# Lösungen

## 1. Drehende Scheibe

```
Remove["Global`*"]
```

```
x (z'[x] - 3/10 y'[x]) + (1 + 3/10) (z[x] - y[x]) // Simplify
```

$$\frac{13}{10}\,(-y[x]+z[x])+x\left(-\frac{3}{10}\,y'[x]+z'[x]\right)$$

```
solv = DSolve[{x^2 ρ ω^2 + y[x] - z[x] + x y'[x] == 0,
    -13/10 y[x] + 13/10 z[x] + x (-3/10  y'[x] + z'[x]) == 0}, {y, z}, x] // Flatten
```

$$\left\{z\to \text{Function}\left[\{x\},\ -\frac{19}{80}\,x^2\,\rho\,\omega^2-\frac{C[1]}{x^2}+C[2]\right],\ y\to \text{Function}\left[\{x\},\ -\frac{33}{80}\,x^2\,\rho\,\omega^2+\frac{C[1]}{x^2}+C[2]\right]\right\}$$

```
y1[x_, h_] := (y[x] /. solv[[2]]) /. ω → h;
y1[x, h]
```

$$-\frac{33}{80}\,h^2\,x^2\,\rho+\frac{C[1]}{x^2}+C[2]$$

```
z1[x_,h_]:=(z[x]/.solv[[1]])/.ω→h;
z1[x,h]
```

$$-\frac{19}{80}\,h^2\,x^2\,\rho-\frac{C[1]}{x^2}+C[2]$$

**Für muss y1[r] und y2[R]=0 sein..**

```
solv1 = Solve[{y1[r, ω] == 0, y1[R, ω] == 0}, {C[1], C[2]}] // Simplify // Flatten
```

$$\left\{C[1]\to -\frac{33}{80}\,r^2\,R^2\,\rho\,\omega^2,\ C[2]\to \frac{33}{80}\,(r^2+R^2)\,\rho\,\omega^2\right\}$$

```
y2[x_,ω_]:=y1[x,ω]/.solv1; y2[x,ω]
```

$$\frac{33}{80}\,(r^2+R^2)\,\rho\,\omega^2-\frac{33\,r^2\,R^2\,\rho\,\omega^2}{80\,x^2}-\frac{33}{80}\,x^2\,\rho\,\omega^2$$

```
z2[x_, ω_] := z1[x, ω] /. solv1; z2[x, ω]
```

$$\frac{33}{80}\,(r^2+R^2)\,\rho\,\omega^2+\frac{33\,r^2\,R^2\,\rho\,\omega^2}{80\,x^2}-\frac{19}{80}\,x^2\,\rho\,\omega^2$$

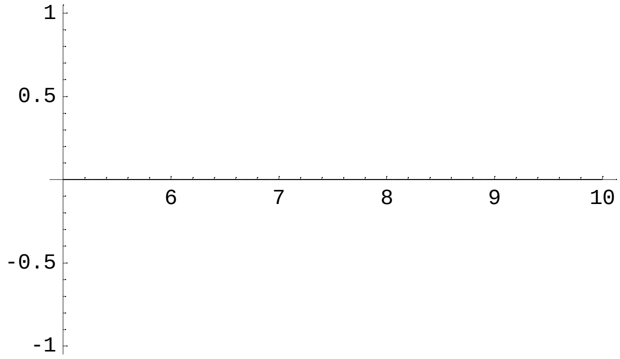
```
y3[x_,ω_]:=y2[x,ω]/.{r->5,R->10, ρ->1};
y3[x,ω]
```

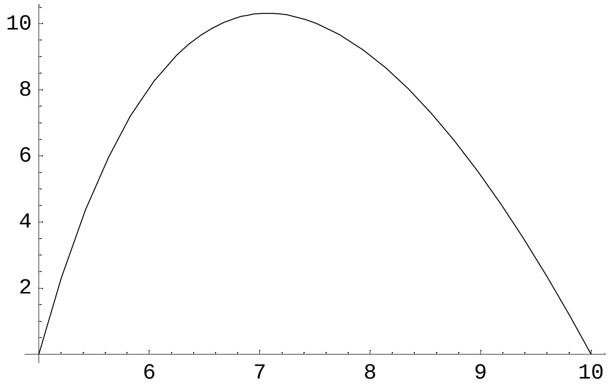
$$\frac{825\,\omega^2}{16}-\frac{4125\,\omega^2}{4\,x^2}-\frac{33\,x^2\,\omega^2}{80}$$

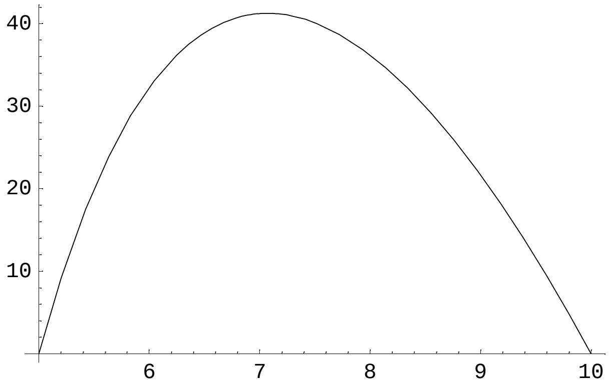
```
z3[x_,ω_]:=z2[x,ω]/.{r->5,R->10, ρ->1};
z3[x,ω]
```

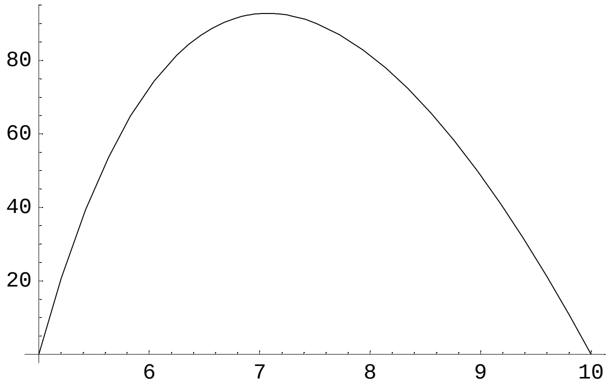
$$\frac{825\,\omega^2}{16}+\frac{4125\,\omega^2}{4\,x^2}-\frac{19\,x^2\,\omega^2}{80}$$

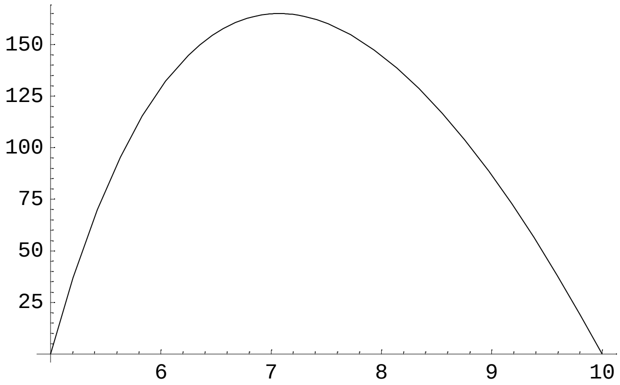
```
Table[Plot[Evaluate[y3[x, ω]], {x, 5, 10}], {ω, 0, 5}];
```

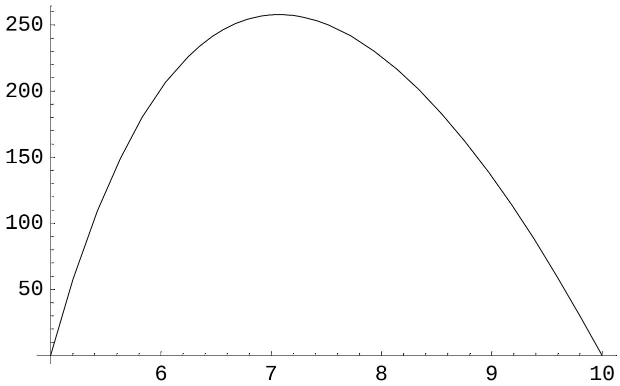




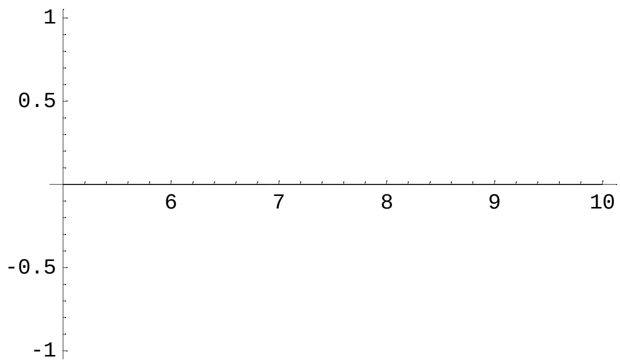


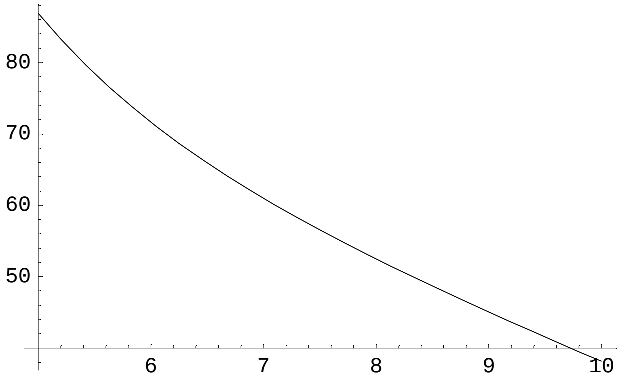


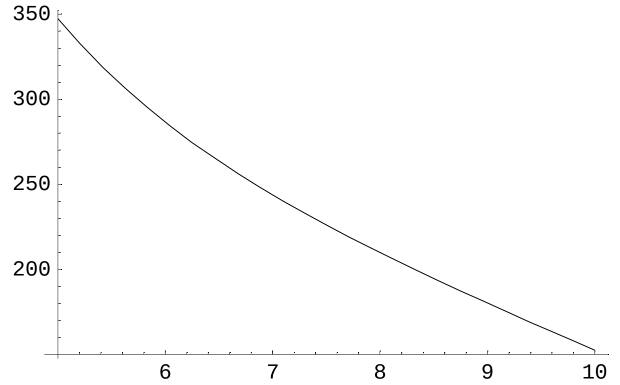


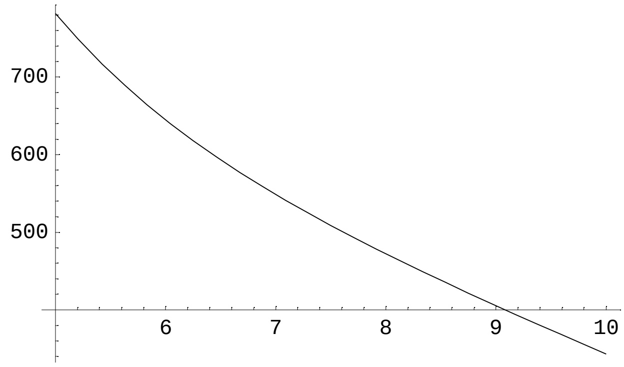



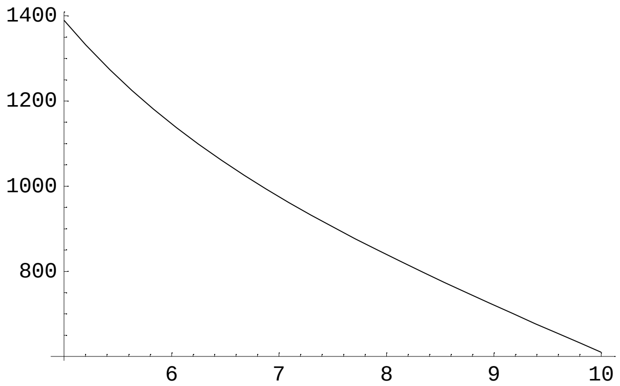

**Table[Plot[Evaluate[z3[x,ω]],{x,5,10}],{ω,0,5}];**

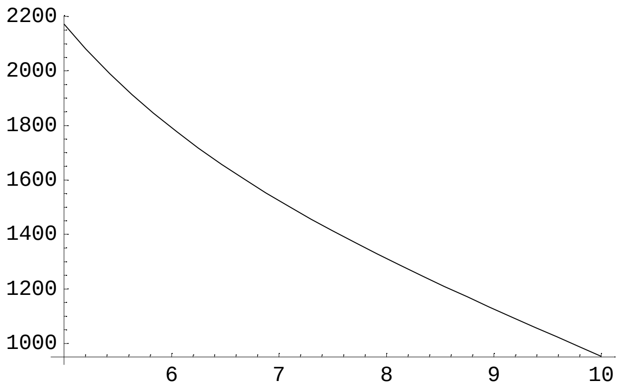



350
300
250
200
6 7 8 9 10


700
600
500
6 7 8 9 10


1400
1200
1000
800
6 7 8 9 10


2200
2000
1800
1600
1400
1200
1000
6 7 8 9 10

**Die letzten Graphen zeigen, dass die Tangentialspannung innen weitaus am grössten ist.**

**In welchem Abstand x ist die Radialspannung maximal?**

```
Solve[D[y2[x,ω],x]==0,{x}]
```

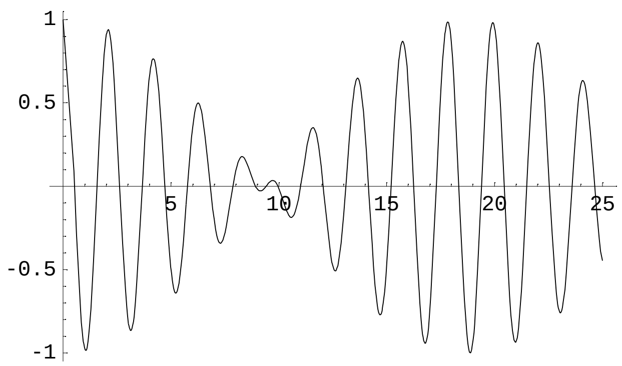
$$\{\{x\to-\sqrt{r}\sqrt{R}\},\ \{x\to-i\sqrt{r}\sqrt{R}\},\ \{x\to i\sqrt{r}\sqrt{R}\},\ \{x\to\sqrt{r}\sqrt{R}\}\}$$

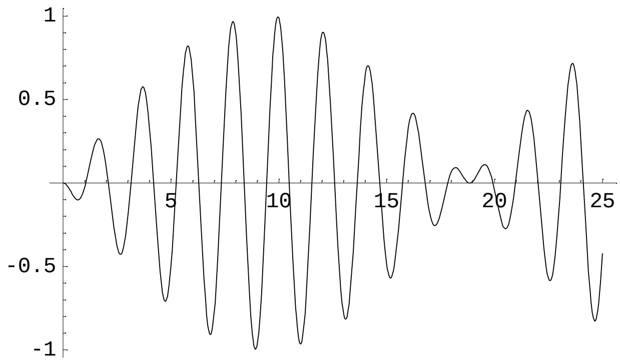
## 2. Gekoppelte Pendel

```
Remove["Global`*"];
links1 = LaplaceTransform[y2''[t] + g/L y2[t] - k/m y2[t] + k/m y1[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->a, y1'[0]->0, y2[0]->0, y2'[0]->0};
links2 = LaplaceTransform[y1''[t] + g/L y1[t] - k/m y1[t] + k/m y2[t], t,s] /.
{LaplaceTransform[y1[t],t,s]->Y1[s],
LaplaceTransform[y2[t],t,s]->Y2[s], y1[0]->a, y1'[0]->0, y2[0]->0, y2'[0]->0};
solv=Solve[{links1 == 0, links2 == 0},{Y1[s],Y2[s]}] // Flatten;
U1[s]:=Y1[s]/. solv[[1]];
U2[s]:=Y2[s]/. solv[[2]];
u1[t_]:=InverseLaplaceTransform[U1[s],s,t]//Simplify; Print["u1(t) = ",u1[t]];
u2[t_]:=InverseLaplaceTransform[U2[s],s,t]//Simplify; Print["u2(t) = ",u2[t]];
u1P[t]:=u1[t]/. {g -> 10, m->1, L->1, k->1, a->1};
u2P[t]:=u2[t]/. {g -> 10, m->1, L->1, k->1, a->1};
Plot[Evaluate[u1P[t]],{t,0,25}];
Plot[Evaluate[u2P[t]],{t,0,25}];
```

$$u1(t) = \frac{1}{4}\,a\left(e^{-\frac{\sqrt{2kL-gm}\,t}{\sqrt{L}\sqrt{m}}} + e^{\frac{\sqrt{2kL-gm}\,t}{\sqrt{L}\sqrt{m}}} + 2\,\mathrm{Cos}\left[\frac{\sqrt{g}\,t}{\sqrt{L}}\right]\right)$$

$$u2(t) = \frac{1}{4}\,a\left(-e^{-\frac{\sqrt{2kL-gm}\,t}{\sqrt{L}\sqrt{m}}}\left(1 + e^{\frac{2\sqrt{2kL-gm}\,t}{\sqrt{L}\sqrt{m}}}\right) + 2\,\mathrm{Cos}\left[\frac{\sqrt{g}\,t}{\sqrt{L}}\right]\right)$$






## 3. Kleinprojekt

**Um die Selbständigkeit nicht zu stören, wird dazu vorläufig keine Lösung ausgegeben.**