Lösungen Laplace-Rückransformationen

```
?*Laplace*
```

System`

```
InverseLaplaceTransform LaplaceTransform
```

```
?Laplace*
```

```
LaplaceTransform[expr, t, s] gives the Laplace
   transform of expr. LaplaceTransform[expr, {t1, t2, ... }, {s1,
   s2, ... }] gives the multidimensional Laplace transform of expr. Mehr…
```

```
?InverseLaplace*
```

```
InverseLaplaceTransform[expr, s, t] gives the inverse Laplace transform
   of expr. InverseLaplaceTransform[expr, {s1, s2, ... }, {t1, t2, ... }]
   gives the multidimensional inverse Laplace transform of expr. Mehr…
```

```
?Limit
```

```
Limit[expr, x->x0] finds the limiting value of expr when x approaches x0. Mehr…
```

```
?Direction
```

```
Direction is an option for Limit. Limit[expr, x -> x0, Direction -> 1] computes the limit as
  x approaches x0 from smaller values. Limit[expr, x -> x0, Direction -> -1] computes
  the limit as x approaches x0 from larger values. Direction -> Automatic uses
  Direction -> -1 except for limits at Infinity, where it is equivalent to Direction -> 1.
```

```
Remove["Global`*"]
```

1

a

```
f[t_]:= t Sin[t]
```

```
transf= LaplaceTransform[f[t],t,s]
```

$$\frac{2\,s}{(1+s^2)^2}$$

```
Limit[f[t],t->Infinity]
```

```
Interval[{-∞, ∞}]
```

```
Limit[s transf,s->0,Direction→-1]
```

```
0
```

!! Ungleiches Resultat - Wieso? !!

```
Limit[f[t],t->0,Direction→-1]
```

0

```
Limit[s transf,s->Infinity]
```

0

b

```
f[t_]:= 1/t Sin[t]
```

```
transf= LaplaceTransform[f[t],t,s]
```

$\text{ArcTan}\left[\frac{1}{s}\right]$

```
Limit[f[t],t->Infinity]
```

0

```
Limit[s transf,s->0,Direction→-1]
```

0

```
Limit[f[t],t->0,Direction→-1]
```

1

```
Limit[s transf,s->Infinity]
```

1

c

```
f[t_]:= Integrate[1/τ Sin[τ],{τ,0,t}]
```

```
transf= LaplaceTransform[Evaluate[f[t]],t,s]
```

$\frac{\text{ArcTan}\left[\frac{1}{s}\right]}{s}$

```
Limit[f[t],t->Infinity]
```

$\frac{\pi}{2}$

```
Limit[s transf,s->0,Direction→-1]
```

$\frac{\pi}{2}$

```
Limit[f[t],t->0,Direction→-1]
```

0

```
Limit[s transf,s->Infinity]
```

0

2

a

```
InverseLaplaceTransform[(6 s + 2)/(2 s^2+4 s+8),s,t]
```

$$\frac{1}{3} e^{-t} \left(9 \operatorname{Cos}\left[\sqrt{3}\, t\right] - 2 \sqrt{3} \operatorname{Sin}\left[\sqrt{3}\, t\right]\right)$$

b

```
InverseLaplaceTransform[2/(2 s^2+4 s+8),s,t]
```

$$\frac{e^{-t} \operatorname{Sin}\left[\sqrt{3}\, t\right]}{\sqrt{3}}$$

c

```
InverseLaplaceTransform[(6 s - 2)/(2 s^2-4 s-8),s,t]
```

$$\frac{1}{10} e^{t-\sqrt{5}\, t} \left(15 - 2 \sqrt{5} + \left(15 + 2 \sqrt{5}\right) e^{2 \sqrt{5}\, t}\right)$$

d

```
InverseLaplaceTransform[(6 s - 2)/(2 s^2+4 s+8)^2,s,t]
```

$$\frac{1}{36} e^{-t} \left(12\, t \operatorname{Cos}\left[\sqrt{3}\, t\right] + \sqrt{3}\, (-4 + 9\, t) \operatorname{Sin}\left[\sqrt{3}\, t\right]\right)$$

3

a

```
InverseLaplaceTransform[(3 s + 4)/(s^2+2 s-3),s,t]
```

$$\frac{5\, e^{-3 t}}{4} + \frac{7\, e^{t}}{4}$$

b

```
InverseLaplaceTransform[2/(s+2)-2s/(s^2+16)+3/(s^2+15),s,t]
```

$$2\, e^{-2\,t} - 2\,\mathrm{Cos}[4\,t] + \sqrt{\frac{3}{5}}\ \mathrm{Sin}\left[\sqrt{15}\ t\right]$$

c

```
InverseLaplaceTransform[1/(s^2+4 s+3),s,t]
```

$$\frac{1}{2}\, e^{-3\,t}\,(-1 + e^{2\,t})$$

d

```
InverseLaplaceTransform[e^(- Pi/4 s) s/(s^2-4),s,t]
```

$$\mathrm{InverseLaplaceTransform}\left[\frac{e^{-\frac{\pi s}{4}}\ s}{-4 + s^2},\ s,\ t\right]$$

(Handrechnung, Verschiebung der Originalfunktion um, f(t)=0 für t< /4 , sonst um /4 verschobene Rücktransformierte von s/(s^2-4)). Diese ist von der Verschiebung:

```
InverseLaplaceTransform[s/(s^2-4),s,t]
```

$$\frac{1}{2}\, e^{-2\,t}\,(1 + e^{4\,t})$$

Verschiebung:

```
1/2 e^(-2 t) (1 + e^(4 t)) /. t → (t - Pi / 4)
```

$$\frac{1}{2}\, e^{-2\,\left(-\frac{\pi}{4}+t\right)}\,\left(1 + e^{4\,\left(-\frac{\pi}{4}+t\right)}\right)$$

e

```
InverseLaplaceTransform[(2s-6)/(s^2+4s-20),s,t]
```

$$\frac{1}{12}\, e^{-2\,\left(1+\sqrt{6}\right)\,t}\,\left(12 + 5\,\sqrt{6} + \left(12 - 5\,\sqrt{6}\right)\, e^{4\,\sqrt{6}\,t}\right)$$

f

```
InverseLaplaceTransform[1/(s (s^2-ω^2)),s,t]
```

$$\frac{e^{-t\,\omega}\,\left(-1 + e^{t\,\omega}\right)^2}{2\,\omega^2}$$

g

```
InverseLaplaceTransform[1/(s^2-2s),s,t]
```

$$\frac{1}{2}\left(-1+e^{2\,t}\right)$$