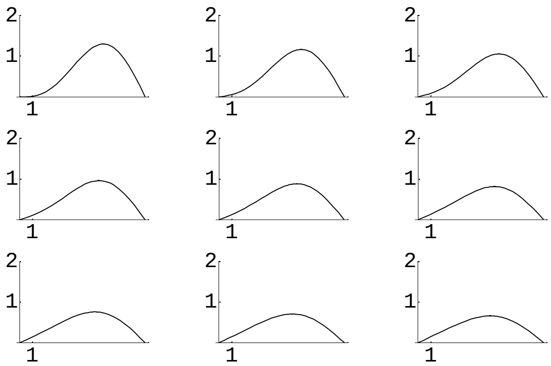
Lösungen

1

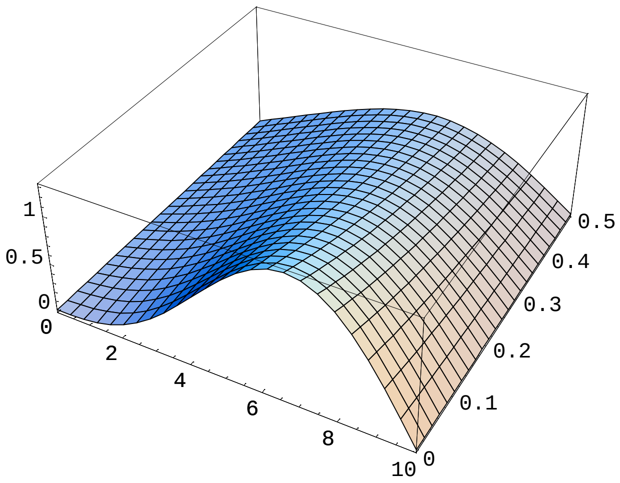
```
Remove["Global`*"];
xL = 10; k = 9;
f[x_] := Sin[Pi/10 x]-Sin[Pi/10 2x]/2;
mm = 10; r = 2;
Cc[n_] := 2/xL Integrate[f[x] Sin[n Pi x /xL], {x, 0, xL}];
u[x_, t_, n_] := Cc[n]  Sin[n Pi x /xL] E^(-k t (n Pi/xL)^2);
uApprox[x_, t_, n_] := Sum[u[x, t, j], {j, 1, n}];
uApprox[x, t, 10]
```

$$e^{-\frac{9\pi^2 t}{100}} \operatorname{Sin}\left[\frac{\pi x}{10}\right] - \frac{1}{2} e^{-\frac{9\pi^2 t}{25}} \operatorname{Sin}\left[\frac{\pi x}{5}\right]$$

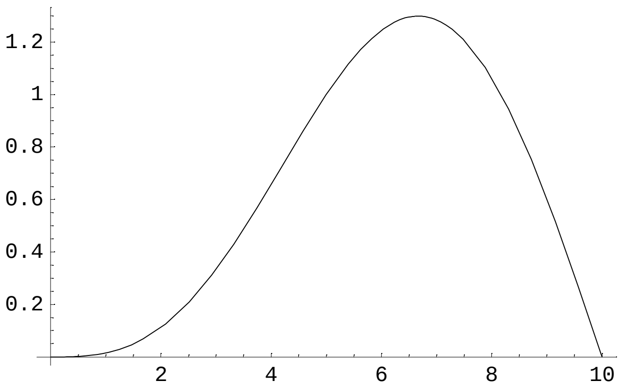
```
graphs = Table[Plot[uApprox[x, t, mm], {x, 0, xL},
        PlotRange -> {0, r}, Ticks -> {{0, 1},
            Range[0, Floor[r]]},
         DisplayFunction -> Identity], {t, 0, 1/2, 1/16}];
graphsarray = Partition[graphs, 3];
Show[GraphicsArray[graphsarray],
    DisplayFunction -> $DisplayFunction];
```



```
Plot3D[uApprox[x, t, mm], {x, 0, xL},{t,0,1/2}];
```




```
Plot[f[x],{x,0,xL}];
```

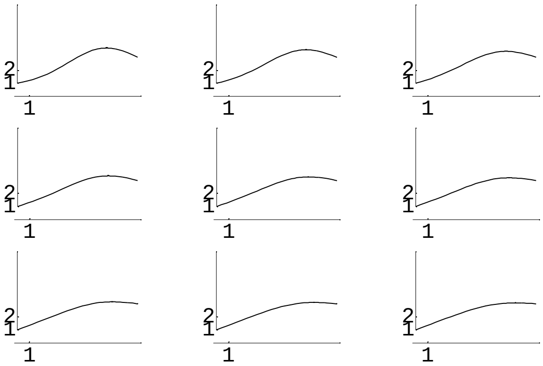


2

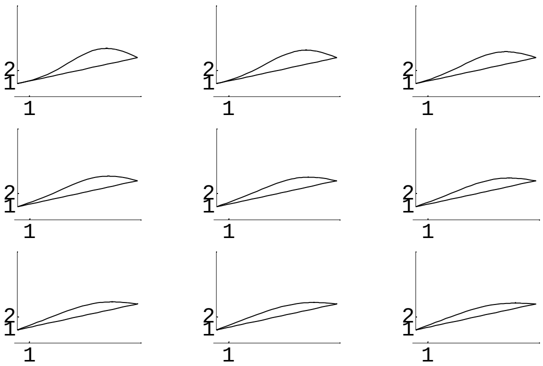
```
h[x_]:= (3-1)/10 * x + 1
vApprox[x_,t_,10]:= uApprox[x, t, 10]+ h[x];
vApprox[x,t,10]
```

$$1 + \frac{x}{5} + e^{-\frac{9\pi^2 t}{100}} \operatorname{Sin}\left[\frac{\pi x}{10}\right] - \frac{1}{2} e^{-\frac{9\pi^2 t}{25}} \operatorname{Sin}\left[\frac{\pi x}{5}\right]$$

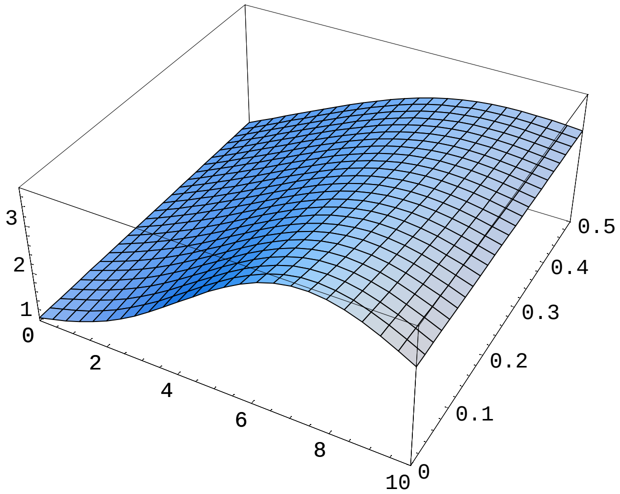
```
mm = 10;
r1=1; r2 = 7;
graphs = Table[Plot[vApprox[x, t, mm], {x, 0, xL},
        PlotRange -> {r1, r2}, Ticks -> {{0, 1},
            Range[0, Floor[r]]},
         DisplayFunction -> Identity], {t, 0, 1/2, 1/16}];
graphsarray = Partition[graphs, 3];
Show[GraphicsArray[graphsarray],
    DisplayFunction -> $DisplayFunction];
```



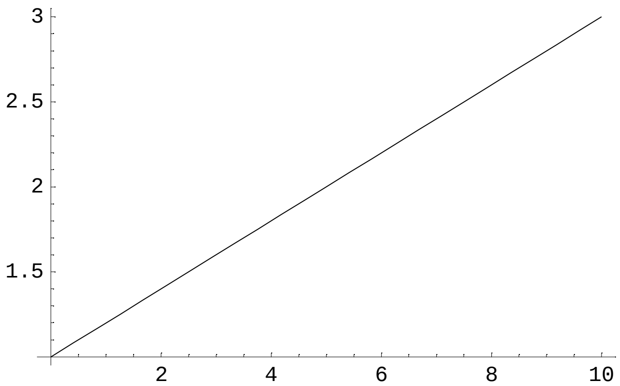
```
mm = 10;
r1=1; r2 = 7;
graphs = Table[Plot[{vApprox[x, t, mm],h[x]}, {x, 0, xL},
        PlotRange -> {r1, r2}, Ticks -> {{0, 1},
            Range[0, Floor[r]]},
         DisplayFunction -> Identity], {t, 0, 1/2, 1/16}];
graphsarray = Partition[graphs, 3];
Show[GraphicsArray[graphsarray],
    DisplayFunction -> $DisplayFunction];
```



```
Plot3D[vApprox[x, t, 10], {x, 0, xL},{t,0,1/2}];
```




```
Plot[h[x],{x,0,xL}];
```



3

```
Remove["Global`*"];
```

Berechnungen voraus:

```
ds = DSolve[{v'[t] == g - g v[t]^2 /vu^2}, v[t], t]//Flatten//Chop
```

$\left\{v[t] \to vu\,\mathrm{Tanh}\left[\frac{g\,t}{vu} + vu\,C[1]\right]\right\}$

```
h[t_]=v[t]/.ds
```

$vu\,\mathrm{Tanh}\left[\frac{g\,t}{vu} + vu\,C[1]\right]$

```
v0==h[0]
```

$v0 == vu\,\mathrm{Tanh}[vu\,C[1]]$

```
Solve[v0==vu Tanh[c1],{c1}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

$$\left\{\left\{c1 \to \text{ArcTanh}\left[\frac{v0}{vu}\right]\right\}\right\}$$

```
v0==(vu (E^c1-E^(-c1))/(E^c1+E^(-c1))/.{E^c1->c2,E^-c1->1/c2})
```

$$v0 == \frac{\left(-\frac{1}{c2}+c2\right) vu}{\frac{1}{c2}+c2}$$

```
solv=(Solve[v0==Evaluate[vu
(E^c1-E^(-c1))/(E^c1+E^(-c1))/.{E^c1->c2,E^-c1->1/c2}],{c2}]//Flatten)[[2]]
```

$$c2 \to \frac{\sqrt{-v0-vu}}{\sqrt{v0-vu}}$$

```
h[t]
```

$$vu\,\text{Tanh}\left[\frac{g\,t}{vu}+vu\,C[1]\right]$$

```
h[t_]:= (vu (E^(2g t/vu) c2^2 -1)/(E^(2g t/vu) c2^2 +1) /. solv ); h[t]
```

$$\frac{\left(-1+\frac{e^{\frac{2 g t}{vu}}(-v0-vu)}{v0-vu}\right) vu}{1+\frac{e^{\frac{2 g t}{vu}}(-v0-vu)}{v0-vu}}$$

```
h[t_]:= (vu (E^(2g t/vu) c2^2 -1)/(E^(2g t/vu) c2^2 +1) /. solv ) //Simplify; h[t]
```

$$\frac{vu\left(\left(1+e^{\frac{2 g t}{vu}}\right) v0+\left(-1+e^{\frac{2 g t}{vu}}\right) vu\right)}{\left(-1+e^{\frac{2 g t}{vu}}\right) v0+\left(1+e^{\frac{2 g t}{vu}}\right) vu}$$

Eingabe der Konstanten:

```
(*
g=9.81; v0=0; m=80; t1=10; ρ=1.2;
A1=0.8; A2=25; cw1=1; cw2=1.33;
c[cw_,A_]:= cw ρ A/2;
*)

c[cw_,A_]:= cw ρ A/2;
a[t_,c_]:=g-c v[t]^2 / m;

ersetzen = {g->9.81, m->80, t1->10, ρ->1.2,
A1->0.8, A2->25, cw1->1, cw2->1.33};
```

Eingabe der Konstanten:

Bei einem zeitlich unendlich langen Sinkflug muss sich ein Gleichgewicht einstellen zwischen g und c v^2/r) / m. Denn solange nach unten beschleunigt wird, steigt v und damit v^2. c v^2/r) / m kann aber nicht grösser als g werden, sonst hätten wir eine Aufwärtsbewegung.
Weiter ist a[t]=v'[t]. Sei c/m = q.

Berechnungen 1:

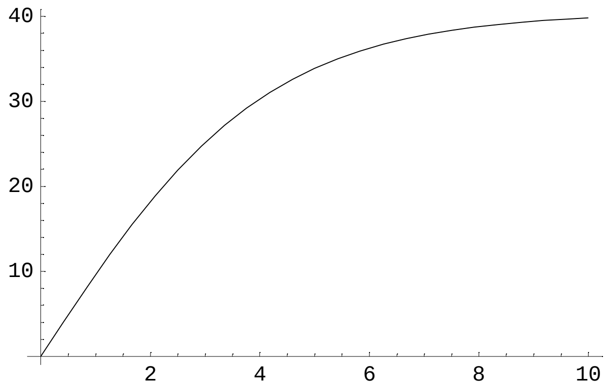
```
vu1 = Sqrt[g m/c[cw1,A1]]/.ersetzen
```

40.4351

```
w1[t_]:=(h[t]/.ersetzen)/.{vu->vu1,v0->0};w1[t]
```

$$\frac{40.4351\,(-1+e^{0.485222\,t})}{1+e^{0.485222\,t}}$$

```
p1=Plot[w1[t],{t,0,10}];
```




```
w1[10]
```

39.8083

Berechnungen 2:

```
vu2 = Sqrt[g m/c[cw2,A2]]/.ersetzen
```

6.27203

```
h[t-10]
```

$$\frac{vu\left(-1+\frac{e^{\frac{2\,g\,(-10+t)}{vu}}\,(v0+vu)}{-v0+vu}\right)}{1+\frac{e^{\frac{2\,g\,(-10+t)}{vu}}\,(v0+vu)}{-v0+vu}}$$

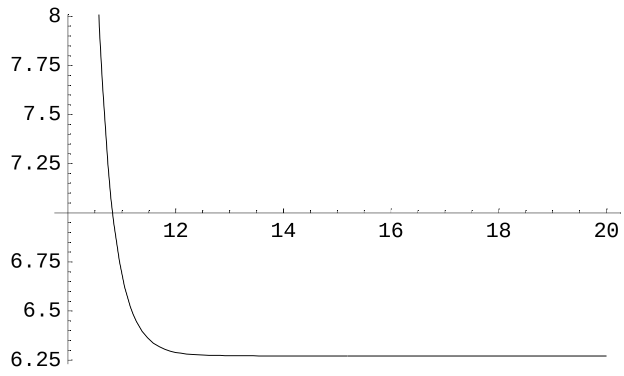
```
h[t-10]/.ersetzen
```

$$\frac{vu\left(-1+\frac{e^{\frac{19.62\,(-10+t)}{vu}}\,(v0+vu)}{-v0+vu}\right)}{1+\frac{e^{\frac{19.62\,(-10+t)}{vu}}\,(v0+vu)}{-v0+vu}}$$

```
w2[t_]:=(h[t-10]/.ersetzen)/.{vu->vu2,v0->w1[10]};w2[t]
```

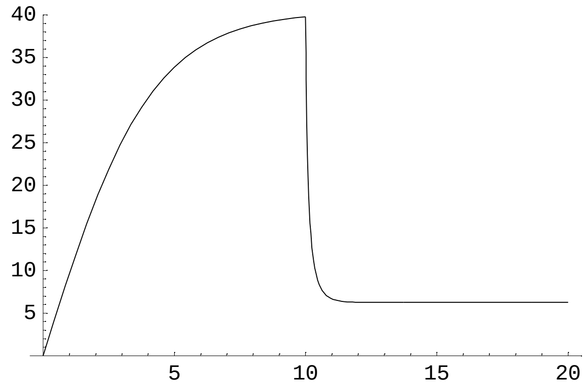
$$\frac{6.27203\,(-1-1.37404\,e^{3.12817\,(-10+t)})}{1-1.37404\,e^{3.12817\,(-10+t)}}$$

```
p2=Plot[w2[t],{t,10,20}];
```



Zusammen:

```
Show[p2,p1,PlotRange->{0,40}];
```




```
vu2
```

6.27203

4. Kleinprojekt

Um die Selbständigkeit nicht zu stören, wird dazu vorläufig keine Lösung ausgegeben.