit der Wahrscheinlichkeit 0.988182 (Konvidenzniveau) kommen bei einer solchen Ziehung mindestens 5 rote und maximal 15 rote Bonbons, d. h maximal 15 blaue und minimal 5 blaue in eine Tüte zu liegen. Das Konvidenzintervall zum Niveau 0.988182 ist damit [5, 15].

7.

```
M1 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12};
M2 = {56, 202, 381, 526, 530, 404, 270, 56, 137, 44, 26, 9, 3};
```

```
{a1, b1, c1}.{a2, b2, c2}
```

```
a1 a2 + b1 b2 + c1 c2
```

```
Length[M1] == Length[M2]
```

```
True
```

```
n = Apply[Plus, M2]
```

```
2644
```

```
μ = M1.M2 / n  // N
```

```
4.00189
```

```
({a1, b1, c1} - d) ^2
```

$\{(a1 - d)^2, (b1 - d)^2, (c1 - d)^2\}$

```
({a1, b1, c1} - d) ^2 .{a2, b2, c2}
```

$a2 (a1 - d)^2 + b2 (b1 - d)^2 + c2 (c1 - d)^2$

```
1 / (n - 1) (M2.((M1 - μ) ^2))
```

4.39387

```
σ = Sqrt[1 / (n - 1) (M2.((M1 - μ) ^2))]
```

2.09616

```
σQuadrat = σ^2
```

4.39387

```
RelativerFehler = (σ^2 - μ) / σ^2
```

0.0892098

a

```
f1[k_] := μ^k / k ! E^(-μ)
```

p12 := P(X >12)=1-P(X 12)

```
p12 = 1 - Sum[f1[k], {k, 0, 12}]
```

0.000274932

Das ist weniger als ein Drittel Promill.

b

```
f2[k_] := σQuadrat^k / k ! E^(-σQuadrat)
p12 = 1 - Sum[f2[k], {k, 0, 12}]
```

0.000649747

Das ist weniger als zwei Drittel Promill.