Lösungen / Statistik 2/08

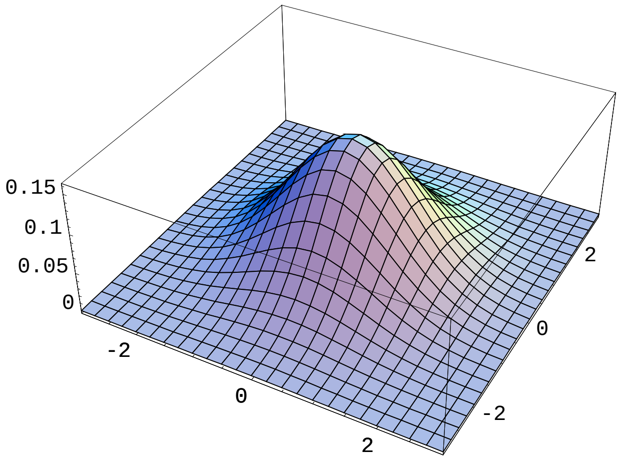
```
Remove["Global`*"]
```

1.

■ a

```
f[x_, y_] := 1 / (2 Pi) E^(-1 / 2 (x^2 + y^2));
```

```
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}];
```




```
μX = 3;  μY = 2;  σX = 3;  σY = 2 / 3;  ϱXY = 1 / 2;
```

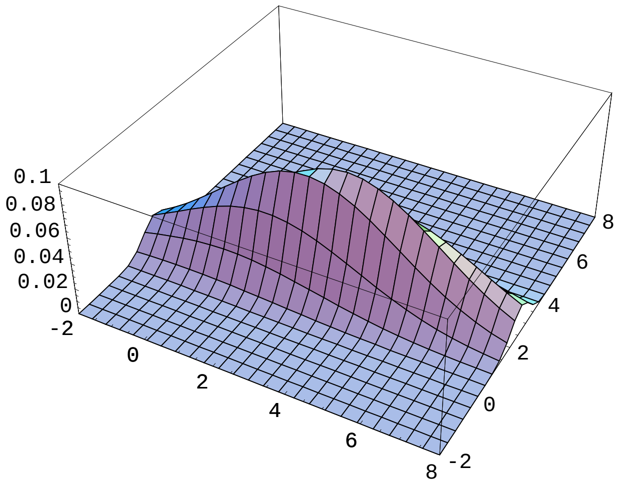
```
f1[x_, y_] := 1 / (2 Pi σX σY Sqrt[1 - ϱXY^2]) E^((-1 / (2 (1 - ϱXY^2))
        (((x - μX)^2) / (σX^2) + ((y - μY)^2) / (σY^2) - 2 ϱXY (x - μX) (y - μY) / (σX σY))));
```

```
? f1
```

Global`f1

f1[x_, y_] := $\dfrac{e^{-\frac{\frac{(x-\mu X)^2}{\sigma X^2}+\frac{(y-\mu Y)^2}{\sigma Y^2}-\frac{2\,\varrho XY\,(x-\mu X)\,(y-\mu Y)}{\sigma X\,\sigma Y}}{2\,(1-\varrho XY^2)}}}{2\,\pi\,\sigma X\,\sigma Y\,\sqrt{1-\varrho XY^2}}$

```
Plot3D[f1[x, y], {x, -2, 8}, {y, -2, 8}, PlotRange → {0, 0.1}];
```



■ b

```
Integrate[Integrate[f[x, y], {x, -a, a}], {y, -a, a}]
```

$\text{Erf}\left[\frac{a}{\sqrt{2}}\right]^2$

```
Integrate[Integrate[f[x, y], {x, -Infinity, Infinity}], {y, -Infinity, Infinity}]
```

1

```
Integrate[Integrate[f[x, y], {x, -a, a}], {y, -a, a}] /. {a → Infinity}
```

1

```
Integrate[Integrate[f[x, y], {x, -Infinity, Infinity}], {y, -Infinity, Infinity}]
```

1

```
Integrate[Integrate[f1[x, y], {x, -Infinity, Infinity}], {y, -Infinity, Infinity}]
```

1

```
Integrate[f1[x, y], {x, -Infinity, Infinity}]
```

$\frac{3\, e^{-\frac{9}{8}(-2+y)^2}}{2\sqrt{2\pi}}$

```
Integrate[f1[x, y], {y, -Infinity, Infinity}]
```

$\frac{e^{-\frac{1}{18}(-3+x)^2}}{3\sqrt{2\pi}}$

■ c

```
F[x_, y_] := Integrate[Integrate[f[u, v], {u, -Infinity, x}], {v, -Infinity, y}]
```

```
F[0, 0]
```

$\frac{1}{4}$

```
F[0, Infinity]
```

$\frac{1}{2}$

```
F[Infinity, 0]
```

$\frac{1}{2}$

```
F[Infinity, Infinity]
```

1

```
F1[x_, y_] := Integrate[Integrate[f1[u, v], {u, -Infinity, x}], {v, -Infinity, y}]

NF1[x_, y_] := NIntegrate[Integrate[f1[u, v], {u, -Infinity, x}], {v, -Infinity, y}]

NF1[3, 2]
```

0.333333

2.

▪ a

Xbar=X1+X2;
μ=μ1=μ2;
σ^2=σ1^2 / 2 = σ2^2 / 2;

▪ b

```
Remove[f2, F2]

f2[x_, σ_, μ_] := 1 / (Sqrt[2 Pi] σ) E^(-1 / (2) ((x - μ)^2 / σ^2));

F2[x_, σ_, μ_] := Integrate[f2[u, σ, μ], {u, -Infinity, x}]

F2[Infinity, σ, μ] // Simplify
```

$$\frac{1}{\sqrt{2\pi}\,\sigma}\text{If}\left[\text{Re}[\sigma^2] > 0,\ \sqrt{2\pi}\,\sqrt{\sigma^2},\ \text{Integrate}\left[e^{-\frac{(u-\mu)^2}{2\sigma^2}},\ \{u, -\infty, \infty\},\ \text{Assumptions} \to \text{Re}[\sigma^2] \le 0\right]\right]$$

```
F2[Infinity, 4, 5]
```

1

```
F2[μ]
```

F2[μ]

```
f3[x_, σ_, μ_] := 1 / (Sqrt[2 Pi] c1 σ) E^ (-1 / (2) ((c1 x + c2 - (c1 μ + c2)) ^2 / (c1 σ) ^2));

F3[x_, σ_, μ_] := Integrate[f3[u, σ, μ] * Evaluate[D[c1 u + c2, u]], {u, -Infinity, x}]

F3[Infinity, 4, 5]
```

1

3.

a

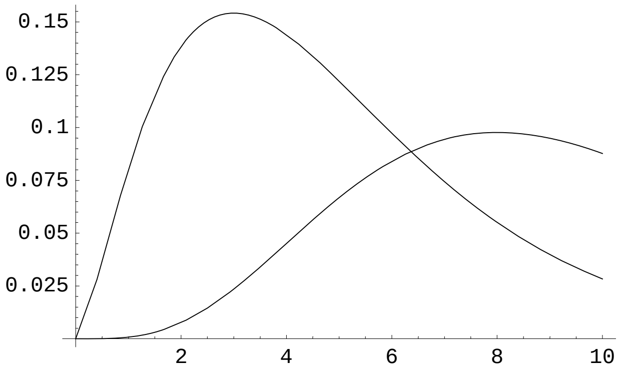
```
k[n_] := 1 / (2^ (n / 2) Gamma[n / 2])

Table[k[n], {n, 0, 10}]
```

$$\left\{0, \frac{1}{\sqrt{2\pi}}, \frac{1}{2}, \frac{1}{\sqrt{2\pi}}, \frac{1}{4}, \frac{1}{3\sqrt{2\pi}}, \frac{1}{16}, \frac{1}{15\sqrt{2\pi}}, \frac{1}{96}, \frac{1}{105\sqrt{2\pi}}, \frac{1}{768}\right\}$$

```
f4[x_, n_] := k[n] x^ ((n - 2) / 2) E^ (-x / 2)

Plot[{f4[x, 10], f4[x, 5]}, {x, 0, 10}];
```




```
F4[x_, n_] := Integrate[f4[u, n], {u, 0, x}]

F4[2, 5]
```

$$-\frac{10}{3 e \sqrt{\pi}} + \text{Erf}[1]$$

```
F4[2, 5] // N
```

0.150855

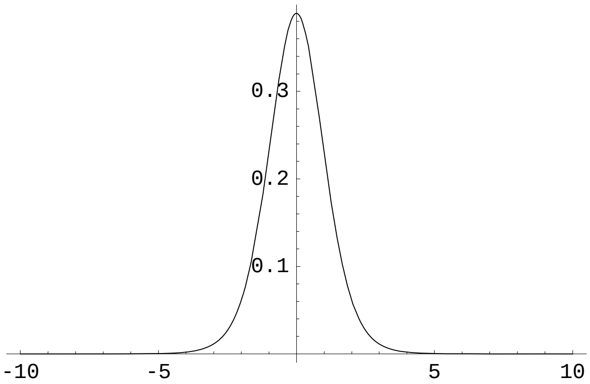
b

```
Remove[f5, f6, F5, F6]

f5[z_, n_] := Gamma[(n + 1) / 2] / Sqrt[n Pi] / Gamma[n / 2] / (1 + z^2 / n) ^ ((n + 1) / 2)

F5[z_, n_] := Integrate[f4[u, n], {u, -Infinity, z}]
```

```
Plot[{f5[x, 10]}, {x, -10, 10}];
```




```
F6[z_, a_, n_] := Integrate[f5[u, n], {u, a, z}]
```

```
?F6
```

Global`F6

$F6[z\_, a\_, n\_] := \int_a^z f5[u, n]\, du$

```
F6[z, a, n]
```

$$\Bigg((-a+z)\,\text{Gamma}\Big[\frac{1+n}{2}\Big]\ \text{If}\Big[\Big(\text{Im}\Big[\frac{\sqrt{n}}{a-z}\Big]+\text{Re}\Big[\frac{a}{a-z}\Big]\ge 1 \;||\; \text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]+\text{Re}\Big[\frac{a}{-a+z}\Big]\ge 0 \;||\; \text{Im}\Big[\frac{a-i\sqrt{n}}{a-z}\Big]\neq 0\Big)\ \&\&\ \Big(\text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]+\text{Re}\Big[\frac{a}{a-z}\Big]\ge 1 \;||\; \text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]==\text{Re}\Big[\frac{a}{-a+z}\Big] \;||\; \text{Im}\Big[\frac{\sqrt{n}}{a-z}\Big]\ge \text{Re}\Big[\frac{a}{a-z}\Big] \;||\; \text{Im}\Big[\frac{a+i\sqrt{n}}{a-z}\Big]\neq 0\Big),\ \frac{1}{a-z}\Big(a\,\text{Hypergeometric2F1}\Big[\frac{1}{2},\frac{1+n}{2},\frac{3}{2},-\frac{a^2}{n}\Big]-z\,\text{Hypergeometric2F1}\Big[\frac{1}{2},\frac{1+n}{2},\frac{3}{2},-\frac{z^2}{n}\Big]\Big),\ \text{Integrate}\Big[\Big(\frac{n+(a+u\,(-a+z))^2}{n}\Big)^{\frac{1}{2}(-1-n)},\{u,0,1\},\text{Assumptions}\to\ !\Big(\Big(\text{Im}\Big[\frac{\sqrt{n}}{a-z}\Big]+\text{Re}\Big[\frac{a}{a-z}\Big]\ge 1 \;||\; \text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]+\text{Re}\Big[\frac{a}{-a+z}\Big]\ge 0 \;||\; \text{Im}\Big[\frac{a-i\sqrt{n}}{a-z}\Big]\neq 0\Big)\ \&\&\ \Big(\text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]+\text{Re}\Big[\frac{a}{a-z}\Big]\ge 1 \;||\; \text{Im}\Big[\frac{\sqrt{n}}{-a+z}\Big]==\text{Re}\Big[\frac{a}{-a+z}\Big] \;||\; \text{Im}\Big[\frac{\sqrt{n}}{a-z}\Big]\ge\text{Re}\Big[\frac{a}{a-z}\Big] \;||\; \text{Im}\Big[\frac{a+i\sqrt{n}}{a-z}\Big]\neq 0\Big)\Big)\Big]\Big]\Bigg)\Big/\Big(\sqrt{n}\,\sqrt{\pi}\,\text{Gamma}\Big[\frac{n}{2}\Big]\Big)$$

```
F6[4, a, 6]
```

$$\frac{(4-a)\left(-1127\sqrt{22}+\frac{2662\,a\,(135+30\,a^2+2\,a^4)}{(6+a^2)^{5/2}}\right)}{10648\,(-4+a)}$$

```
Limit[Evaluate[F6[4, a, 6]], a → -Infinity]
```

$$\frac{1}{2}+\frac{1127}{484\sqrt{22}}$$

```
N[%]
```

0.996441

```
Limit[Evaluate[F6[1, a, 6]], a → -Infinity] // N
```

0.822041

```
Limit[Evaluate[F6[0, a, 6]], a → -Infinity] // N
```

0.5

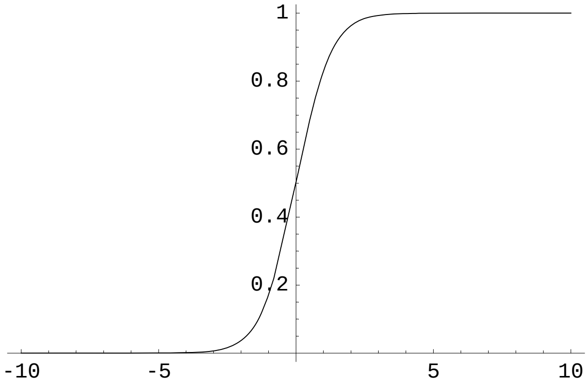
```
Limit[Evaluate[F6[-4, a, 6]], a → -Infinity] // N
```

0.00355949

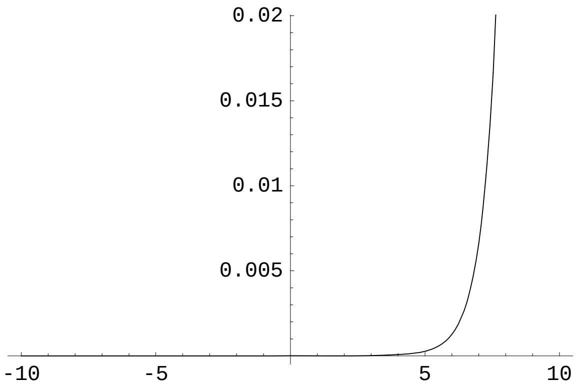
```
Limit[Evaluate[F6[20, a, 6]], a → -Infinity] // N
```

0.999999

```
Plot[{F6[x, -1000, 10]}, {x, -10, 10}];
```




```
Plot[{F6[x, -10^10, 10]}, {x, -10, 10}];
```



- **C**

```
F6[2, a, 5]
```

$$\left((2-a)\left(-370+\frac{405\,a\,(25+3\,a^2)}{(5+a^2)^2}-243\sqrt{5}\,\mathrm{ArcTan}\left[\frac{2}{\sqrt{5}}\right]+243\sqrt{5}\,\mathrm{ArcTan}\left[\frac{a}{\sqrt{5}}\right]\right)\right)\Big/\left(243\sqrt{5}\,(-2+a)\,\pi\right)$$

```
Limit[F6[2, a, 5], a → -Infinity]
```

$$\frac{1}{2}+\frac{\frac{74\sqrt{5}}{243}+\mathrm{ArcTan}\left[\frac{2}{\sqrt{5}}\right]}{\pi}$$

```
N[%]
```

0.94903

4

```
Remove["Global`*"]
```

```
f[z_, n_] := Gamma[(n + 1) / 2] / (Sqrt[n Pi] Gamma[n / 2]) * 1 / (1 + z^2 / n) ^ ((n + 1) / 2);
f[z, n]
```

$$\frac{\left(1+\frac{z^2}{n}\right)^{\frac{1}{2}(-1-n)}\,\mathrm{Gamma}\left[\frac{1+n}{2}\right]}{\sqrt{n}\,\sqrt{\pi}\,\mathrm{Gamma}\left[\frac{n}{2}\right]}$$

```
F[z_, a_, n_] := Gamma[(n + 1) / 2] / (Sqrt[n Pi] Gamma[n / 2])
  Integrate[1 / (1 + u^2 / n) ^ (n + 1), {u, a, z}]; F[z, a, n]
```

$$\Bigg((-a+z)\,\mathrm{Gamma}\left[\frac{1+n}{2}\right]$$

$$\mathrm{If}\Big[\Big(\mathrm{Im}\left[\frac{\sqrt{n}}{a-z}\right]+\mathrm{Re}\left[\frac{a}{a-z}\right]\geq 1\ ||\ \mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]+\mathrm{Re}\left[\frac{a}{-a+z}\right]\geq 0\ ||\ \mathrm{Im}\left[\frac{a-i\sqrt{n}}{a-z}\right]\neq 0\Big)\ \&\&$$

$$\Big(\mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]+\mathrm{Re}\left[\frac{a}{a-z}\right]\geq 1\ ||\ \mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]==\mathrm{Re}\left[\frac{a}{-a+z}\right]\ ||$$

$$\mathrm{Im}\left[\frac{\sqrt{n}}{a-z}\right]\geq\mathrm{Re}\left[\frac{a}{a-z}\right]\ ||\ \mathrm{Im}\left[\frac{a+i\sqrt{n}}{a-z}\right]\neq 0\Big),\ \frac{1}{a-z}$$

$$\left(a\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},1+n,\frac{3}{2},-\frac{a^2}{n}\right]-z\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},1+n,\frac{3}{2},-\frac{z^2}{n}\right]\right),$$

$$\mathrm{Integrate}\left[\left(\frac{n+(a+u(-a+z))^2}{n}\right)^{-1-n},\ \{u,0,1\},\ \mathrm{Assumptions}\to\right.$$

$$!\ \Big(\Big(\mathrm{Im}\left[\frac{\sqrt{n}}{a-z}\right]+\mathrm{Re}\left[\frac{a}{a-z}\right]\geq 1\ ||\ \mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]+\mathrm{Re}\left[\frac{a}{-a+z}\right]\geq 0\ ||\ \mathrm{Im}\left[\frac{a-i\sqrt{n}}{a-z}\right]\neq 0\Big)\ \&\&$$

$$\Big(\mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]+\mathrm{Re}\left[\frac{a}{a-z}\right]\geq 1\ ||\ \mathrm{Im}\left[\frac{\sqrt{n}}{-a+z}\right]==\mathrm{Re}\left[\frac{a}{-a+z}\right]\ ||$$

$$\left.\mathrm{Im}\left[\frac{\sqrt{n}}{a-z}\right]\geq\mathrm{Re}\left[\frac{a}{a-z}\right]\ ||\ \mathrm{Im}\left[\frac{a+i\sqrt{n}}{a-z}\right]\neq 0\Big)\Big)\Big]\Big]\Bigg)\Big/\left(\sqrt{n}\,\sqrt{\pi}\,\mathrm{Gamma}\left[\frac{n}{2}\right]\right)$$

```
F[z, -Infinity, n]
```

$$\Bigg(\mathrm{Gamma}\left[\frac{1+n}{2}\right]$$

$$\mathrm{If}\left[\mathrm{Re}[n]>-\frac{1}{2},\ \frac{\sqrt{\frac{1}{n}}\,\sqrt{\pi}\,\mathrm{Gamma}\left[\frac{1}{2}+n\right]}{2\,\mathrm{Gamma}[n]}+z\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},1+n,\frac{3}{2},-\frac{z^2}{n}\right],\right.$$

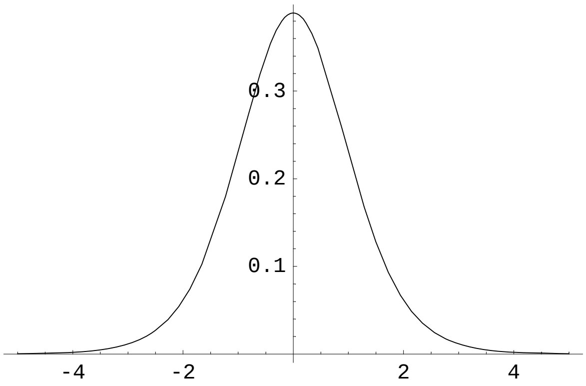
$$\left.\mathrm{Integrate}\left[\left(\frac{n+u^2}{n}\right)^{-1-n},\ \{u,-\infty,z\},\ \mathrm{Assumptions}\to\mathrm{Re}[n]\leq-\frac{1}{2}\right]\right]\Bigg)\Big/\left(\sqrt{n}\,\sqrt{\pi}\,\mathrm{Gamma}\left[\frac{n}{2}\right]\right)$$

▪ a

```
f[z, 10]
```

$$\frac{63\sqrt{\frac{5}{2}}}{256\left(1+\frac{z^2}{10}\right)^{11/2}}$$

```
Plot[f[z, 10], {z, -5, 5}];
```

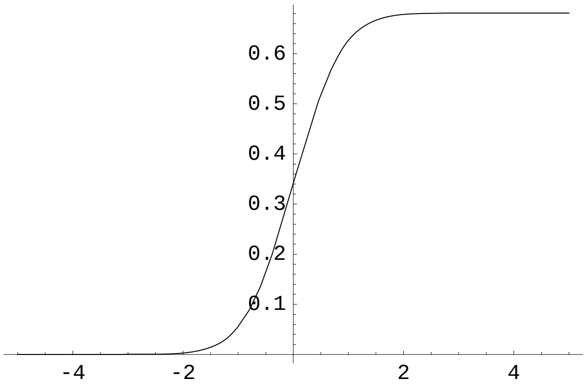


▪ b

```
F[z, -Infinity, 10]
```

$$\left(\sqrt{\frac{5}{2}}\left(2721033000000000000\,z+2909907\sqrt{10}\,\pi\,(10+z^2)^{10}+380\,z^3\,(402454500000000+17\,z^2\,(7238280000000+z^2\,(1396620000000+13\,z^2\,(13901000000+11\,z^2\,(111940000+6732000\,z^2+263760\,z^4+6090\,z^6+63\,z^8)))))+5819814\sqrt{10}\,(10+z^2)^{10}\,\mathrm{ArcTan}\left[\frac{z}{\sqrt{10}}\right]\right)\right)\Big/\left(134217728\,(10+z^2)^{10}\right)$$

```
Plot[F[z, -Infinity, 10], {z, -5, 5}];
```



5

```
Remove["Global`*"]
```

■ a

```
<< Statistics`DescriptiveStatistics`

data = {4.6, 4.5, 4.3, 4.7, 4.5, 4.6, 4.7, 4.5, 4.8}

{4.6, 4.5, 4.3, 4.7, 4.5, 4.6, 4.7, 4.5, 4.8}

Length[data]

9

locRep = LocationReport[data]

{Mean → 4.57778, HarmonicMean → 4.57347, Median → 4.6}

dispRep = DispersionReport[data]

{Variance → 0.0219444, StandardDeviation → 0.148137, SampleRange → 0.5,
 MeanDeviation → 0.11358, MedianDeviation → 0.1, QuartileDeviation → 0.1}
```

■ b

Der Mittelwert der Grundgesamtheit müsste also mit μ = 4.5777... und die Standardabweichung der Grundgesamtheit mit σ = 0.14813657... eingesetzt werden.

```
μ = Mean /. locRep

4.57778

σ = StandardDeviation /. dispRep

0.148137
```

■ c

```
f[X_] := Sin[X - X^2] / (1 - X^2) - 1 / X;

f[X]
```

$$-\frac{1}{X} + \frac{\text{Sin}[X - X^2]}{1 - X^2}$$

```
f[μ]

-0.249576

D[f[X], X]
```

$$\frac{1}{X^2} + \frac{(1 - 2X)\,\text{Cos}[X - X^2]}{1 - X^2} + \frac{2X\,\text{Sin}[X - X^2]}{(1 - X^2)^2}$$

```
Abs[D[f[X], X]] Abs[ΔX] /. {X → μ, ΔX → σ}

0.0382563
```

μY +/- σY = -0.24957639255372532` +/- 0.03825625460156701`