Lösungen

1. Dirachsche Deltafunktion und Einheitssprung

a

$$\int_{-3\pi}^{3\pi} \text{DiracDelta}[x] \, dx$$

1

b

$$\int_{-3\pi}^{3\pi} \text{DiracDelta}[x] \, E\char`^(x+1) \, dx$$

e

c

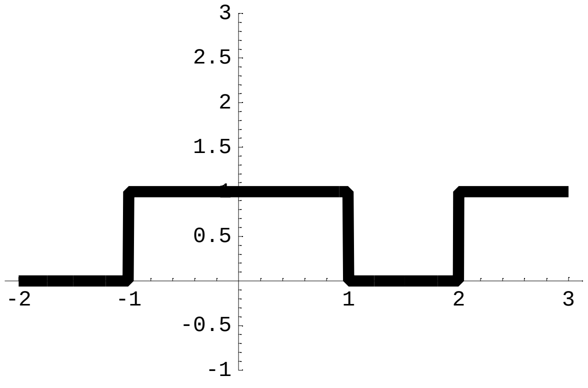
$$\int_{-3\pi}^{3\pi} \text{DiracDelta}\left[x - \frac{\pi}{2}\right] \text{Sin}[x] \, dx$$

1

d

```
u[x_]=UnitStep[(x+1)(x-1)(x-2)];
```

```
Plot[u[x],{x,-2,3},PlotRange->{-1,3},AxesOrigin->{0,0},PlotStyle->{Thickness[0.02]}
];
```



e

$$\int_{-3}^{5} u[x]\, dx$$

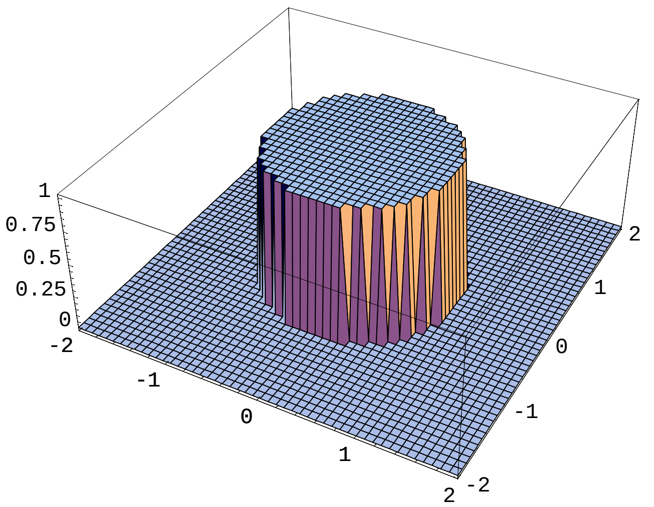
5

```
u[x] // FunctionExpand
```

```
UnitStep[-2 + x] - UnitStep[-1 + x] + UnitStep[1 + x]
```

f

```
Plot3D[UnitStep[1 - x^2 - y^2], {x, -2, 2}, {y, -2, 2}, PlotPoints -> 50];
```



2. D'Gl. 2. Ordnung

a

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

```
General::spell1 :
 Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr…
```

$-2 - s + Y[s] + s\,Y[s] + s^2\,Y[s]$

```
rechts = LaplaceTransform[DiracDelta[t] , t, s]
```

1

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{3+s}{1+s+s^2}\right\}$$

```
U[s] := Y[s] /. solv;  U[s]
```

$$\frac{3+s}{1+s+s^2}$$

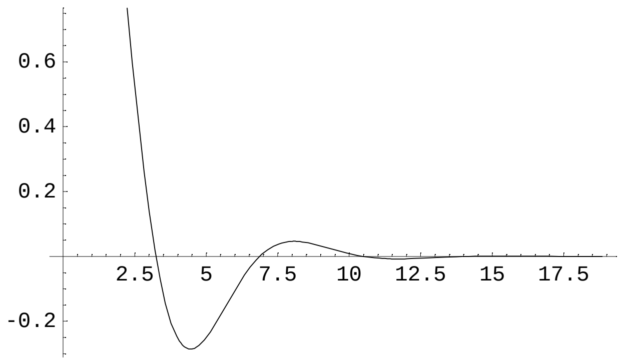
```
U[s] // Apart
```

$$\frac{3+s}{1+s+s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3}\, e^{-t/2} \left(3\,\mathrm{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + 5\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



b

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
   {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

General::spell1 :
 Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr…

$$-2 - s + Y[s] + s\,Y[s] + s^2\,Y[s]$$

```
LaplaceTransform[DiracDelta[t-1],t,s]
```

$$e^{-s}$$

```
rechts = LaplaceTransform[DiracDelta[t] + DiracDelta[t - 1] + DiracDelta[t - 2] , t, s]
```

$$1 + e^{-2s} + e^{-s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{e^{-2s}\,(1 + e^{s} + 3\,e^{2s} + e^{2s}\,s)}{1+s+s^2}\right\}$$

```
U[s] := Y[s] /. solv;  U[s]
```

$$\frac{e^{-2s}\,(1+e^{s}+3\,e^{2s}+e^{2s}\,s)}{1+s+s^2}$$

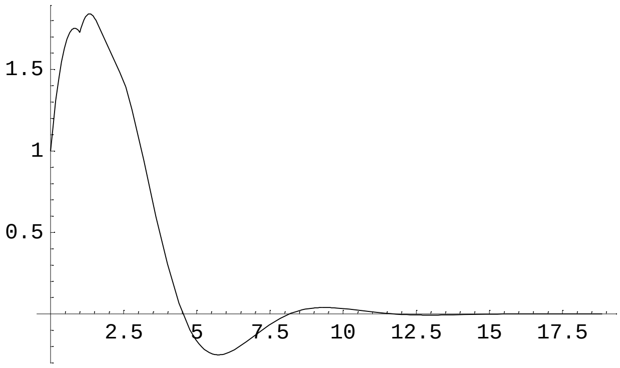
```
U[s] // Apart
```

$$\frac{e^{-2s}}{1+s+s^2}+\frac{e^{-s}}{1+s+s^2}+\frac{3+s}{1+s+s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3}\,e^{-t/2}\left(3\,\mathrm{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+5\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]+2\sqrt{3}\,e\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,(-2+t)\right]\mathrm{UnitStep}[-2+t]+2\sqrt{3\,e}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,(-1+t)\right]\mathrm{UnitStep}[-1+t]\right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



c

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

```
General::spell1 :
 Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr…
```

$$-2-s+Y[s]+s\,Y[s]+s^2\,Y[s]$$

```
rechts=LaplaceTransform[UnitStep[t] ,t,s]
```

$$\frac{1}{s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s]\to\frac{1+2\,s+s^2}{s\,(1+s+s^2)}\right\}$$

```
U[s] := Y[s] /. solv;  U[s]
```

$$\frac{1+2\,s+s^2}{s\,(1+s+s^2)}$$

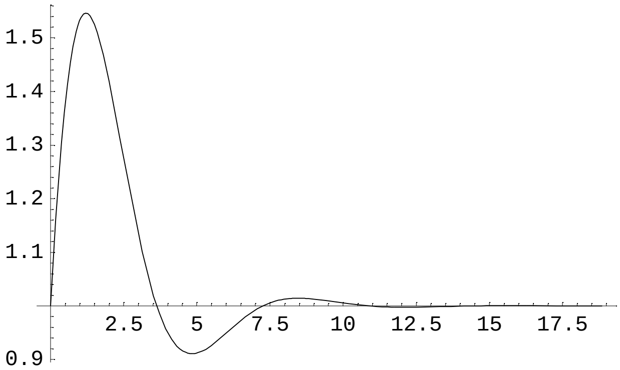
```
U[s] // Apart
```

$$\frac{1}{s} + \frac{1}{1+s+s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$1 + \frac{2\, e^{-t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



d

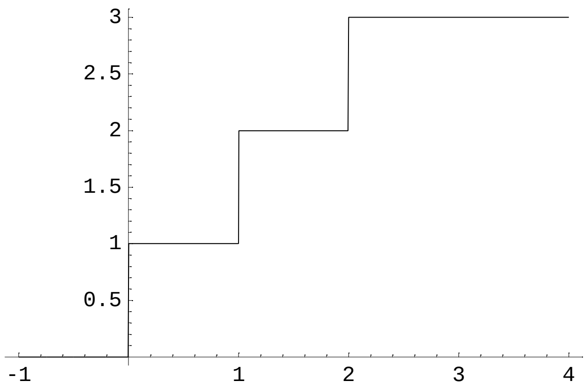
```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

General::spell1 :
Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr…

$$-2 - s + Y[s] + s\, Y[s] + s^2\, Y[s]$$

```
Plot[UnitStep[t]+UnitStep[t-1]+UnitStep[t-2],{t,-1,4}];
```




```
rechts=LaplaceTransform[UnitStep[t]+UnitStep[t-1]+UnitStep[t-2] ,t,s]
```

$$\frac{1}{s} + \frac{e^{-2 s}}{s} + \frac{e^{-s}}{s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{Y[s] \to \frac{e^{-2s}\,(1 + e^{s} + e^{2s} + 2\,e^{2s}\,s + e^{2s}\,s^2)}{s\,(1 + s + s^2)}\right\}$$

```
U[s] := Y[s] /. solv;  U[s]
```

$$\frac{e^{-2s}\,(1 + e^{s} + e^{2s} + 2\,e^{2s}\,s + e^{2s}\,s^2)}{s\,(1 + s + s^2)}$$

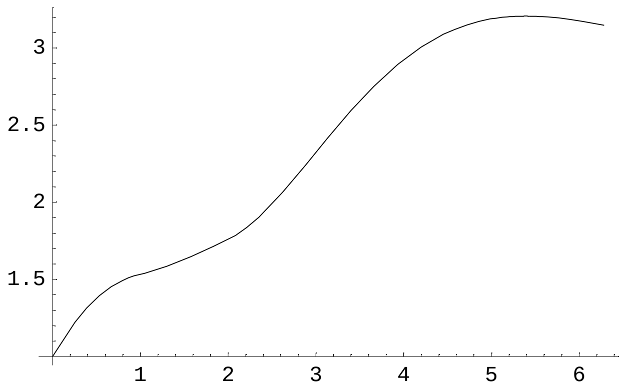
```
U[s] // Apart
```

$$\frac{e^{-2s}}{s\,(1 + s + s^2)} + \frac{e^{-s}}{s\,(1 + s + s^2)} + \frac{(1 + s)^2}{s\,(1 + s + s^2)}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3}\,e^{-t/2}\left(3\,e^{t/2} + 2\sqrt{3}\,\mathrm{Sin}\left[\frac{\sqrt{3}\,t}{2}\right] + \left(3\,e^{t/2} - 3\,e\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,(-2 + t)\right] - \sqrt{3}\,e\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,(-2 + t)\right]\right)\mathrm{UnitStep}[-2 + t] + \left(3\,e^{t/2} - 3\sqrt{e}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,(-1 + t)\right] - \sqrt{3\,e}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,(-1 + t)\right]\right)\mathrm{UnitStep}[-1 + t]\right)$$

```
Plot[Evaluate[{u0[t]}],{t,0,2Pi}];
```




```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```