Lösungen Laplace-Transformationen

```
?*Laplace*
```

System`

```
InverseLaplaceTransform LaplaceTransform
```

```
?Laplace*
```

```
LaplaceTransform[expr, t, s] gives the Laplace
   transform of expr. LaplaceTransform[expr, {t1, t2, ... }, {s1,
   s2, ... }] gives the multidimensional Laplace transform of expr. Mehr…
```

```
?InverseLaplace*
```

```
InverseLaplaceTransform[expr, s, t] gives the inverse Laplace transform
   of expr. InverseLaplaceTransform[expr, {s1, s2, ... }, {t1, t2, ... }]
   gives the multidimensional inverse Laplace transform of expr. Mehr…
```

1

a

```
transf = LaplaceTransform[(y''[t]+ ω
y[t]==Sin[t]),t,s]/.{LaplaceTransform[y[t],t,s]->Y[s]}
```

$$-s\,y[0] + s^2\,Y[s] + \omega\,Y[s] - y'[0] == \frac{1}{1+s^2}$$

b

```
solv = Solve[transf,{Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{1 + s\,y[0] + s^3\,y[0] + y'[0] + s^2\,y'[0]}{(1+s^2)\,(s^2+\omega)}\right\}$$

```
Y1[s_]:=Y[s] /. solv
```

```
Y1[s]
```

$$\frac{1 + s\,y[0] + s^3\,y[0] + y'[0] + s^2\,y'[0]}{(1+s^2)\,(s^2+\omega)}$$

```
Y2[s_]:=Y1[s] /. {y[0]->1,y'[0]->1}
```

```
Y2[s]
```

$$\frac{2 + s + s^2 + s^3}{(1+s^2)\,(s^2+\omega)}$$

c

```
InverseLaplaceTransform[Y2[s],s,t]
```

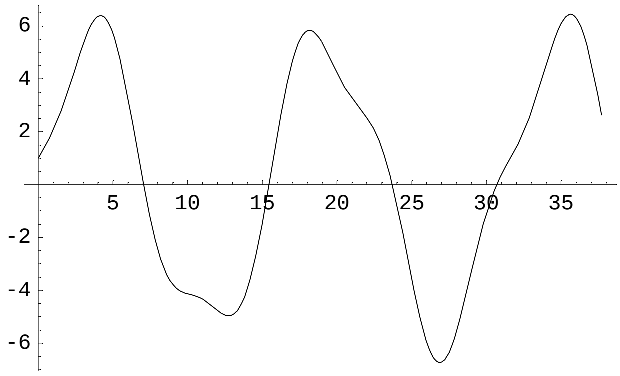
$$\frac{(-1+\omega)\sqrt{\omega}\,\mathrm{Cos}[t\sqrt{\omega}]+\sqrt{\omega}\,\mathrm{Sin}[t]+(-2+\omega)\,\mathrm{Sin}[t\sqrt{\omega}]}{(-1+\omega)\sqrt{\omega}}$$

d

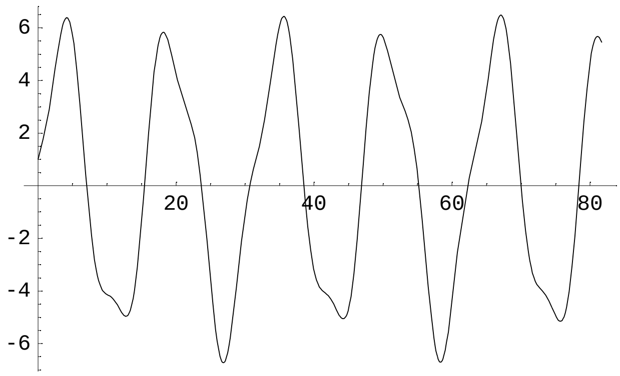
```
(InverseLaplaceTransform[Y2[s],s,t] /. ω-> 1/(2 π))//Simplify
```

$$\frac{1}{\sqrt{2}\,(-1+2\pi)}\left(\sqrt{2}\,(-1+2\pi)\,\mathrm{Cos}\left[\frac{t}{\sqrt{2\pi}}\right]+2\sqrt{\pi}\left(-\sqrt{2\pi}\,\mathrm{Sin}[t]+(-1+4\pi)\,\mathrm{Sin}\left[\frac{t}{\sqrt{2\pi}}\right]\right)\right)$$

```
Plot[%,{t,0,12 π}];
```




```
Plot[%%,{t,0,26 π}];
```



d

Ueberlagerung von periodischen Funktionen.

2

a

```
E^(-s Pi/2) LaplaceTransform[Cos[t],t,s]
```

$$\frac{e^{-\frac{\pi s}{2}}\, s}{1+s^2}$$

b

```
LaplaceTransform[E^(t Pi/2) Cos[t],t,s]
```

$$\frac{-\frac{\pi}{2}+s}{1+\left(-\frac{\pi}{2}+s\right)^2}$$

c

```
LaplaceTransform[t^3 Cos[t],t,s]
```

$$\frac{6\,(1-6\,s^2+s^4)}{(1+s^2)^4}$$

d

```
LaplaceTransform[Sin[2t]/t,t,s]
```

$$\text{ArcTan}\left[\frac{2}{s}\right]$$

e

```
LaplaceTransform[(Cos[t-1])/t,t,s]
```

$$-\frac{1}{2}\,\text{Cos}[1]\,(2\,\text{EulerGamma}+\text{Log}[1+s^2])+\text{ArcCot}[s]\,\text{Sin}[1]$$

So weit gehen die bei uns behandelten Regeln nicht... ==> Handarbeit!

e1

```
LaplaceTransform[(Cos[t]-1)/t,t,s]
```

$$-\infty$$

f

```
LaplaceTransform[Evaluate[Integrate[Sin[λ] Cos[t-λ],{λ,0,t}]],t,s] // Simplify
```

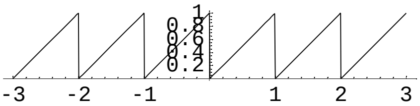
$$\frac{s}{(1+s^2)^2}$$

```
LaplaceTransform[Sin[t],t,s]*LaplaceTransform[Cos[t],t,s] // Simplify
```

$$\frac{s}{(1+s^2)^2}$$

g

```
f[t_]:=t-Floor[t]; Plot[f[t],{t,-3,3},AspectRatio->Automatic];
```




```
LaplaceTransform[f[t],t,s]
```

$$\frac{1}{s^2} - \text{LaplaceTransform}[\text{Floor}[t], t, s]$$

```
ρ[s_] := E^(-s*1);
1/(1-ρ[s]) Integrate[E^(-s τ) * τ, {τ, 0, 1}]
```

$$\frac{1-e^{-s}(1+s)}{(1-e^{-s})\, s^2}$$