Uebungen

D'Gl. 2. Ordnung

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

$-2 - s + Y[s] + s\,Y[s] + s^2\,Y[s]$

```
rechts = LaplaceTransform[Sin[t], t, s]
```

$\frac{1}{1+s^2}$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$\left\{Y[s] \to \frac{3 + s + 2\,s^2 + s^3}{(1+s^2)\,(1+s+s^2)}\right\}$

```
U[s] := Y[s] /. solv;  U[s]
```

$\frac{3 + s + 2\,s^2 + s^3}{(1+s^2)\,(1+s+s^2)}$

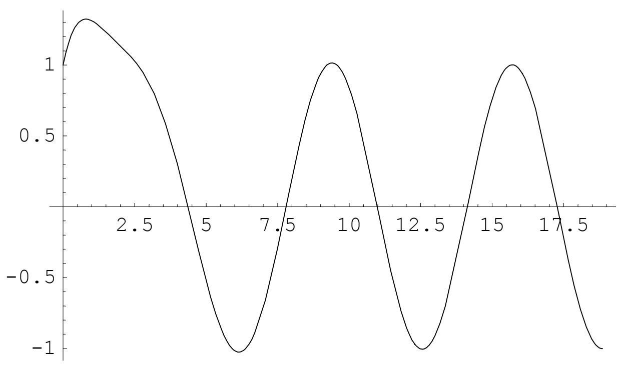
```
U[s] // Apart
```

$-\frac{s}{1+s^2} + \frac{3+2\,s}{1+s+s^2}$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$-\mathrm{Cos}[t] + \frac{2}{3}\,e^{-t/2}\left(3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + 2\,\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



System

```
Remove["Global`*"]
```

```
links1 = LaplaceTransform[y''[t] + z[t], t, s] /. {LaplaceTransform[y[t], t, s] → Y[s],
   y[0] → 1, y'[0] → 0, LaplaceTransform[z[t], t, s] → Z[s], z[0] → 1, z'[0] → 0}
```

$-s + s^2\, Y[s] + Z[s]$

```
links2 = LaplaceTransform[y[t] - z'[t], t, s] /. {LaplaceTransform[y[t], t, s] → Y[s],
   y[0] → 1, y'[0] → 0, LaplaceTransform[z[t], t, s] → Z[s], z[0] → 1, z'[0] → 0}
```

$1 + Y[s] - s\, Z[s]$

```
rechts1 = LaplaceTransform[Sin[t], t, s]
```

$\frac{1}{1+s^2}$

```
rechts2 = LaplaceTransform[Cos[t], t, s]
```

$\frac{s}{1+s^2}$

```
solv = Solve[{links1 == rechts1, links2 == rechts2}, {Y[s], Z[s]}] // Flatten
```

$\left\{Y[s] \to -\frac{1-2s-s^4}{(1+s^2)\,(1+s^3)},\ Z[s] \to -\frac{-1-s-s^2-s^4}{(1+s^2)\,(1+s^3)}\right\}$

```
U[s] := Y[s] /. solv[[1]]; U[s]
```

$-\frac{1-2s-s^4}{(1+s^2)\,(1+s^3)}$

```
U[s] // Apart
```

$-\frac{1}{3\,(1+s)} + \frac{-1+s}{1+s^2} + \frac{1+s}{3\,(1-s+s^2)}$

```
V[s] := Z[s] /. solv[[2]]; V[s]
```

$-\frac{-1-s-s^2-s^4}{(1+s^2)\,(1+s^3)}$

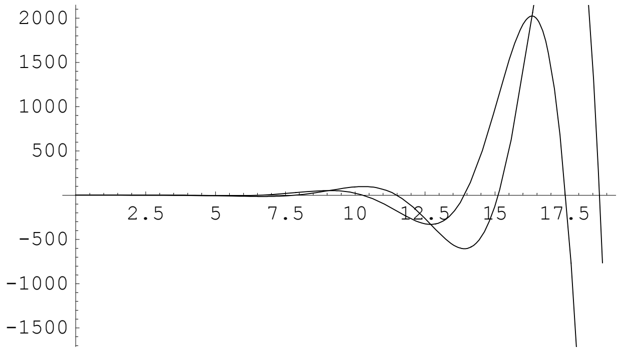
```
u[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u[t]
```

$-\frac{e^{-t}}{3} + \mathrm{Cos}[t] - \mathrm{Sin}[t] + \frac{1}{3}\, e^{t/2} \left(\mathrm{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$

```
v[t_] := InverseLaplaceTransform[V[s], s, t] // Simplify; v[t]
```

$\frac{1}{3} \left(e^{-t} + 3\, \mathrm{Cos}[t] + e^{t/2} \left(-\mathrm{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)\right)$

```
Plot[Evaluate[{u[t], v[t]}], {t, 0, 6 Pi}];
```



Faltung

```
Remove["Global`*"]
```

■ Rechnung gewöhnlich

```
links = LaplaceTransform[y''[t] + 3 y'[t] + 4 y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 0, y'[0] → 0}
```

General::spell1 :
 Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr…

$4\,Y[s] + 3\,s\,Y[s] + s^2\,Y[s]$

```
rechts = LaplaceTransform[Sin[t], t, s]
```

$$\frac{1}{1+s^2}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{1}{4+3\,s+5\,s^2+3\,s^3+s^4}\right\}$$

```
U[s] := Y[s] /. solv;  U[s]
```

$$\frac{1}{4+3\,s+5\,s^2+3\,s^3+s^4}$$

```
U[s] // Apart
```

$$\frac{1-s}{6\,(1+s^2)} + \frac{2+s}{6\,(4+3\,s+s^2)}$$

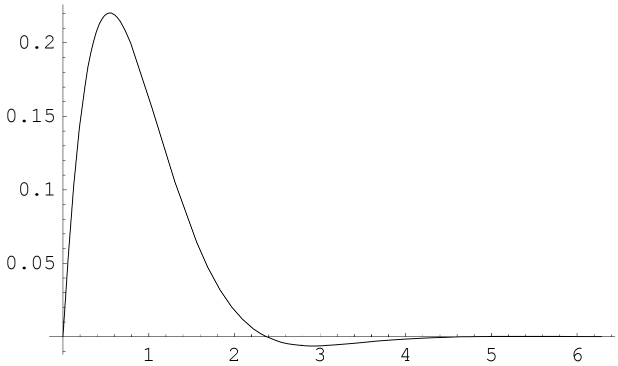
```
InverseLaplaceTransform[U[s], s, t] // Simplify
```

$$\frac{1}{42}\left(7\,(-\mathrm{Cos}[t] + \mathrm{Sin}[t]) + e^{-3\,t/2}\left(7\,\mathrm{Cos}\left[\frac{\sqrt{7}\,t}{2}\right] + \sqrt{7}\,\mathrm{Sin}\left[\frac{\sqrt{7}\,t}{2}\right]\right)\right)$$

```
g[t_] := InverseLaplaceTransform[1 / (s^2 + 3 s + 4), s, t] // Simplify; g[t]
```

$$\frac{2\,e^{-3\,t/2}\,\mathrm{Sin}\left[\frac{\sqrt{7}\,t}{2}\right]}{\sqrt{7}}$$

```
Plot[Evaluate[{g[t]}], {t, 0, 2 Pi}];
```



■ Rechnung mit Faltung

```
faltung[u_, v_, t_] := Integrate[u[λ] v[t - λ], {λ, 0, t}];
```

```
u[t_] := g[t]; v[t_] := Sin[t];
faltung[u, v, t]
```

$$\frac{1}{42}\left(-7\,\mathrm{Cos}[t] + 7\,\mathrm{Sin}[t] + e^{-3t/2}\left(7\,\mathrm{Cos}\left[\frac{\sqrt{7}\,t}{2}\right] + \sqrt{7}\,\mathrm{Sin}\left[\frac{\sqrt{7}\,t}{2}\right]\right)\right)$$

■ Faltung allgemein

```
Remove["Global`*"]
```

```
(* Zur Information *)LaplaceTransform[a y''[t] + b y'[t] + c y[t], t, s] /.
 {LaplaceTransform[y[t], t, s] → Y[s], y[0] → y0, y'[0] → y1}
```

$$c\,Y[s] + b\,(-y0 + s\,Y[s]) + a\,(-s\,y0 - y1 + s^2\,Y[s])$$

■ Beispiel 1

```
a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := Sin[t];

mo[k_] := Module[{ }, r[t_] :=
    InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
   Print["r[t] = ", r[t]];
   g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
   Print["g[t] = ", g[t]];
   faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
     GenerateConditions → False] // Simplify;
   Print["y[t] = ", faltung[f, g, t]];
  ];
mo[k]
```

r[t] = $e^{-t/2}\left(\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\sqrt{3}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

g[t] = $\frac{2\,e^{-t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$

y[t] = $-\text{Cos}[t]+\frac{1}{3}\,e^{-t/2}\left(3\,\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\sqrt{3}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

■ Beispiel 2

```
a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := Cos[t];

mo[k_] := Module[{ }, r[t_] :=
    InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
   Print["r[t] = ", r[t]];
   g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
   Print["g[t] = ", g[t]];
   faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
     GenerateConditions → False] // Simplify;
   Print["y[t] = ", faltung[f, g, t]];
  ];
mo[k]
```

r[t] = $e^{-t/2}\left(\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\sqrt{3}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

g[t] = $\frac{2\,e^{-t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$

y[t] = $\text{Sin}[t]-\frac{2\,e^{-t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$

■ Beispiel 3

```
a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := E^(-t);

mo[k_] := Module[{ }, r[t_] :=
    InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
   Print["r[t] = ", r[t]];
   g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
   Print["g[t] = ", g[t]];
   faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
      GenerateConditions → False] // Simplify;
   Print["y[t] = ", faltung[f, g, t]];
  ];
mo[k]
```

$r[t] = e^{-t/2}\left(\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

$g[t] = \frac{2\,e^{-t/2}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$

$y[t] = \frac{1}{3}\,e^{-t}\left(3 + e^{t/2}\left(-3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)\right)$

■ Beispiel 4

```
a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := t;

mo[k_] := Module[{ }, r[t_] :=
    InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
   Print["r[t] = ", r[t]];
   g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
   Print["g[t] = ", g[t]];
   faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
      GenerateConditions → False] // Simplify;
   Print["y[t] = ", faltung[f, g, t] // Simplify];
  ];
mo[k]
```

$r[t] = e^{-t/2}\left(\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$

$g[t] = \frac{2\,e^{-t/2}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$

$y[t] = -1 + t - \frac{1}{3}\,e^{-t/2}\left(-3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$