Lösungen / Statistik 2/02

```
Remove["Global`*"]
```

1.

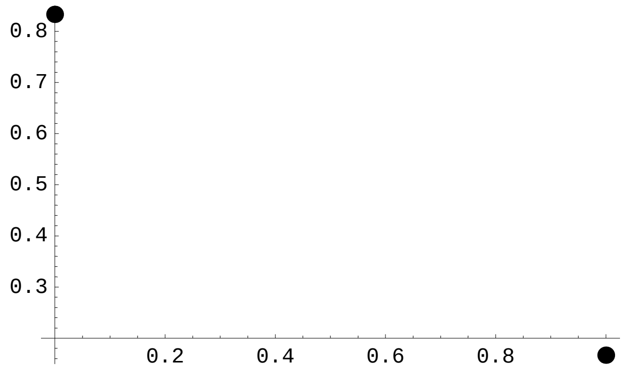
```
<< Statistics`DiscreteDistributions`
```

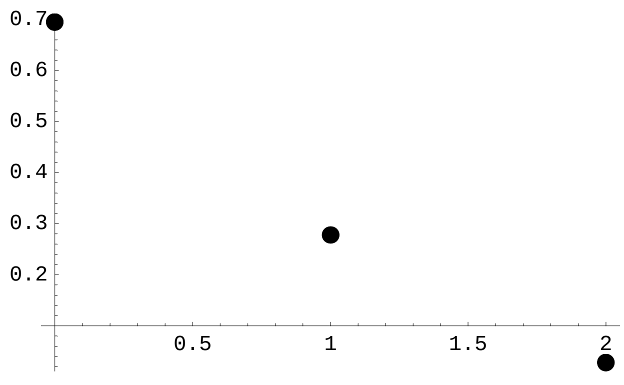
```
bdist[n_] := BinomialDistribution[n, 1 / 6]
```

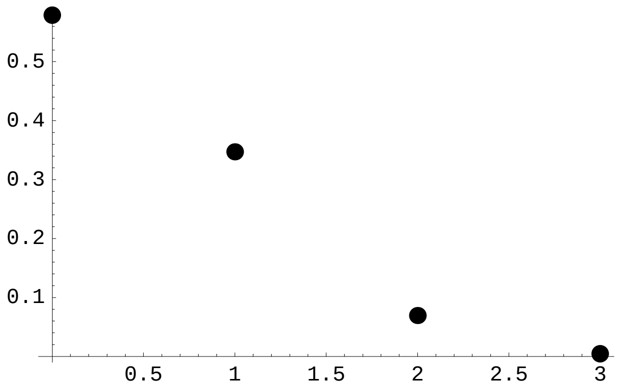
```
pdf[n_, x_] := PDF[bdist[n], x];
cdf[n_, x_] := CDF[bdist[n], x];
```

```
lpPDF[n_] := ListPlot[Table[{x, pdf[n, x]}, {x, 0, n}], PlotStyle → {PointSize[0.03]}];
lpCDF[n_] := ListPlot[Table[{x, cdf[n, x]}, {x, 0, n}], PlotStyle → {PointSize[0.03]}];
```

```
Table[lpPDF[n], {n, 1, 20}];
```



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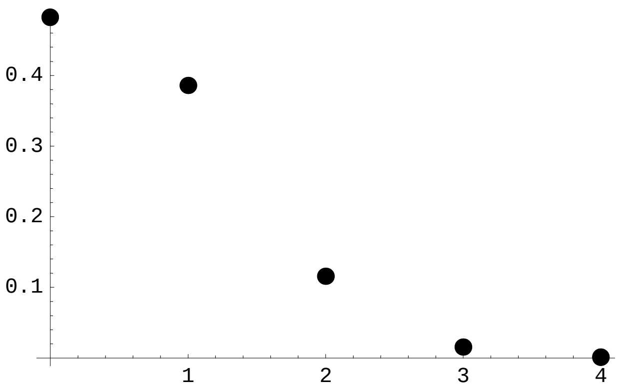
0.3
0.2
0.1
0.5 1 1.5 2 2.5 3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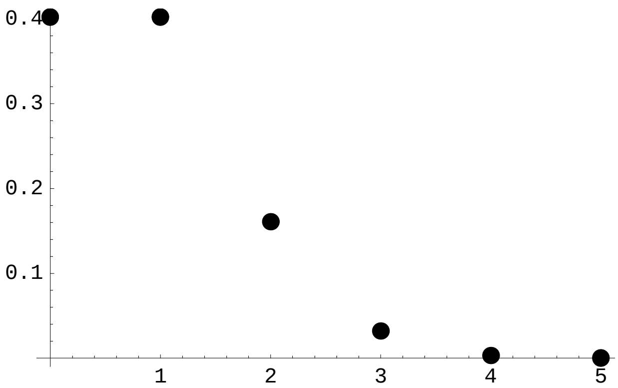
0.3
0.2
0.1
1 2 3 4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0.2
0.1
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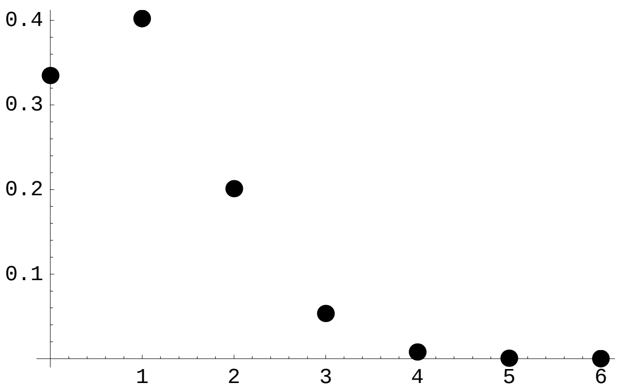
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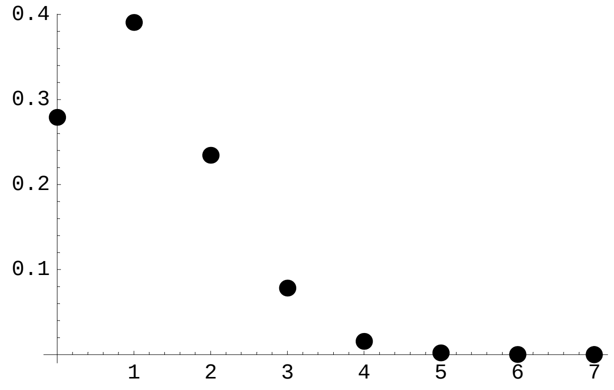
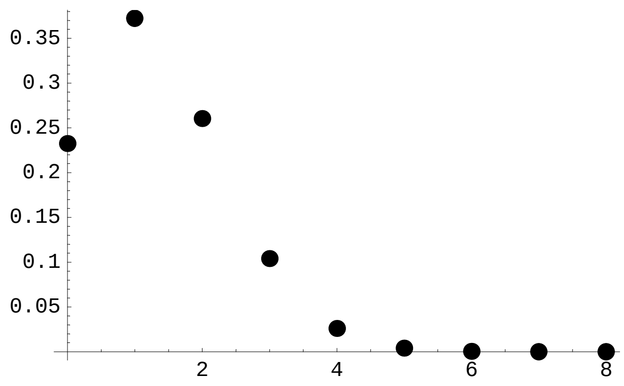
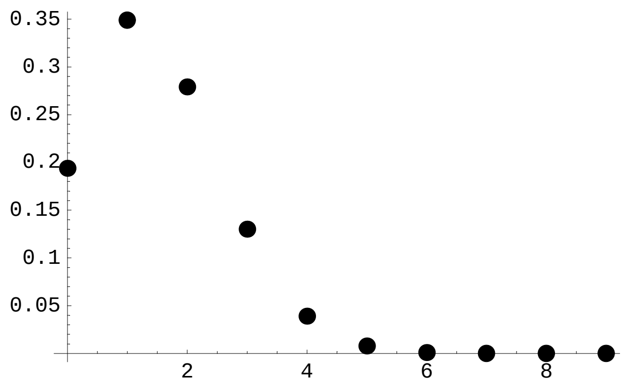
0.3
0.2
0.1
1 2 3 4 5 6


0.4
0.3
0.2
0.1
1 2 3 4 5 6 7


0.35
0.3
0.25
0.2
0.15
0.1
0.05
2 4 6 8


0.35
0.3
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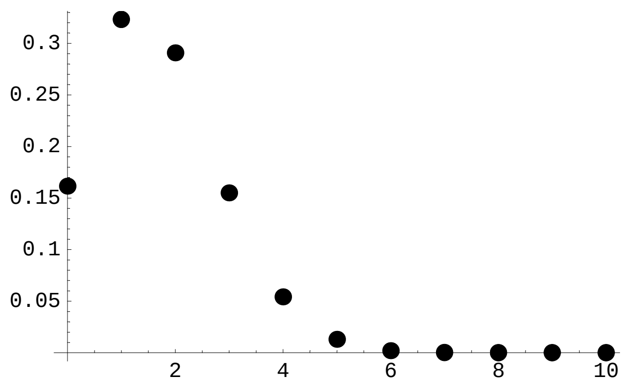
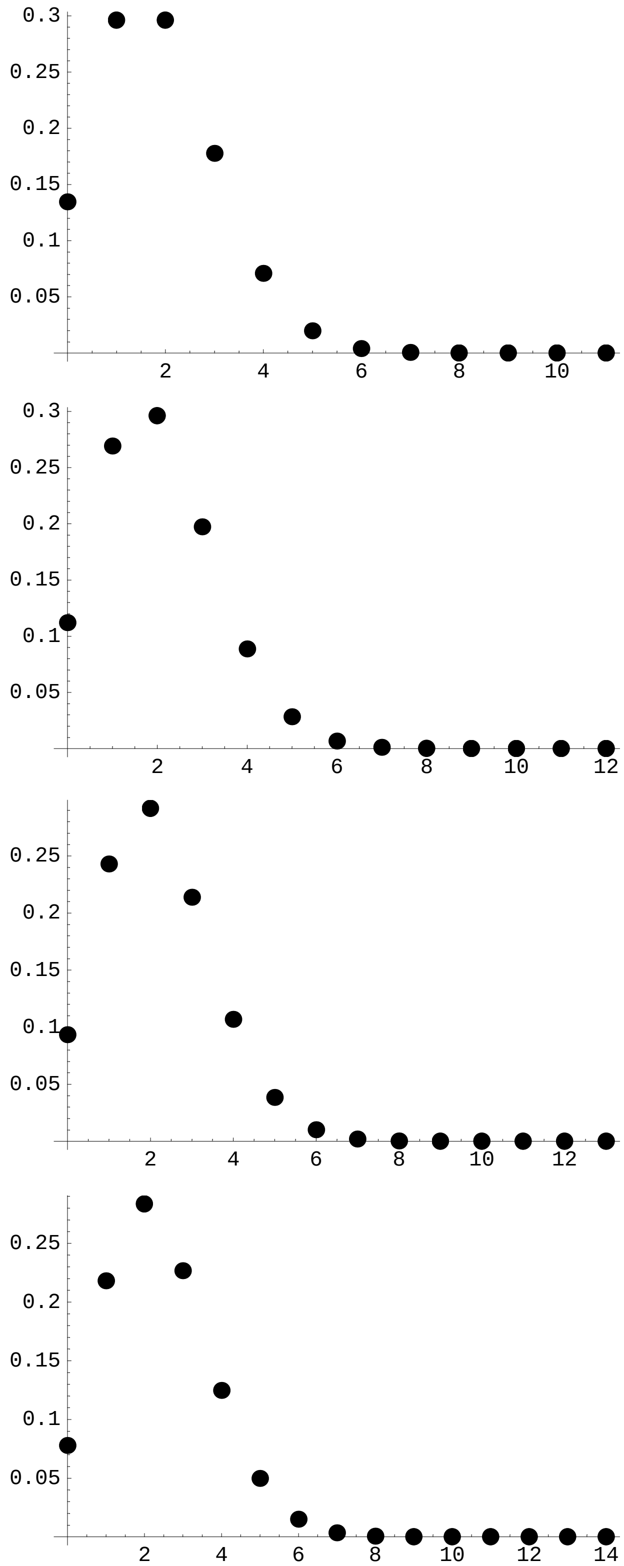
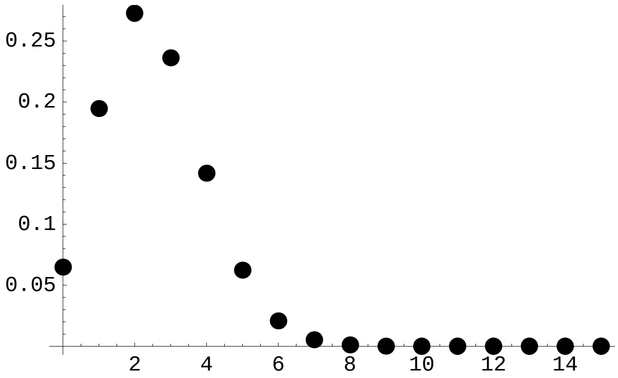
0.2
0.15
0.1
0.05
2 4 6 8


0.3
0.25
0.2
0.15
0.1
0.05
2 4 6 8 10


0.3
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10
0.3
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10
12
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10
12
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10
12
14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0.15
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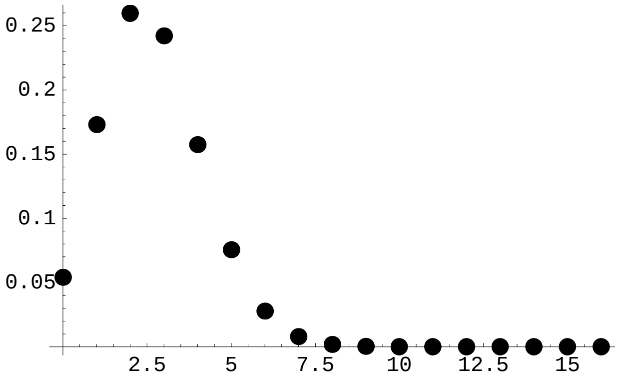
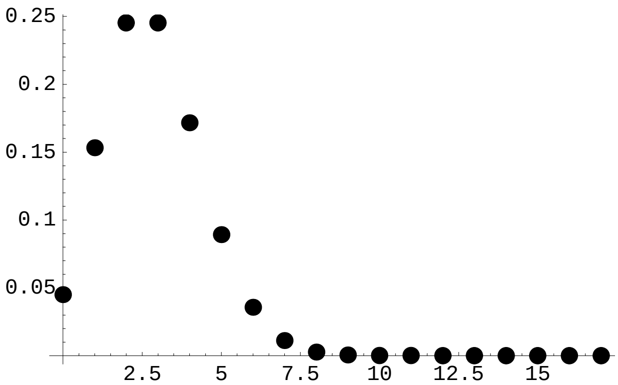
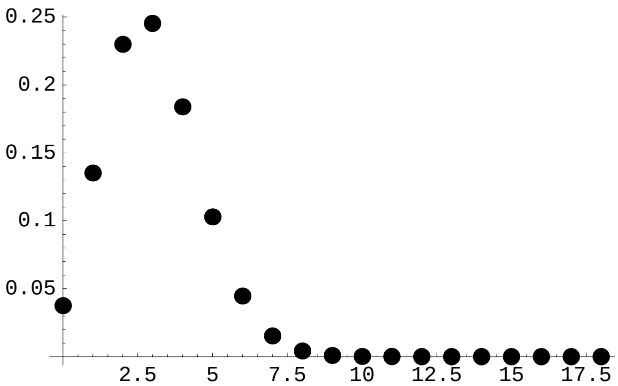
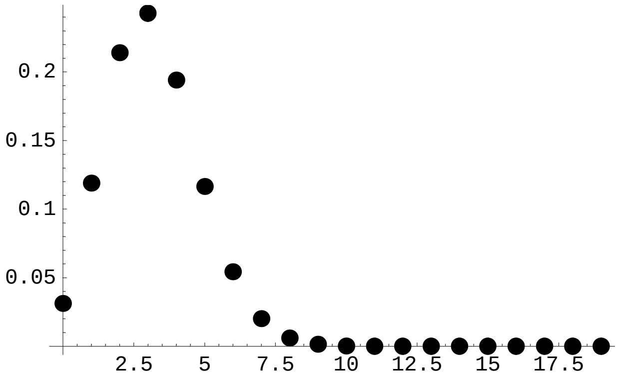
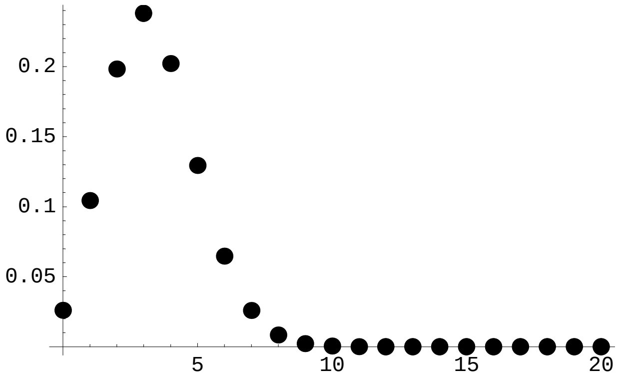
0.05
2 4 6 8 10 12 14


0.25
0.2
0.15
0.1
0.05
2.5 5 7.5 10 12.5 15

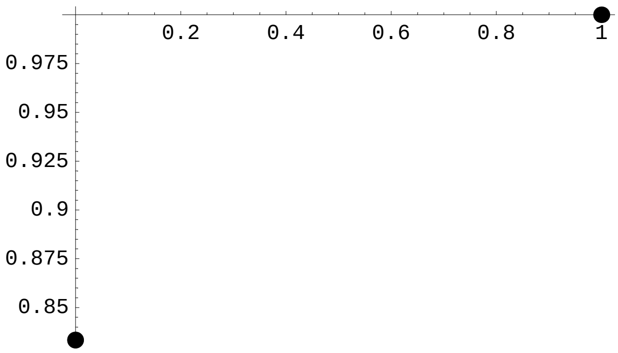

0.25
0.2
0.15
0.1
0.05
2.5 5 7.5 10 12.5 15

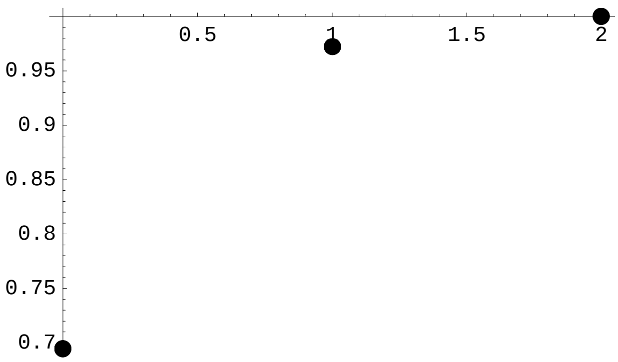

0.25
0.2
0.15
0.1
0.05
2.5 5 7.5 10 12.5 15 17.5

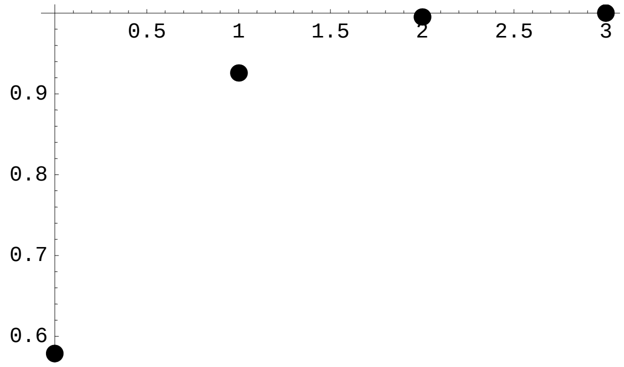


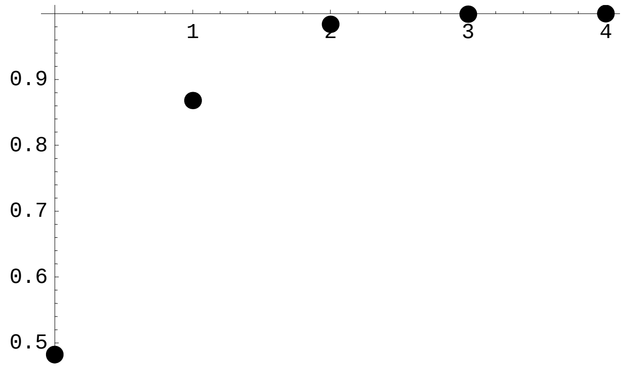


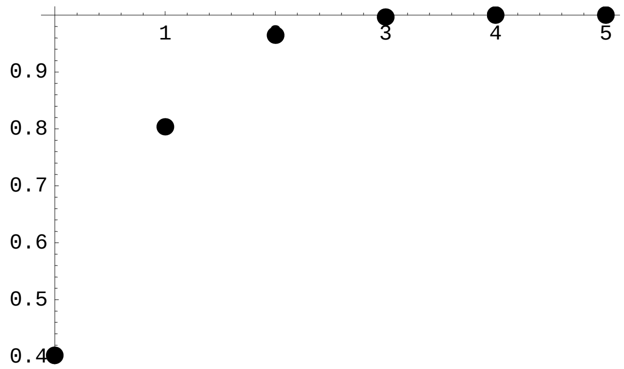

```
Table[lpCDF[n], {n, 1, 20}];
```

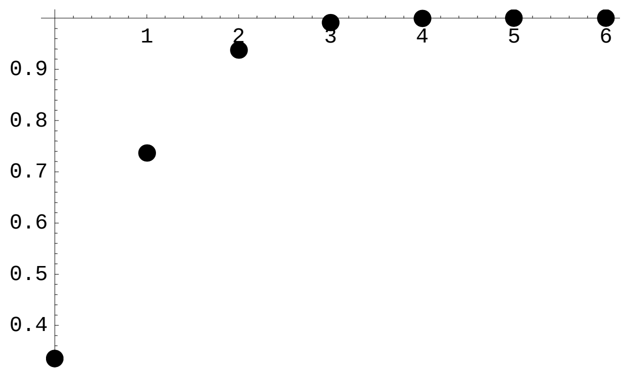


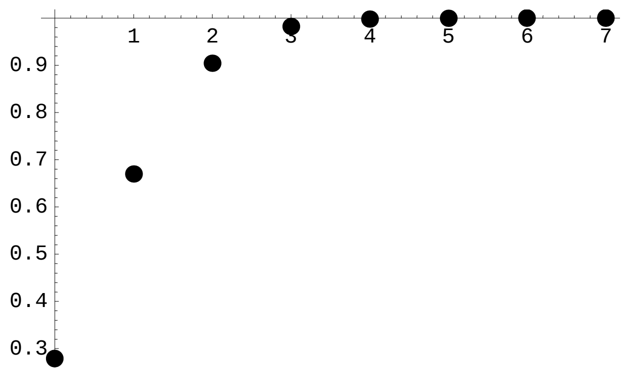



0.5
1
1.5
2
2.5
3
0.9
0.8
0.7
0.6


1
2
3
4
0.9
0.8
0.7
0.6
0.5


1
3
4
5
0.9
0.8
0.7
0.6
0.5
0.4


1
2
3
4
5
6
0.9
0.8
0.7
0.6
0.5
0.4


1
2
3
4
5
6
7
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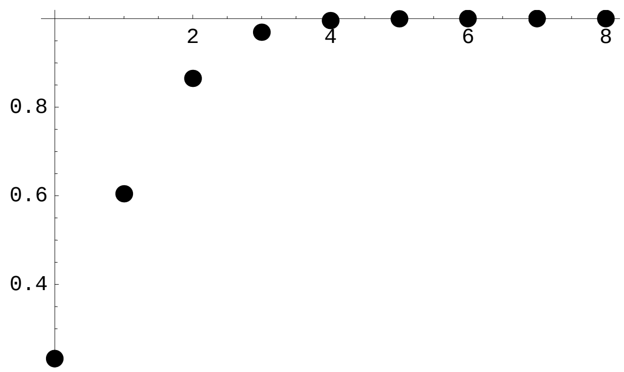
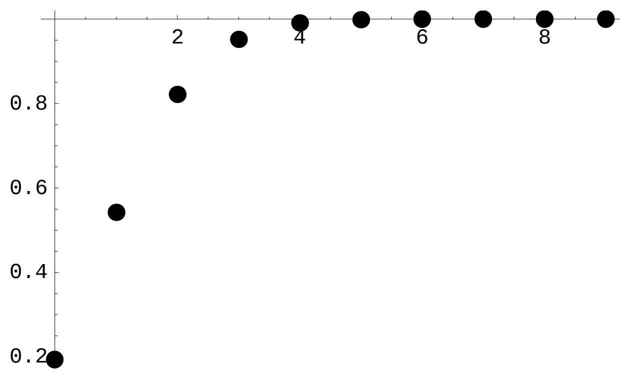
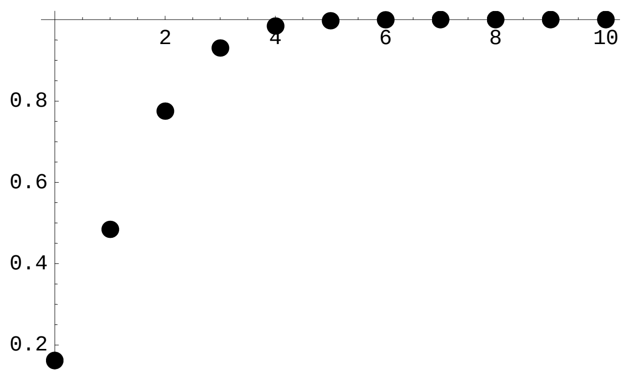
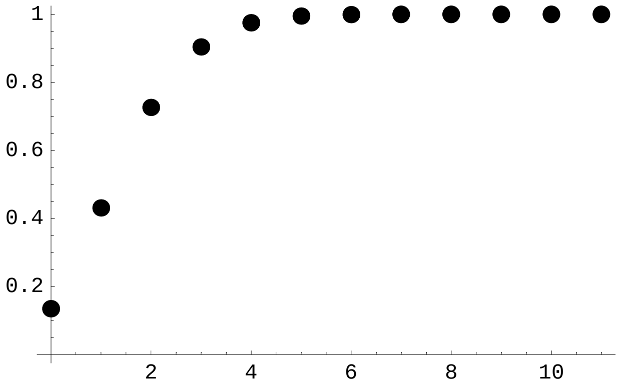
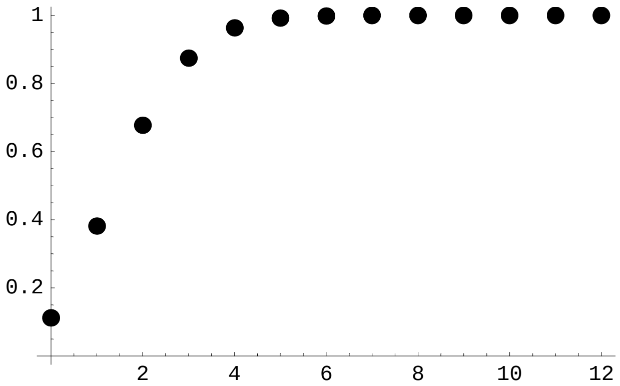
0.8
0.7
0.6
0.5
0.4
0.3

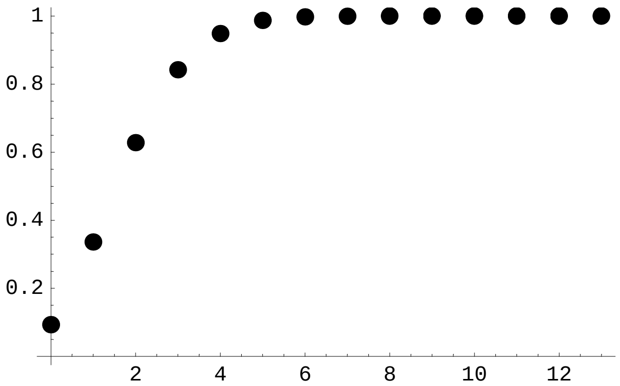

2
4
6
8
0.8
0.6
0.4

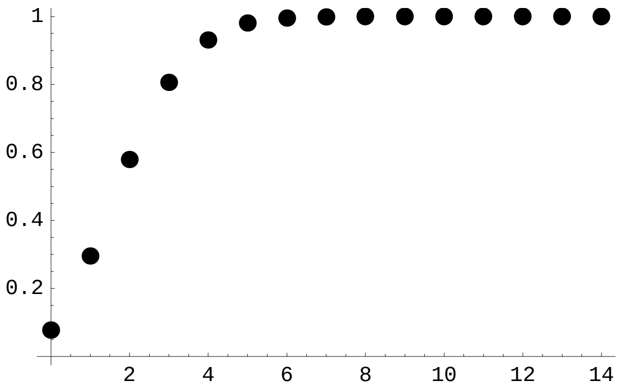

2
4
6
8
0.8
0.6
0.4
0.2

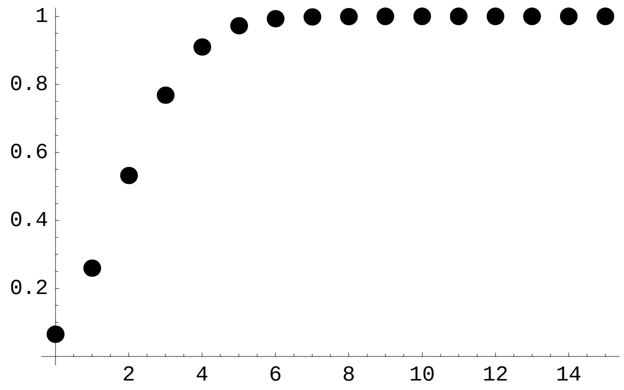

2
4
6
8
10
0.8
0.6
0.4
0.2

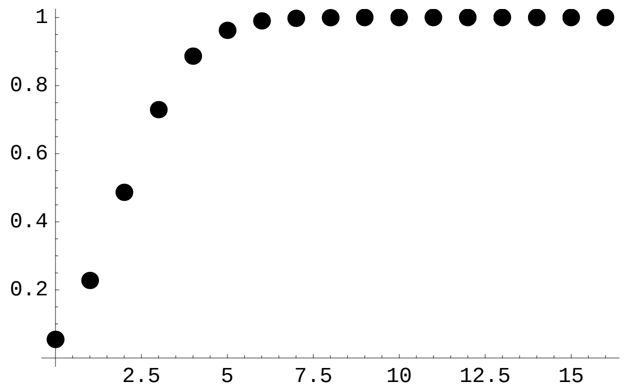

1
0.8
0.6
0.4
0.2
2 4 6 8 10

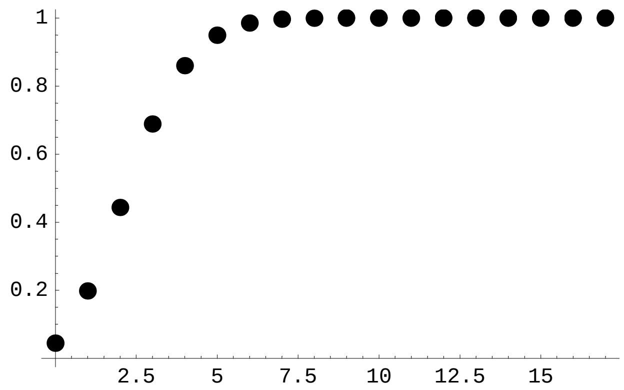

1
0.8
0.6
0.4
0.2
2 4 6 8 10 12

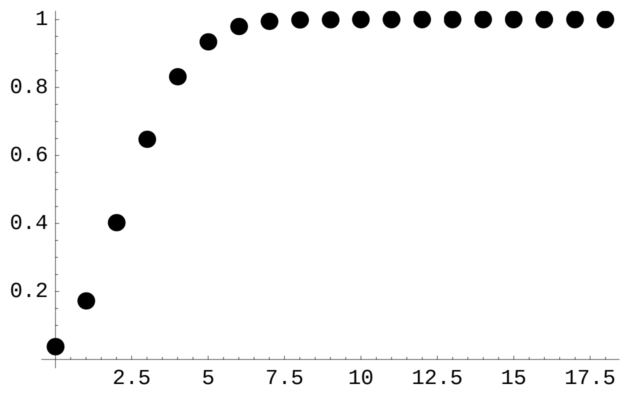

1
0.8
0.6
0.4
0.2
2 4 6 8 10 12

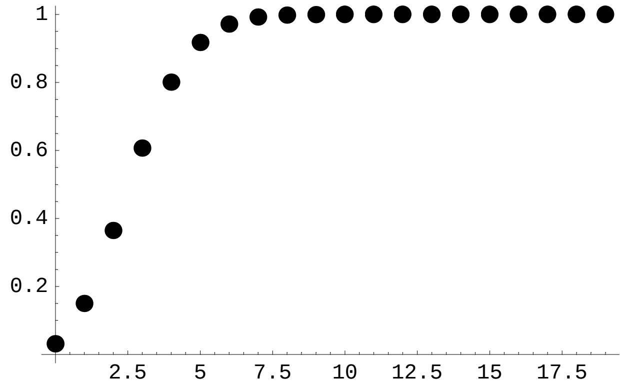

1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14

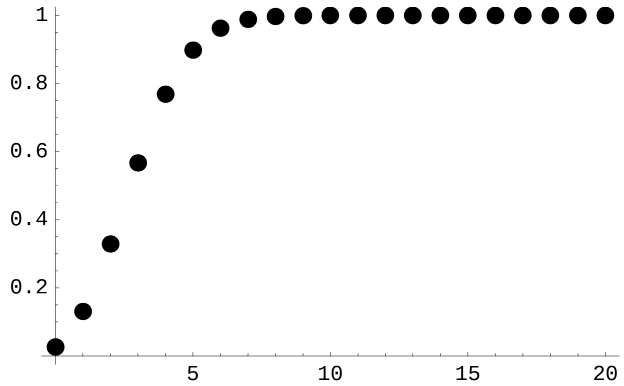

1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14


1
0.8
0.6
0.4
0.2
2.5 5 7.5 10 12.5 15


1
0.8
0.6
0.4
0.2
2.5 5 7.5 10 12.5 15


1
0.8
0.6
0.4
0.2
2.5 5 7.5 10 12.5 15 17.5

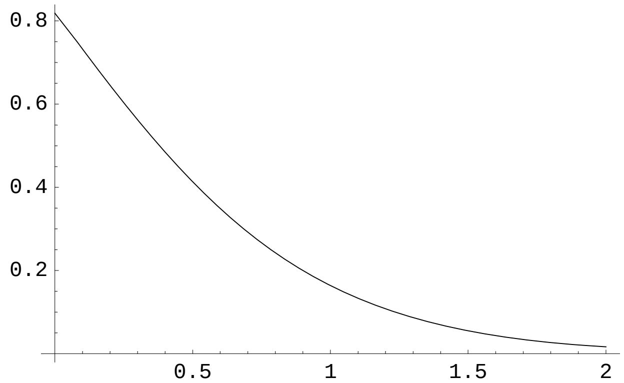




2.

```
p = 0.002; μ[n_] := n * p;
f[x_, n_] := (μ[n]^x) / (x!) * E^(-μ[n])
```

```
Plot[f[x, 100], {x, 0, 2}];
```



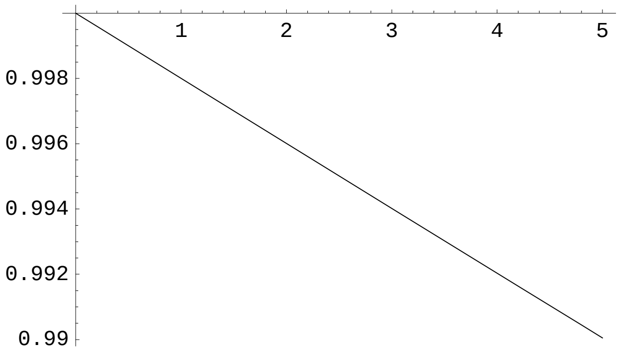

```
f[0, 100]
```

0.818731

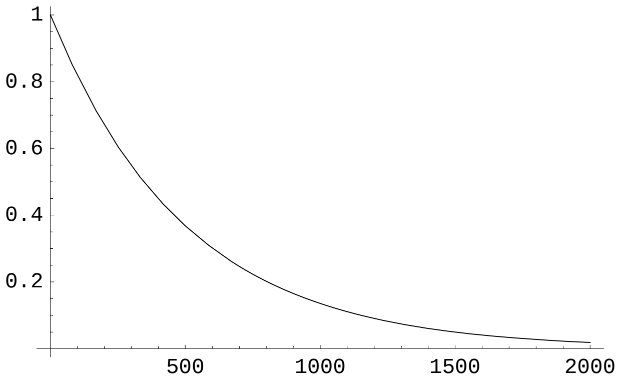
```
f[0, 1000]
```

0.135335

```
Plot[f[0, n], {n, 0, 5}];
```



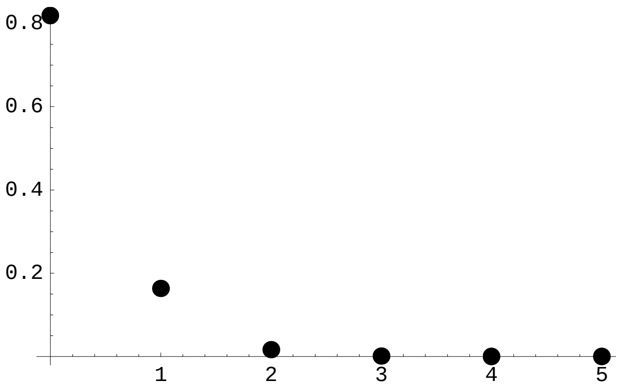

```
Plot[f[0, n], {n, 0, 2000}];
```



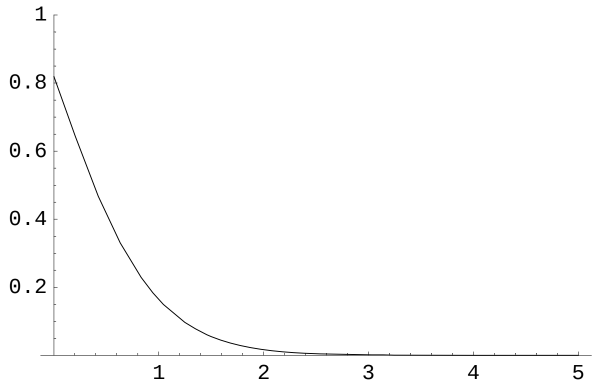

```
pD100[x_] := PDF[PoissonDistribution[μ[100]], x]

{pD100[0], f[0, 100]}

{0.818731, 0.818731}

{pD100[3], f[3, 100]}

{0.00109164, 0.00109164}

Table[{k, pD100[k]}, {k, 0, 5}]
```

{{0, 0.818731}, {1, 0.163746}, {2, 0.0163746}, {3, 0.00109164}, {4, 0.0000545821}, {5, 2.18328×10^{-6}}}

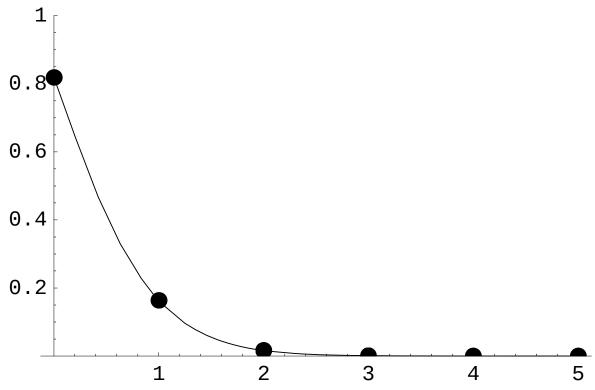
```
g1 =
  ListPlot[Evaluate[Table[{k, pD100[k]}, {k, 0, 5}]], PlotStyle → {PointSize[0.03]}];
```



```
g2 = Plot[f[x, 100], {x, 0, 5}, PlotRange → {0, 1}];
```



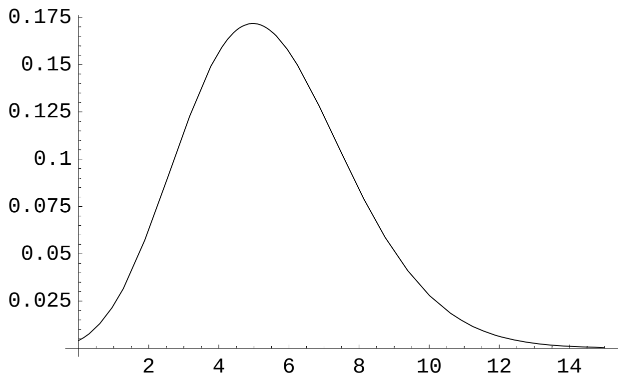

```
Plot[f[x, 100], {x, 0, 5}, PlotRange → {0, 1},
  Epilog → {PointSize[0.03], Map[Point, Table[{k, pD100[k]}, {k, 0, 5}]]}];
```



3.

■ a

```
n = 2000; p = 1 / 365;  μ[n_] := n * p; f[x_, n_] := (μ[n]^x) / (x!) * E^(-μ[n]);
Plot[f[x, n], {x, 0, 15}];
```



```
Table[f[x, n], {x, 0, 15}] // N
```

```
{0.00417161, 0.0228582, 0.0626251, 0.114384, 0.15669,
 0.171715, 0.156817, 0.122753, 0.0840777, 0.0511888, 0.0280487,
 0.0139719, 0.00637988, 0.0026891, 0.00105248, 0.000384469}
```

```
f[2, n] // N
```

```
0.0626251
```

▪ b

```
1 - f[1, n] - f[0, n] // N
```

```
0.97297
```

4.

```
t1 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13};
t = {57, 203, 383, 525, 532, 408, 273, 139, 45, 27, 10, 4, 2, 0}
```

```
{57, 203, 383, 525, 532, 408, 273, 139, 45, 27, 10, 4, 2, 0}
```

```
t2 = Transpose[{t1, t}]
```

```
{{0, 57}, {1, 203}, {2, 383}, {3, 525}, {4, 532}, {5, 408},
 {6, 273}, {7, 139}, {8, 45}, {9, 27}, {10, 10}, {11, 4}, {12, 2}, {13, 0}}
```

```
s = Apply[Plus, t]
```

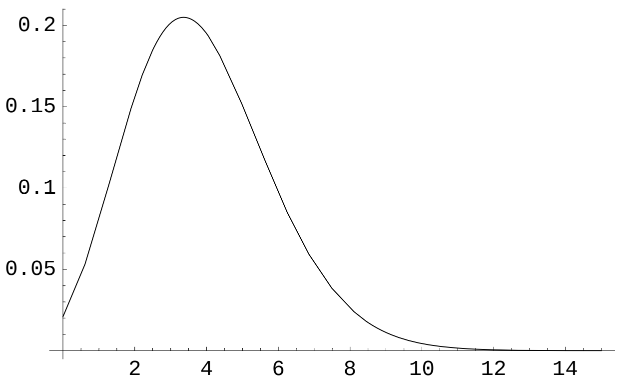
```
2608
```

```
μ4 = Sum[t1[[k]] t[[k]], {k, 1, Length[t]}] / s // N
```

```
3.8704
```

```
f4[x_] := (μ4^x) / (x!) * E^(-μ4);
```

```
Plot[f4[x], {x, 0, 15}];
```



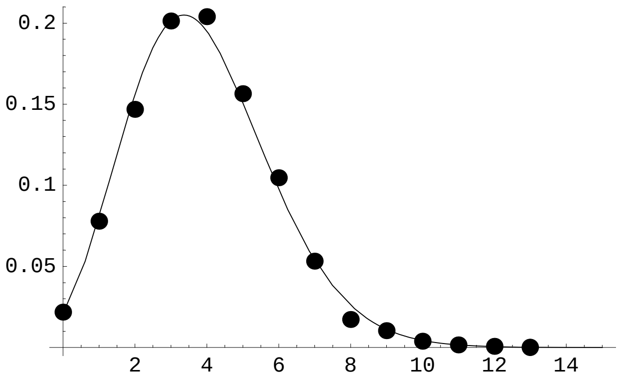
```
t3 = t / s
```

$\{\frac{57}{2608}, \frac{203}{2608}, \frac{383}{2608}, \frac{525}{2608}, \frac{133}{652}, \frac{51}{326}, \frac{273}{2608}, \frac{139}{2608}, \frac{45}{2608}, \frac{27}{2608}, \frac{5}{1304}, \frac{1}{652}, \frac{1}{1304}, 0\}$

```
t4 = Transpose[{t1, t3}] // N
```

```
{{0., 0.0218558}, {1., 0.0778374}, {2., 0.146856}, {3., 0.201304}, {4., 0.203988},
 {5., 0.156442}, {6., 0.104678}, {7., 0.0532975}, {8., 0.0172546}, {9., 0.0103528},
 {10., 0.00383436}, {11., 0.00153374}, {12., 0.000766871}, {13., 0.}}
```

```
Plot[f4[x], {x, 0, 15}, Epilog → {PointSize[0.03], Map[Point, t4]}];
```



5.

```
f5[Nn_, M_, n_, x_] := Binomial[M, x] Binomial[Nn - M, n - x] / Binomial[Nn, n]
```

```
Table[f5[11 + 3, 11, 4, x], {x, 0, 4}] // N
```

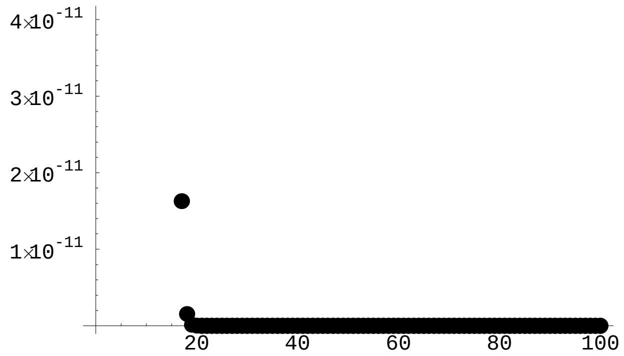
```
{0., 0.010989, 0.164835, 0.494505, 0.32967}
```

6.

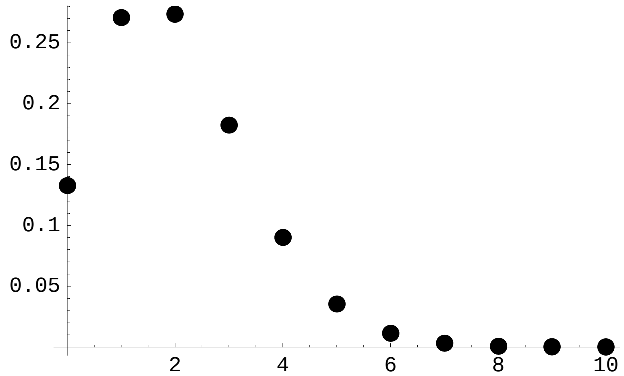
```
Nn = 1000; M = 20; n = 100; p = M / Nn;
```

```
bdist[n_, p_] := BinomialDistribution[n, p];
pdf[n_, x_, p_] := PDF[bdist[n, p], x];
```

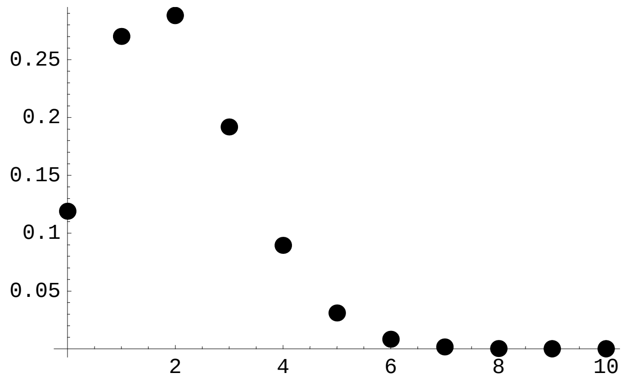
```
ListPlot[Table[{x, pdf[n, x, p]}, {x, 0, n}], PlotStyle → {PointSize[0.03]}];
```



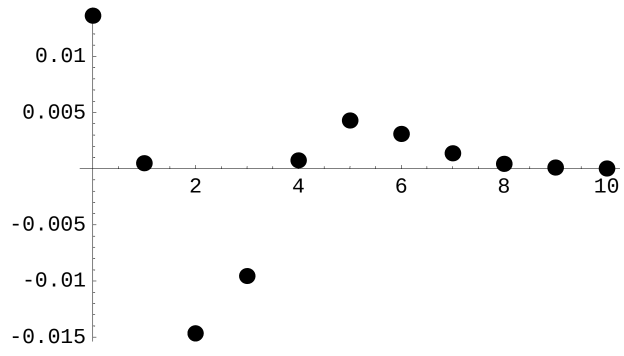

```
p1 = ListPlot[Table[{x, pdf[n, x, p]}, {x, 0, 10}], PlotStyle → {PointSize[0.03]}];
```



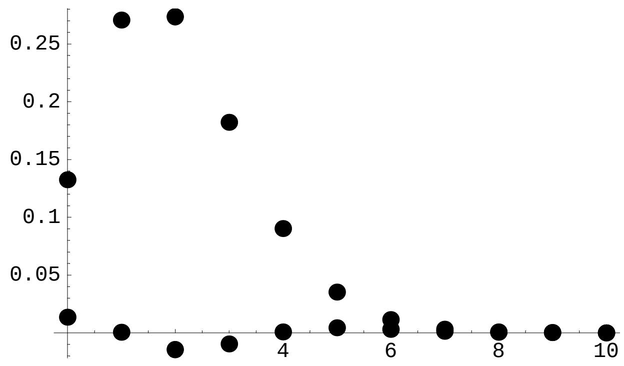

```
ListPlot[Table[{x, f5[Nn, M, n, x]}, {x, 0, 10}], PlotStyle → {PointSize[0.03]}];
```




```
p2 = ListPlot[Table[{x, pdf[n, x, p] - f5[Nn, M, n, x]}, {x, 0, 10}],
   PlotStyle → {PointSize[0.03]}];
```

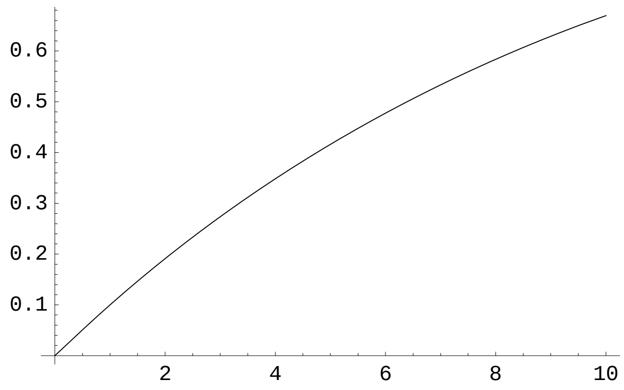


```
Show[p1, p2];
```



7.

```
Nn = 100; M = 100 * 0.1; n = 10;

f5[Nn, M, n, 0] // N

0.330476

1 - f5[Nn, M, n, 0] // N

0.669524

1 - f5[Nn, 100 * (0.1 - 0.01), n, 0] // N

0.628724

1 - f5[Nn, 100 * (0.1 - 0.05), n, 0] // N

0.416248

1 - f5[Nn, 100 * (0), n, 0] // N

0.

Plot[1 - f5[Nn, k, n, 0], {k, 0, M}];
```



8.

```
Remove["Global`*"]
```

Konvention: 2 cm +/- 0.02 cm = 2.00 cm +/- 0.02 cm

```
d1 = 2.00; Δd1 = 0.02;
p1 = 11.2; Δp1 = 0.1;
VpSec1 = 5.00; ΔVpSec1 = 0.01;
ρ1 = 0.83; Δρ1 = 0.01;

d2 = 1.20; Δd2 = 0.02;
p2       ; Δp2      ;
VpSec2 = VpSec1; ΔVpSec2 = ΔVpSec1;
ρ2 = 0.83; Δρ2 = 0.01;
```

```
solv1 = Solve[v1 d1^2 / 4 Pi == VpSec1, {v1}] // Flatten
```

{v1 → 1.59155}

```
v1 = v1 /. solv1
```

1.59155

```
v[VpSec_, d_] := VpSec / d^2 4 / Pi
```

```
Δv1 = (Abs[D[v[VpSec, d], VpSec]] ΔVpSec1 + Abs[D[v[VpSec, d], d]] Δd1) /.
  {VpSec -> VpSec1, d → d1}
```

0.0350141

```
solv2 = Solve[v2 d2^2 / 4 Pi == VpSec2, {v2}] // Flatten
```

{v2 → 4.42097}

```
v2 = v2 /. solv2
```

4.42097

```
Δv2 = (Abs[D[v[VpSec, d], VpSec]] ΔVpSec2 + Abs[D[v[VpSec, d], d]] Δd2) /.
  {VpSec -> VpSec2, d → d2}
```

0.156208

```
solv3 = Solve[p1 / ρ1 + (v1^2) / 2 == p2 / ρ2 + (v2^2) / 2, {p2}] // Flatten
```

{p2 → 4.14004}

```
p2 = p2 /. solv3
```

4.14004

```
solv3 = Solve[p[1] / ρ[1] + (v[1]^2) / 2 == p[2] / ρ[2] + (v[2]^2) / 2, {p[2]}] // Flatten
```

$$\left\{p[2] \to \frac{(2\,p[1] + v[1]^2\,\rho[1] - v[2]^2\,\rho[1])\,\rho[2]}{2\,\rho[1]}\right\}$$

```
pN[p_, v_, ρ_, vN_, ρN_] := (2 p + v^2 ρ - vN^2 ρ) ρN / (2 ρ)

Δp2 = Abs[D[pN[p, v, ρ, vN, ρN], p]] Δp1 +
  Abs[D[pN[p, v, ρ, vN, ρN], v]] Δv1 +
  Abs[D[pN[p, v, ρ, vN, ρN], ρ]] Δρ1 +
  Abs[D[pN[p, v, ρ, vN, ρN], vN]] Δv2 +
  Abs[D[pN[p, v, ρ, vN, ρN], ρN]] Δρ2
```

$$0.005\,\mathrm{Abs}\left[\frac{2\,p + v^2\,\rho - vN^2\,\rho}{\rho}\right] + 0.0350141\,\mathrm{Abs}[v\,\rho N] + 0.156208\,\mathrm{Abs}[vN\,\rho N] + 0.1\,\mathrm{Abs}\left[\frac{\rho N}{\rho}\right] + 0.01\,\mathrm{Abs}\left[\frac{(v^2 - vN^2)\,\rho N}{2\,\rho} - \frac{(2\,p + v^2\,\rho - vN^2\,\rho)\,\rho N}{2\,\rho^2}\right]$$

```
Δp2 /. {p → p1, v → v1, ρ → ρ1, vN → v2, ρN -> ρ2}

0.904262
```

Interessant:
Bei p1=11.2 ist Δp1=0.1.
Bei p2=4.14 hingegen ist Δp2=0.90.