Lösungen / Statistik 2/07

```
Remove["Global`*"]
```

1.

a

```
<< Statistics`DescriptiveStatistics`
```

Def.

```
p[x_, y_] := 1 / 36; p[x_] := 1 / 6;
```

```
Sum[p[x, y], {x, 1, 6}, {y, 1, 6}]
```

```
1
```

```
fS[x_, y_] := x + y; fP[x_, y_] := x * y;
```

```
e[g_, x_, y_] := Sum[g * p[x, y], {x, 1, 6}, {y, 1, 6}];
e[g_, x_] := Sum[g * p[x], {x, 1, 6}];
```

```
var[g_, x_] := e[(g - e[g, x])^2, x]; var[g_, x_, y_] := e[(g - e[g, x, y])^2, x, y];
```

```
cov[x_, y_] := e[x * y, x, y] - e[x, x] * e[y, y]
```

```
corr[x_, y_] := cov[x, y] / Sqrt[var[x, x] var[y, y]]
```

```
tab1 = Flatten[Table[fS[x, y], {x, 1, 6}, {y, 1, 6}]]
```

```
{2, 3, 4, 5, 6, 7, 3, 4, 5, 6, 7, 8, 4, 5, 6, 7, 8,
 9, 5, 6, 7, 8, 9, 10, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11, 12}
```

```
nTab1 = Length[tab1]
```

```
36
```

Mittelwerte / Moyennes

```
{μX = e[x, x], μY = e[y, y]}
```

$$\left\{\frac{7}{2}, \frac{7}{2}\right\}$$

```
μXSY = e[fS[x, y], x, y]
```

```
7
```

```
Mean[Flatten[Table[fS[x, y], {x, 1, 6}, {y, 1, 6}]]]
```

```
7
```

```
μXPY = e[fP[x, y], x, y]
```

$$\frac{49}{4}$$

```
Mean[Flatten[Table[fP[x, y], {x, 1, 6}, {y, 1, 6}]]]
```

$$\frac{49}{4}$$

Varianzen / Variances

```
varX = e[(x - μX)^2, x]
```

$$\frac{35}{12}$$

```
varY = e[(y - μY)^2, y]
```

$$\frac{35}{12}$$

```
var[x, x]
```

$$\frac{35}{12}$$

```
varXSY = e[(fS[x, y] - μXSY)^2, x, y]
```

$$\frac{35}{6}$$

```
var[fS[x, y], x, y]
```

$$\frac{35}{6}$$

```
varXSYDeskr = Variance[Flatten[Table[fS[x, y], {x, 1, 6}, {y, 1, 6}]]]
```

```
6
```

```
varXSYDeskr * (nTab1 - 1) == varXSY * nTab1
```

```
True
```

```
varXPY = e[(fP[x, y] - μXPY)^2, x, y]
```

$$\frac{11515}{144}$$

```
var[fP[x, y], x, y]
```

$$\frac{11515}{144}$$

```
varXPYDeskr = Variance[Flatten[Table[fP[x, y], {x, 1, 6}, {y, 1, 6}]]]
```

$$\frac{329}{4}$$

```
varXPYDeskr * (nTab1 - 1) == varXPY * nTab1
```

```
True
```

Kovarianz x, y / Covarianc x, y

```
covXSY = cov[x, y]
```

```
0
```

```
varXSY == 2 covXSY + varX + varY
```

```
True
```

```
e[(x - μX) (y - μY), x, y]
```

```
0
```

Korrelation / Corrélation

```
corr[x, x]
```

```
1
```

b

```
Remove["Global`*"]
```

```
<< Statistics`DescriptiveStatistics`
```

Def.

```
p[x_, y_] := 1 / 36; p[x_] := 1 / 6;
```

```
Sum[p[x, y], {x, 1, 6}, {y, 1, 6}]
```

```
1
```

```
z1[x_, y_] := x + y; z2[x_, y_] := x * y;
```

```
e[g_, x_, y_] := Sum[g * p[x, y], {x, 1, 6}, {y, 1, 6}];
e[g_, x_] := Sum[g * p[x], {x, 1, 6}];
```

```
var[g_, x_] := e[(g - e[g, x])^2, x]; var[g_, x_, y_] := e[(g - e[g, x, y])^2, x, y];
```

```
covZ1Z2 := e[z1[x, y] * z2[x, y], x, y] - e[z1[x, y], x, y] * e[z2[x, y], x, y]
```

```
corr := covZ1Z2 / Sqrt[var[z1[x, y], x, y] * var[z2[x, y], x, y]]
```

```
tab1 = Flatten[Table[z1[x, y], {x, 1, 6}, {y, 1, 6}]]
```

```
{2, 3, 4, 5, 6, 7, 3, 4, 5, 6, 7, 8, 4, 5, 6, 7, 8,
 9, 5, 6, 7, 8, 9, 10, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11, 12}
```

```
tab2 = Flatten[Table[z2[x, y], {x, 1, 6}, {y, 1, 6}]]
```

```
{1, 2, 3, 4, 5, 6, 2, 4, 6, 8, 10, 12, 3, 6, 9, 12, 15, 18,
 4, 8, 12, 16, 20, 24, 5, 10, 15, 20, 25, 30, 6, 12, 18, 24, 30, 36}
```

```
nTab1 = Length[tab1]
```

```
36
```

Mittelwerte / Moyennes

```
{μZ1 = e[z1[x, y], x, y], μZ2 = e[z2[x, y], x, y]}
```

$$\left\{7, \frac{49}{4}\right\}$$

```
μXSY = e[z1[x, y], x, y]
```

```
7
```

```
μXPY = e[z2[x, y], x, y]
```

$$\frac{49}{4}$$

```
N[%]
```

```
12.25
```

Varianzen / Variances

```
varZ1 = e[(z1[x, y] - μZ1)^2, x, y]
```

$$\frac{35}{6}$$

```
N[%]
```

```
5.83333
```

```
var[z1[x, y], x, y]
```

$$\frac{35}{6}$$

```
N[%]
```

```
5.83333
```

```
varZ2 = e[(z2[x, y] - μZ2)^2, x, y]
```

$$\frac{11515}{144}$$

```
N[%]
```

```
79.9653
```

```
var[z2[x, y], x, y]
```

$$\frac{11515}{144}$$

```
N[%]
```

```
79.9653
```

Kovarianz x, y / Covarianc x, y

```
covZ1Z2
```

$$\frac{245}{12}$$

```
N[%]
```

```
20.4167
```

```
var[z1[x, y] + z2[x, y], x, y] == 2 covZ1Z2 + var[z1[x, y], x, y] + var[z2[x, y], x, y]
```

```
True
```

Korrelation / Corrélation

```
corr
```

$$\sqrt{\frac{42}{47}}$$

```
N[%]
```

```
0.945313
```

2.

```
Remove["Global`*"]
```

```
<< Statistics`DescriptiveStatistics`
```

Def.

```
k = 3;
```

```
pX[1] := 1 / 2; pX[2] = 1 / 3; pX[3] = 1 / 6;
Sum[pX[x], {x, 1, k}]
```

```
1
```

```
pY[1] := 1 / 3; pY[2] = 1 / 4; pY[3] = 5 / 12;
Sum[pY[x], {x, 1, k}]
```

```
1
```

```
pXY[x_, y_] := 1 / 12;
pXY[1, 1] := 1 / 4; pXY[2, 3] := 2 / 12;
Sum[pXY[x, y], {x, 1, k}, {y, 1, k}]
```

```
1
```

```
tab1 = Table[pXY[x, y], {x, 1, k}, {y, 1, k}] // Flatten
```

$$\left\{\frac{1}{4}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{6}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}\right\}$$

```
fS[x_, y_] := x + y; fP[x_, y_] := x * y;
```

```
e[g_, x_, y_] := Sum[g * pXY[x, y], {x, 1, k}, {y, 1, k}];
eX[g_, x_] := Sum[g * pX[x], {x, 1, k}];
eY[g_, y_] := Sum[g * pY[y], {y, 1, k}];
```

```
varX[g_, x_] := eX[(g - eX[g, x])^2, x];
varY[g_, y_] := eY[(g - eY[g, y])^2, y]; var[g_, x_, y_] := e[(g - e[g, x, y])^2, x, y];
```

```
cov[x_, y_] := e[x * y, x, y] - eX[x, x] * eY[y, y]
```

```
corr[x_, y_] := cov[x, y] / Sqrt[varX[x, x] * varY[y, y]]
```

```
tab2 = Flatten[Table[fS[x, y], {x, 1, k}, {y, 1, k}]]
```

```
{2, 3, 4, 3, 4, 5, 4, 5, 6}
```

```
nTab2 = Length[tab2]
```

```
9
```

Mittelwerte / Moyennes

```
{μX = eX[x, x], μY = eY[y, y]}
```

$$\left\{\frac{5}{3}, \frac{25}{12}\right\}$$

```
μXSY = e[fS[x, y], x, y]
```

$$\frac{15}{4}$$

```
μXPY = e[fP[x, y], x, y]
```

$$\frac{11}{3}$$

Varianzen / Variances

```
vX = eX[(x - μX)^2, x]
```

$$\frac{5}{9}$$

```
vY = eY[(y - μY)^2, y]
```

$$\frac{107}{144}$$

```
vXSY = e[(fS[x, y] - μXSY)^2, x, y]
```

$$\frac{27}{16}$$

```
var[fS[x, y], x, y]
```

$$\frac{27}{16}$$

```
vXPY = e[(fP[x, y] - μXPY)^2, x, y]
```

$$\frac{109}{18}$$

```
var[fP[x, y], x, y]
```

$$\frac{109}{18}$$

Kovarianz x, y / Covarianc x, y

```
covXSY = cov[x, y]
```

$$\frac{7}{36}$$

```
vXSY == 2 covXSY + vX + vY
```

```
True
```

```
e[(x - μX) (y - μY), x, y]
```

$$\frac{1}{8}$$

Korrelation / Corrélation

```
corr[x, y]
```

$$\frac{7}{\sqrt{535}}$$

```
N[%]
```

```
0.302636
```

3.

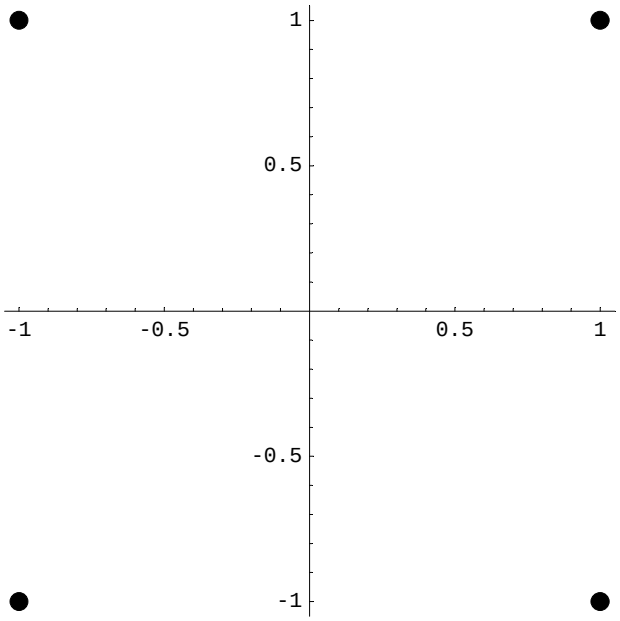
a

```
Remove["Global`*"]
```

```
<< Statistics`LinearRegression`
```

```
m = {{1, 1}, {-1, 1}, {-1, -1}, {1, -1}};
```

```
dPlot = ListPlot[m, PlotStyle → {PointSize[0.03]}, AspectRatio → Automatic];
```




```
Remove[func]
```

```
Fit[m, {1, x}, x]
```

```
0. + 0. x
```

```
fu[x_] := Fit[m, {1, x}, x]
```

```
fu[x]
```

```
0. + 0. x
```

```
Fit[m, {1, x}, x]
```

```
0. + 0. x
```

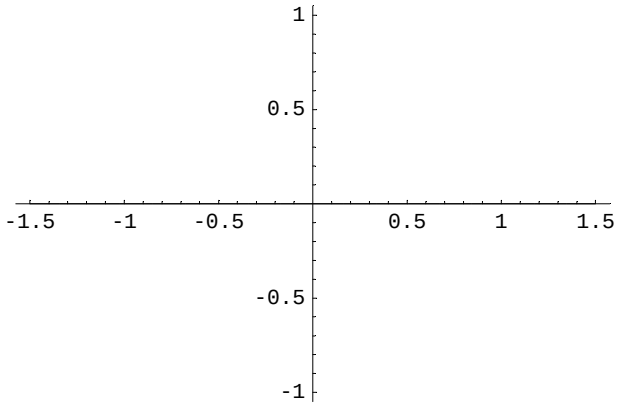
```
fu[x_] = Fit[m, {1, x}, x]
```

```
0. + 0. x
```

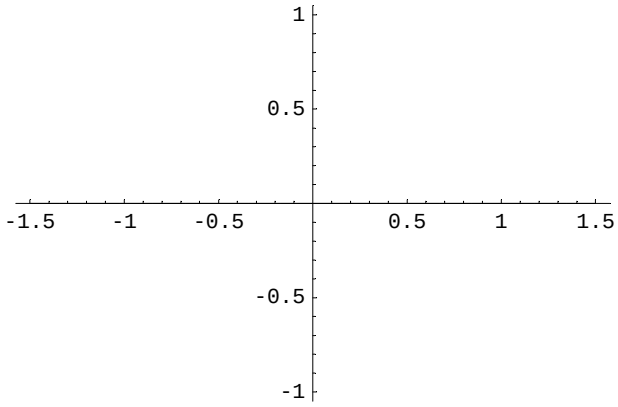
```
fu[x]
```

```
0. + 0. x
```

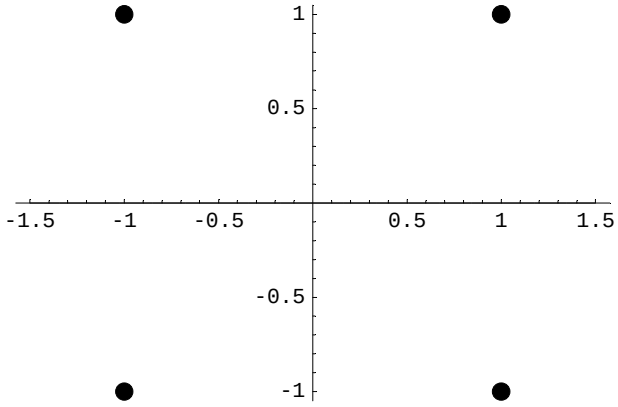
```
fPlot = Plot[fu[x], {x, -1.5, 1.5}, AspectRatio → Automatic];
```




```
fPlot = Plot[Evaluate[fu[x]], {x, -1.5, 1.5}, AspectRatio → Automatic];
```




```
Show[dPlot, fPlot];
```




```
m // MatrixForm
```

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \\ -1 & -1 \\ 1 & -1 \end{pmatrix}$$

```
trp = Transpose[m]; trp // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

```
xListe = trp[[1]]
```

```
{1, -1, -1, 1}
```

```
yListe = trp[[2]]
```

```
{1, 1, -1, -1}
```

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Correlation[xListe, yListe]
```

```
0
```

```
Covariance[xListe, yListe]
```

```
0
```

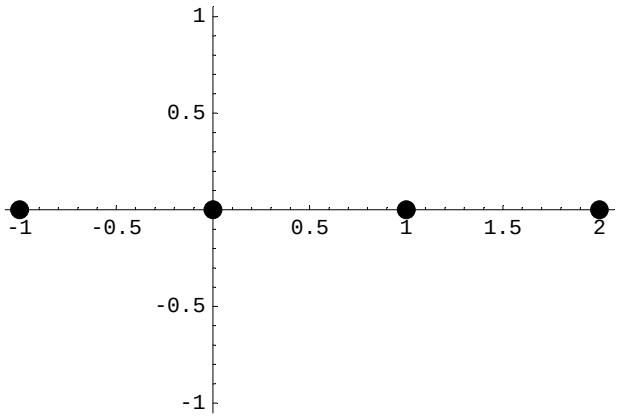
b

```
Remove["Global`*"]
```

```
<< Statistics`LinearRegression`
```

```
m = {{-1, 0}, {0, 0}, {1, 0}, {2, 0}};
```

```
dPlot = ListPlot[m, PlotStyle → {PointSize[0.03]}, AspectRatio → Automatic];
```




```
Remove[func]
```

```
Fit[m, {1, x}, x]
```

```
0. + 0. x
```

```
fu[x_] := Fit[m, {1, x}, x]
```

```
fu[x]
```

```
0. + 0. x
```

```
Fit[m, {1, x}, x]
```

```
0. + 0. x
```

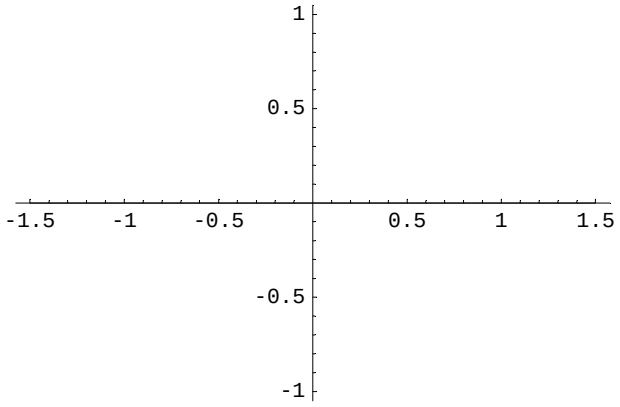
```
fu[x_] = Fit[m, {1, x}, x]
```

```
0. + 0. x
```

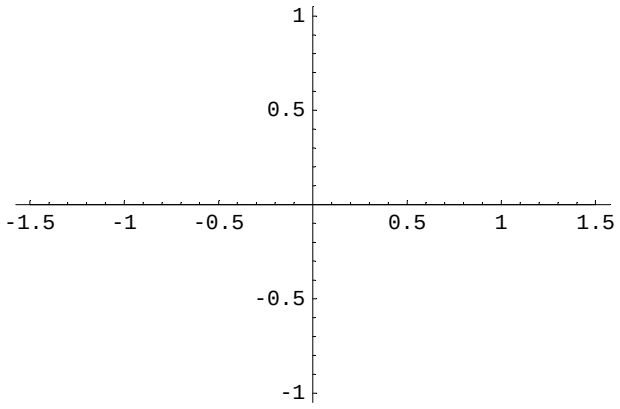
```
fu[x]
```

```
0. + 0. x
```

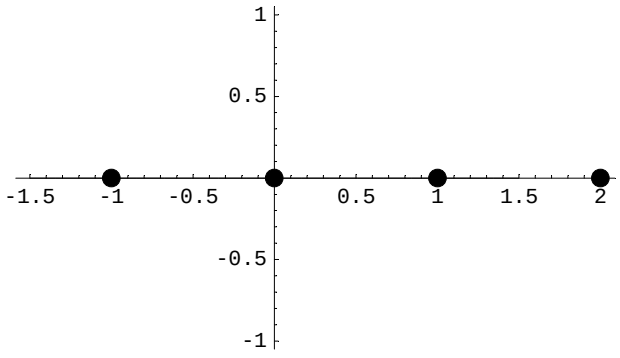
```
fPlot = Plot[fu[x], {x, -1.5, 1.5}, AspectRatio → Automatic];
```




```
fPlot = Plot[Evaluate[fu[x]], {x, -1.5, 1.5}, AspectRatio → Automatic];
```




```
Show[dPlot, fPlot];
```




```
m // MatrixForm
```

$$\begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 2 & 0 \end{pmatrix}$$

```
trp = Transpose[m]; trp // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
xListe = trp[[1]]
```

```
{-1, 0, 1, 2}
```

```
yListe = trp[[2]]
```

```
{0, 0, 0, 0}
```

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Correlation[xListe, yListe]
```

```
Power::infy : Infinite expression 1/0 encountered. Mehr…
```

```
∞::indet : Indeterminate expression 0 ComplexInfinity encountered. Mehr…
```

```
Indeterminate
```

```
Covariance[xListe, yListe]
```

```
0
```

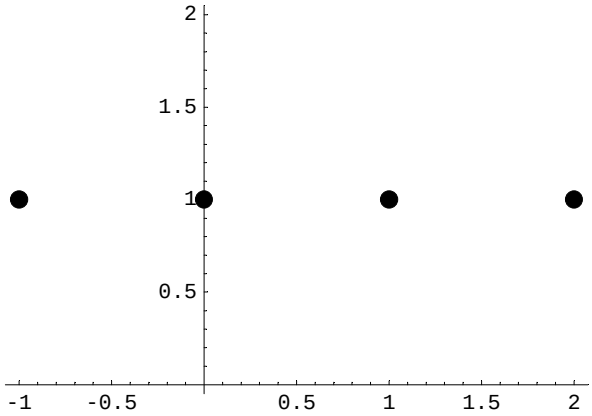
c

```
Remove["Global`*"]
```

```
<< Statistics`LinearRegression`
```

```
m = {{-1, 1}, {0, 1}, {1, 1}, {2, 1}};
```

```
dPlot = ListPlot[m, PlotStyle → {PointSize[0.03]}, AspectRatio → Automatic];
```




```
Remove[func]
```

```
Fit[m, {1, x}, x]
```

```
1. - 1.35974 × 10^-16 x
```

```
fu[x_] := Fit[m, {1, x}, x]
```

```
fu[x]
```

$$1. - 1.35974 \times 10^{-16}\, x$$

```
Fit[m, {1, x}, x]
```

$$1. - 1.35974 \times 10^{-16}\, x$$

```
fu[x_] = Fit[m, {1, x}, x]
```

$$1. - 1.35974 \times 10^{-16}\, x$$

```
fu[x]
```

$$1. - 1.35974 \times 10^{-16}\, x$$

```
fPlot = Plot[fu[x], {x, -1.5, 1.5}, AspectRatio → Automatic];
```

```
fPlot = Plot[Evaluate[fu[x]], {x, -1.5, 1.5}, AspectRatio → Automatic];
```

```
Show[dPlot, fPlot];
```

```
m // MatrixForm
```

$$\begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$$

```
trp = Transpose[m]; trp // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

```
xListe = trp[[1]]
```

$$\{-1, 0, 1, 2\}$$

```
yListe = trp[[2]]
```

$$\{1, 1, 1, 1\}$$

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Correlation[xListe, yListe]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr…

∞::indet : Indeterminate expression 0 ComplexInfinity encountered. Mehr…

Indeterminate

```
Covariance[xListe, yListe]
```

0

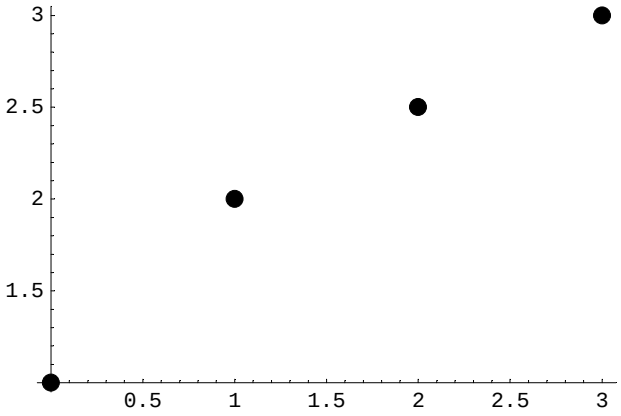
d

```
Remove["Global`*"]
```

```
<< Statistics`LinearRegression`
```

```
m = {{0, 1}, {1, 2}, {2, 2.5}, {3, 3}};
```

```
dPlot = ListPlot[m, PlotStyle → {PointSize[0.03]}, AspectRatio → Automatic];
```




```
Remove[func]
```

```
Fit[m, {1, x}, x]
```

```
1.15 + 0.65 x
```

```
fu[x_] := Fit[m, {1, x}, x]
```

```
fu[x]
```

```
1.15 + 0.65 x
```

```
Fit[m, {1, x}, x]
```

```
1.15 + 0.65 x
```

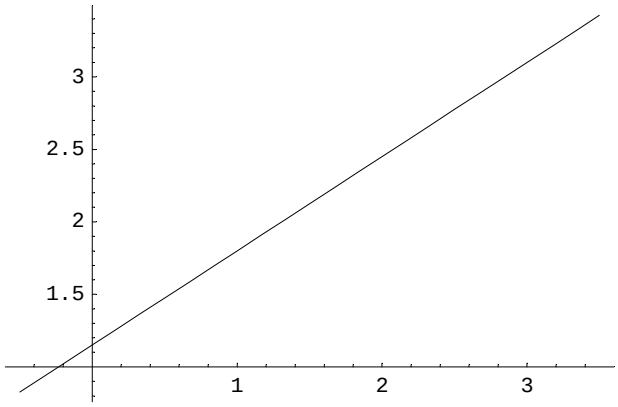
```
fu[x_] = Fit[m, {1, x}, x]
```

```
1.15 + 0.65 x
```

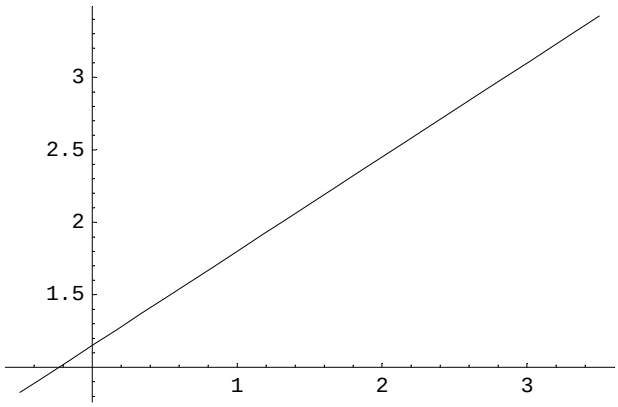
```
fu[x]
```

```
1.15 + 0.65 x
```

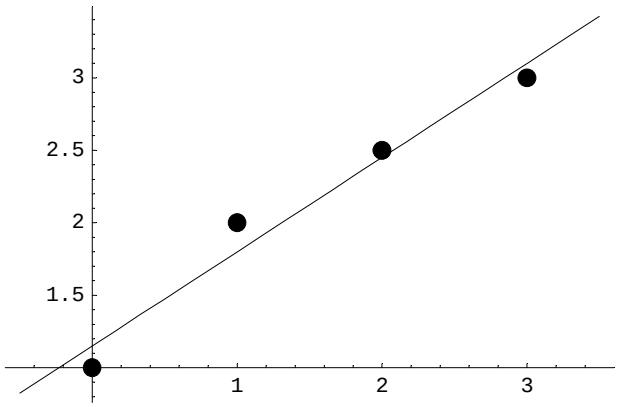
```
fPlot = Plot[fu[x], {x, -0.5, 3.5}, AspectRatio → Automatic];
```




```
fPlot = Plot[Evaluate[fu[x]], {x, -0.5, 3.5}, AspectRatio → Automatic];
```




```
Show[dPlot, fPlot];
```




```
m // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 2.5 \\ 3 & 3 \end{pmatrix}$$

```
trp = Transpose[m]; trp // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 2.5 & 3 \end{pmatrix}$$

```
xListe = trp[[1]]
```

```
{0, 1, 2, 3}
```

```
yListe = trp[[2]]
```

```
{1, 2, 2.5, 3}
```

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Correlation[xListe, yListe]
```

```
0.982708
```

```
Covariance[xListe, yListe]
```

```
1.08333
```

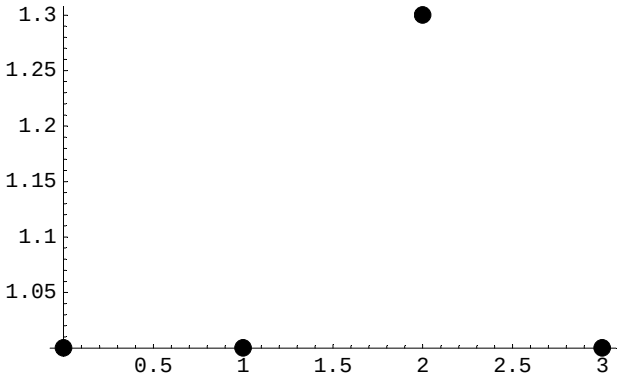
e

```
Remove["Global`*"]
```

```
<< Statistics`LinearRegression`
```

```
m = {{0, 1}, {1, 1}, {2, 1.3}, {3, 1}};
```

```
dPlot = ListPlot[m, PlotStyle → {PointSize[0.03]}];
```




```
Remove[func]
```

```
Fit[m, {1, x}, x]
```

```
1.03 + 0.03 x
```

```
fu[x_] := Fit[m, {1, x}, x]
```

```
fu[x]
```

```
1.03 + 0.03 x
```

```
Fit[m, {1, x}, x]
```

```
1.03 + 0.03 x
```

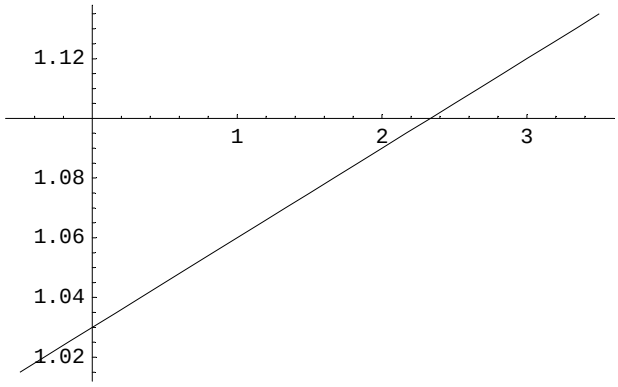
```
fu[x_] = Fit[m, {1, x}, x]
```

```
1.03 + 0.03 x
```

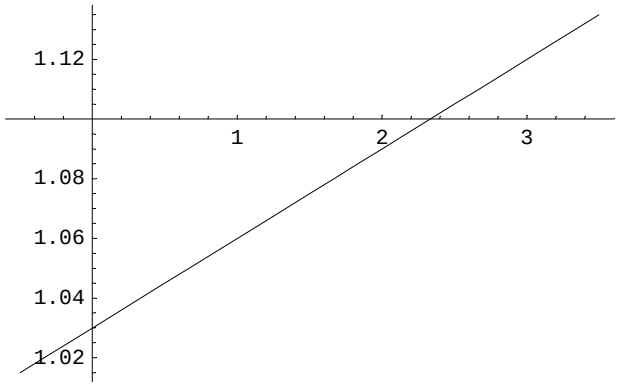
```
fu[x]
```

```
1.03 + 0.03 x
```

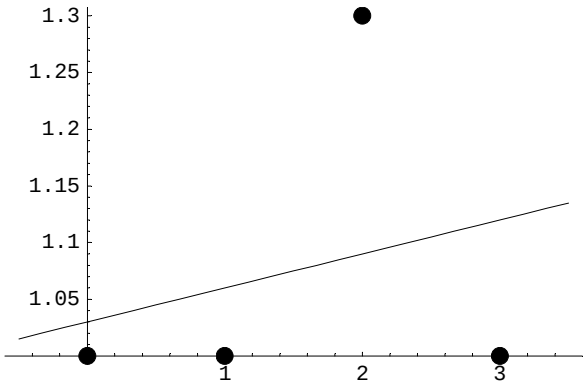
```
fPlot = Plot[fu[x], {x, -0.5, 3.5}];
```




```
fPlot = Plot[Evaluate[fu[x]], {x, -0.5, 3.5}];
```




```
Show[dPlot, fPlot];
```




```
m // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1.3 \\ 3 & 1 \end{pmatrix}$$

```
trp = Transpose[m]; trp // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1.3 & 1 \end{pmatrix}$$

```
xListe = trp[[1]]
```

```
{0, 1, 2, 3}
```

```
yListe = trp[[2]]
```

```
{1, 1, 1.3, 1}
```

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Correlation[xListe, yListe]
```

```
0.258199
```

```
Covariance[xListe, yListe]
```

```
0.05
```