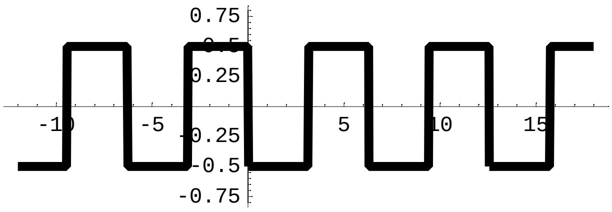
Lösungen

1

```
p1=Plot[-Floor[Sin[t]/2]-0.5,{t,-12,18},
Epilog->{Line[{{0,-0.3-0.5},{0,1.3-0.5}}]},
PlotStyle->{Thickness[0.015]},AspectRatio->1/3,DisplayFunction->Identity];
p2=Show[Graphics[Line[{{0,-0.3-0.5},{0,1.3-0.5}}]],DisplayFunction->Identity];
Show[p1,p2,DisplayFunction->$DisplayFunction];
```



2

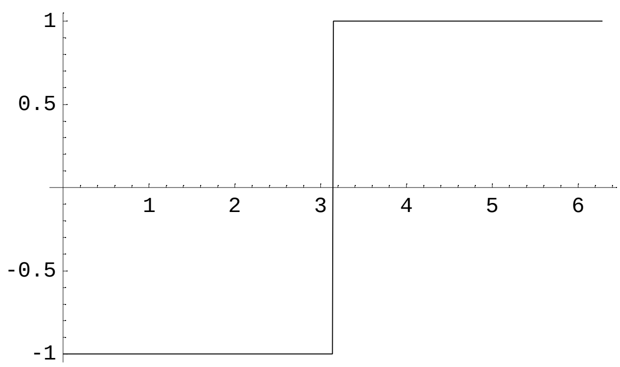
a

Sinusreihe, da ungerade Funktion!!

b

```
Remove["Global`*"]

f[x_ /; 0<=x && x<Pi]:= -1;
f[x_ /; Pi<=x && x<=2Pi]:=1;
Plot[f[t],{t,0,2Pi}];
```



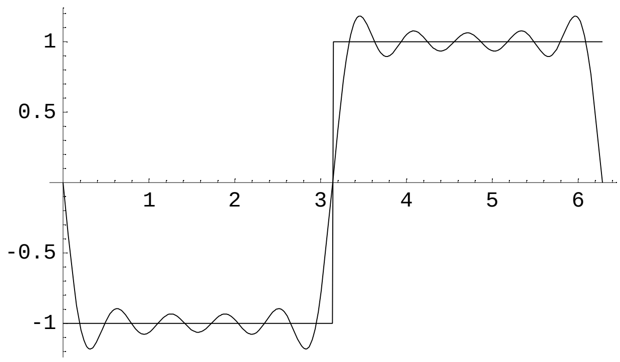
```
b[k_]:=1/Pi NIntegrate[f[t]Sin[k t],{t,0,2Pi}];
b[k]
```

$$\frac{\text{NIntegrate}[f[t]\,\text{Sin}[k\,t],\{t,0,2\pi\}]}{\pi}$$

```
g[t_]=Sum[Evaluate[b[k] Sin[k t]],{k,1,10}];
g[t]
```

$$-1.27324\,\text{Sin}[t]-3.253\times10^{-17}\,\text{Sin}[2\,t]-0.424413\,\text{Sin}[3\,t]-6.24633\times10^{-17}\,\text{Sin}[4\,t]-0.254648\,\text{Sin}[5\,t]+3.10447\times10^{-16}\,\text{Sin}[6\,t]-0.181891\,\text{Sin}[7\,t]-3.43927\times10^{-16}\,\text{Sin}[8\,t]-0.141471\,\text{Sin}[9\,t]+7.38108\times10^{-16}\,\text{Sin}[10\,t]$$

```
Plot[{f[t],g[t]},{t,-0,2Pi}];
```

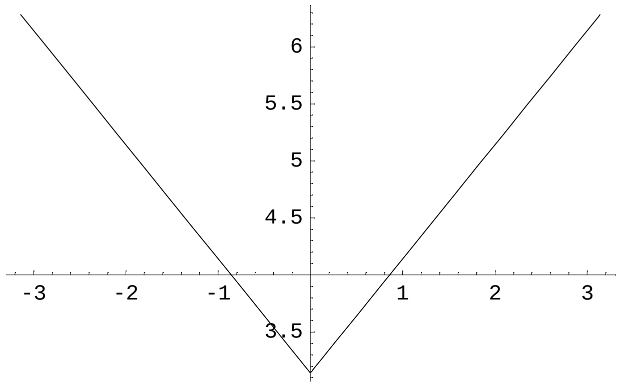


3

a

```
Remove["Global`*"]

f[x_ /; 0<=x && x<=Pi]:= t+Pi;
f[x_ /; -Pi<=x && x<0]:=-t+Pi;
Plot[f[t],{t,-Pi,Pi}];
```



Die Funktion hat eine Cosinusreihe mit konstantem Glied

```
a[k_]:=1/Pi NIntegrate[f[t]Cos[k t],{t,-Pi,Pi}];
a[k]
```

$$\frac{\text{NIntegrate}[f[t]\ \text{Cos}[k\ t],\ \{t,\ -\pi,\ \pi\}]}{\pi}$$

```
a[0]=1/Pi NIntegrate[f[t],{t,-Pi,Pi}]
```

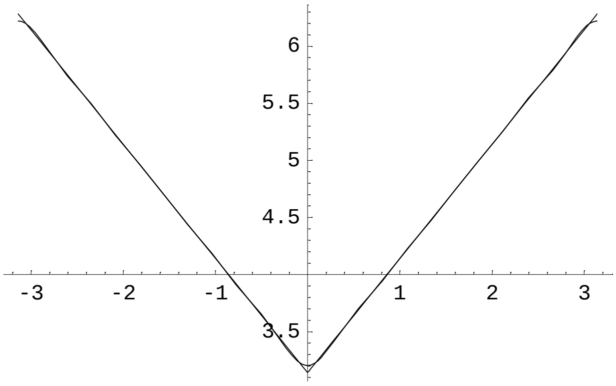
9.42478

```
g[t_]= a[0]/2+Sum[Evaluate[a[k] Cos[k t]],{k,1,10}];
g[t]
```

$$4.71239 - 1.27324\,\text{Cos}[t] - 2.29707\times 10^{-16}\,\text{Cos}[2\,t] - 0.141471\,\text{Cos}[3\,t] - 7.77469\times 10^{-16}\,\text{Cos}[4\,t] - 0.0509296\,\text{Cos}[5\,t] - 1.94367\times 10^{-16}\,\text{Cos}[6\,t] - 0.0259845\,\text{Cos}[7\,t] - 1.1027\times 10^{-15}\,\text{Cos}[8\,t] - 0.015719\,\text{Cos}[9\,t] - 1.45775\times 10^{-16}\,\text{Cos}[10\,t]$$

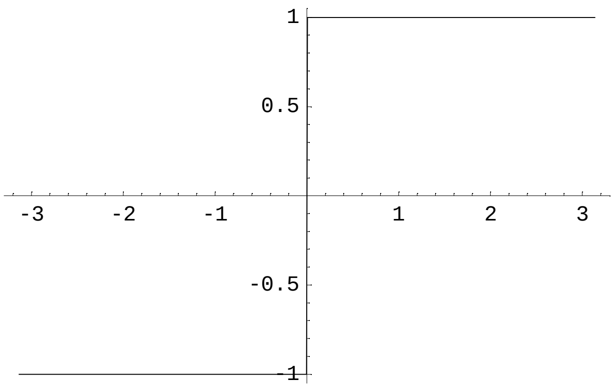
b

```
Plot[{f[t],g[t]},{t,-Pi,Pi}];
```



c

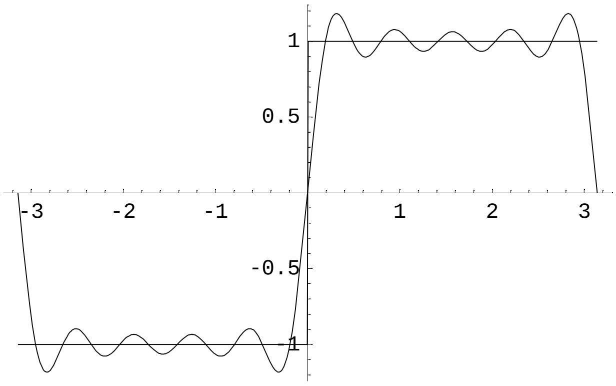
```
fAbl[x_ /; 0<=x && x<=Pi]:= 1;
fAbl[x_ /; -Pi<=x && x<0]:=-1;
Plot[fAbl[t],{t,-Pi,Pi}];
```



```
gAbl[t_]= 0+Sum[Evaluate[-a[k] Sin[k t] k],{k,1,10}];
gAbl[t]
```

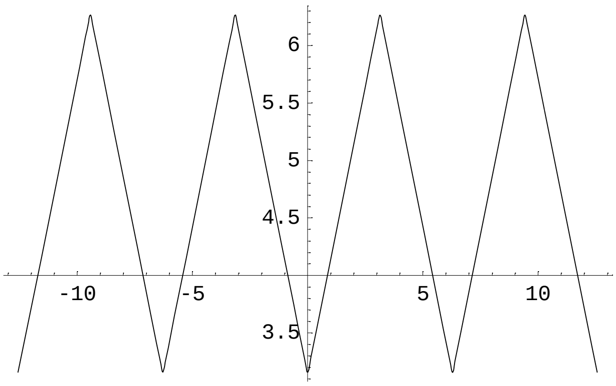
$1.27324\,\mathrm{Sin}[t] + 4.59413\times10^{-16}\,\mathrm{Sin}[2\,t] + 0.424413\,\mathrm{Sin}[3\,t] + 3.10988\times10^{-15}\,\mathrm{Sin}[4\,t] + 0.254648\,\mathrm{Sin}[5\,t] + 1.1662\times10^{-15}\,\mathrm{Sin}[6\,t] + 0.181891\,\mathrm{Sin}[7\,t] + 8.82162\times10^{-15}\,\mathrm{Sin}[8\,t] + 0.141471\,\mathrm{Sin}[9\,t] + 1.45775\times10^{-15}\,\mathrm{Sin}[10\,t]$

```
Plot[{fAbl[t],gAbl[t]},{t,-Pi,Pi}];
```



d Die Reihe bis n = 20 nach Tabelle mit Plot

```
f[t_]:=3 Pi/2-4/Pi Sum[1/(2k-1)^2 Cos[(2k-1)t],{k,1,20}];
Plot[Evaluate[f[t]],{t,-4Pi,4Pi}];
```

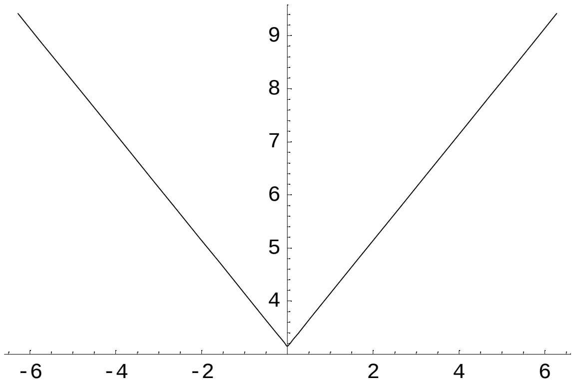


4

a

```
Remove["Global`*"]
```

```
f[x_ /; 0<=x && x<=2Pi]:= t+Pi;
f[x_ /; -2Pi<=x && x<0]:=-t+Pi;
Plot[f[t],{t,-2Pi,2Pi}];
```



Die Funktion hat eine Cosinusreihe mit konstantem Glied

```
a[k_]:=1/(2Pi) NIntegrate[f[t]Cos[k t/2],{t,-2Pi,2Pi}];
a[k]
```

$$\frac{\text{NIntegrate}\left[f[t]\,\text{Cos}\left[\frac{k\,t}{2}\right],\ \{t,\ -2\,\pi,\ 2\,\pi\}\right]}{2\,\pi}$$

```
a[0]=1/(2Pi) NIntegrate[f[t],{t,-2Pi,2Pi}]
```

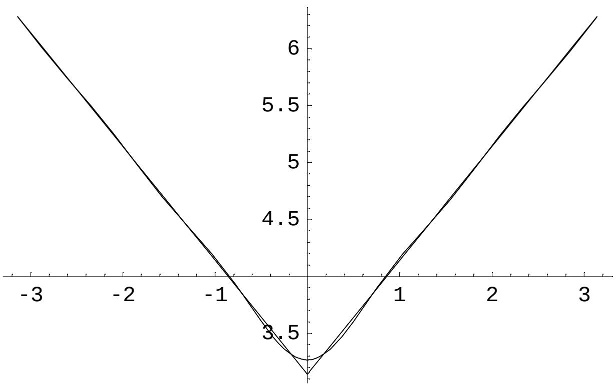
12.5664

```
g[t_]= a[0]/2+Sum[Evaluate[a[k] Cos[k t/2]],{k,1,10}];
g[t]
```

$$6.28319 - 2.54648\,\text{Cos}\left[\frac{t}{2}\right] - 4.24074\times10^{-16}\,\text{Cos}[t] - 0.282942\,\text{Cos}\left[\frac{3\,t}{2}\right] - 1.47211\times10^{-15}\,\text{Cos}[2\,t] - 0.101859\,\text{Cos}\left[\frac{5\,t}{2}\right] - 5.30092\times10^{-16}\,\text{Cos}[3\,t] - 0.051969\,\text{Cos}\left[\frac{7\,t}{2}\right] - 1.48426\times10^{-15}\,\text{Cos}[4\,t] - 0.031438\,\text{Cos}\left[\frac{9\,t}{2}\right] + 2.89342\times10^{-16}\,\text{Cos}[5\,t]$$

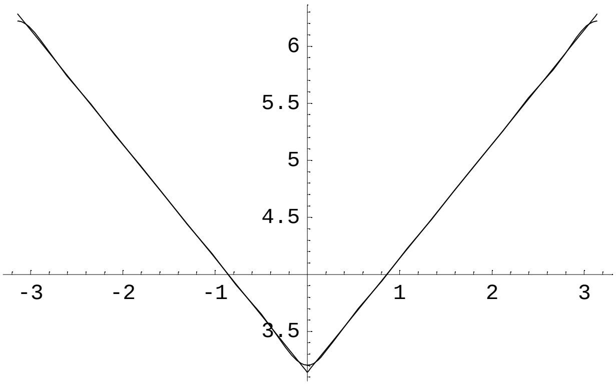
b

```
Plot[{f[t],g[t]},{t,-Pi,Pi}];
```



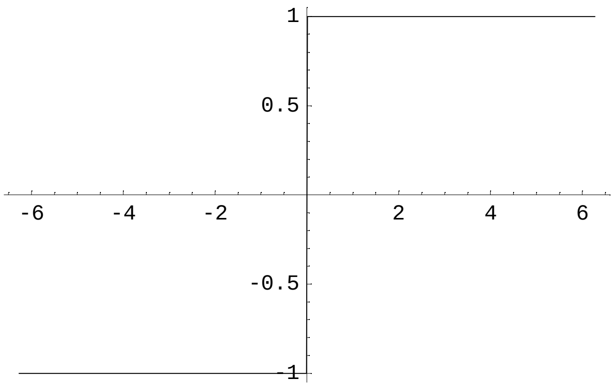
Der Plot zeigt, dass die Approximation an den Enden besser ist. In der Mitte ist sie aber schlecher!

Kopie des oben berechneten Graphen hier anbringen!



c

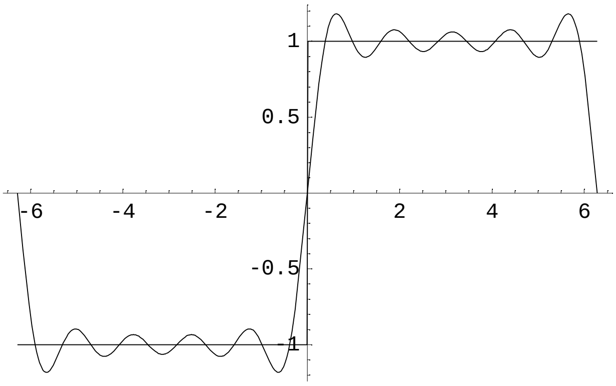
```
fAbl[x_ /; 0<=x && x<=2Pi]:= 1;
fAbl[x_ /; -2Pi<=x && x<0]:=-1;
Plot[fAbl[t],{t,-2Pi,2Pi}];
```



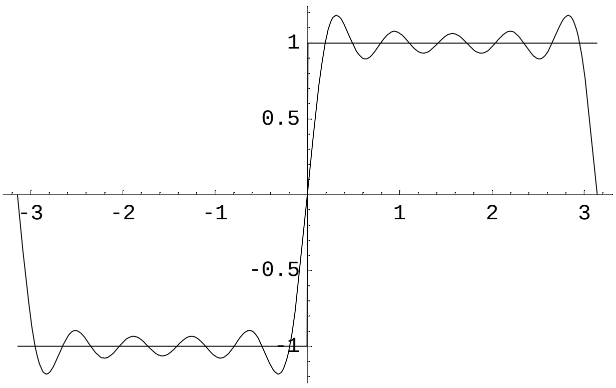

```
gAbl[t_]= 0+Sum[Evaluate[-a[k] Sin[k t/2] k/2],{k,1,10}];
gAbl[t]
```

$$1.27324\,\mathrm{Sin}\left[\frac{t}{2}\right] + 4.24074\times 10^{-16}\,\mathrm{Sin}[t] + 0.424413\,\mathrm{Sin}\left[\frac{3\,t}{2}\right] + 2.94422\times 10^{-15}\,\mathrm{Sin}[2\,t] + 0.254648\,\mathrm{Sin}\left[\frac{5\,t}{2}\right] + 1.59028\times 10^{-15}\,\mathrm{Sin}[3\,t] + 0.181891\,\mathrm{Sin}\left[\frac{7\,t}{2}\right] + 5.93704\times 10^{-15}\,\mathrm{Sin}[4\,t] + 0.141471\,\mathrm{Sin}\left[\frac{9\,t}{2}\right] - 1.44671\times 10^{-15}\,\mathrm{Sin}[5\,t]$$

```
Plot[{fAbl[t],gAbl[t]},{t,-2Pi,2Pi}];
```



Kopie des oben berechneten Graphen hier anbringen: Kein ersichtlicher Unterschied!

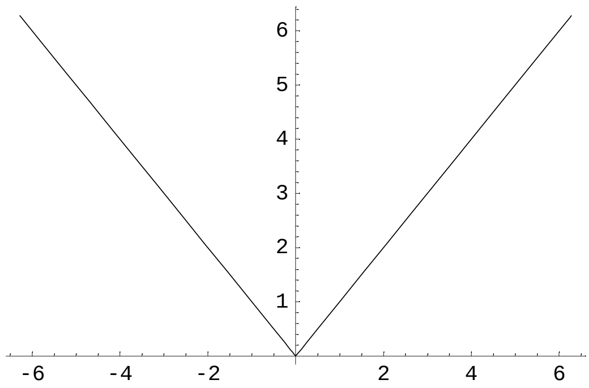


5

a

```
Remove["Global`*"]
```

```
h[x_ /; 0<=x && x<=2Pi]:= t;
h[x_ /; -2Pi<=x && x<0]:=-t;
Plot[h[t],{t,-2Pi,2Pi}];
```



Die Funktion hat eine Cosinusreihe mit konstantem Glied

```
a[k_]:=1/(2Pi) NIntegrate[h[t]Cos[k t/2],{t,-2Pi,2Pi}];
a[k]
```

$$\frac{\text{NIntegrate}\left[h[t]\,\text{Cos}\left[\frac{k\,t}{2}\right],\ \{t,\ -2\,\pi,\ 2\,\pi\}\right]}{2\,\pi}$$

```
a[0]=1/(2Pi) NIntegrate[h[t],{t,-2Pi,2Pi}]
```

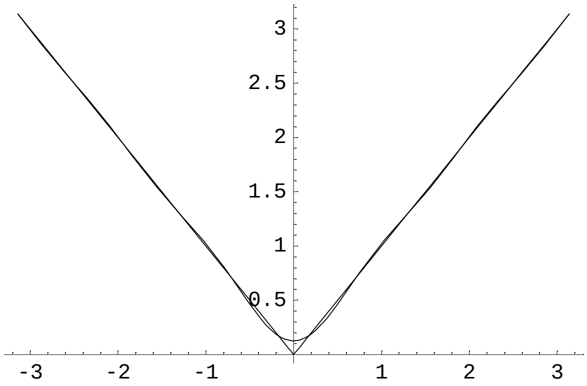
6.28319

```
m[t_]= a[0]/2+Sum[Evaluate[a[k] Cos[k t/2]],{k,1,10}];
m[t]
```

$$3.14159 - 2.54648\,\text{Cos}\left[\frac{t}{2}\right] - 3.00386\times10^{-16}\,\text{Cos}[t] - 0.282942\,\text{Cos}\left[\frac{3\,t}{2}\right] - 9.48092\times10^{-16}\,\text{Cos}[2\,t] - 0.101859\,\text{Cos}\left[\frac{5\,t}{2}\right] - 5.74267\times10^{-16}\,\text{Cos}[3\,t] - 0.051969\,\text{Cos}\left[\frac{7\,t}{2}\right] - 7.50964\times10^{-16}\,\text{Cos}[4\,t] - 0.031438\,\text{Cos}\left[\frac{9\,t}{2}\right] + 5.96354\times10^{-17}\,\text{Cos}[5\,t]$$

b

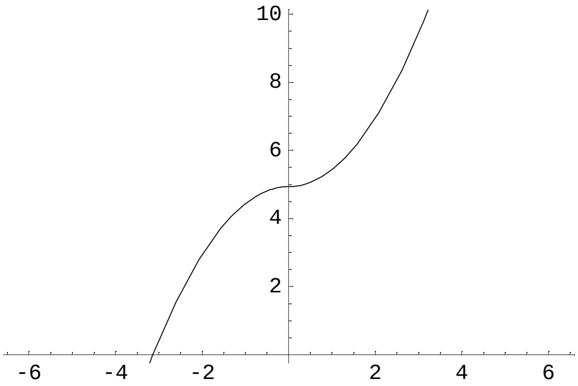
```
Plot[{h[t],m[t]},{t,-Pi,Pi}];
```



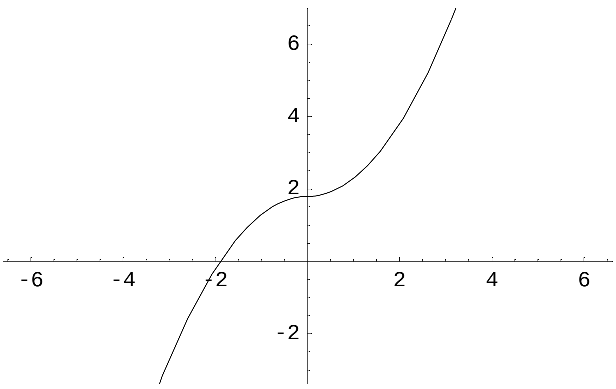
Der Plot zeigt, dass die Approximation an den Enden besser ist. In der Mitte ist sie aber schlecher!

c

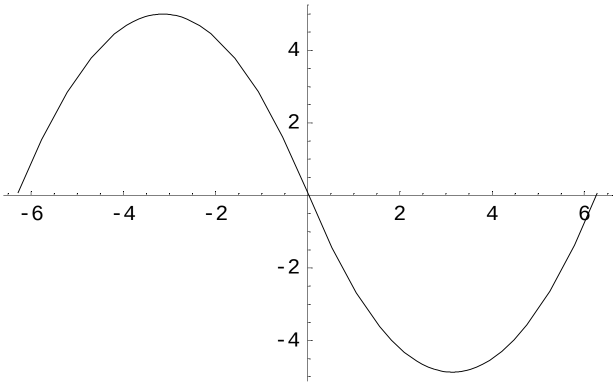
```
hInt[x_,c_]:= NIntegrate[h[t],{t,-Pi,x}]+c;
p1=Plot[hInt[t,0],{t,-2Pi,2Pi}];
```




```
hInt[x_,c_]:= NIntegrate[h[t],{t,-Pi,x}]+c;
p1=Plot[hInt[t,-a[0]/2],{t,-2Pi,2Pi}];
```



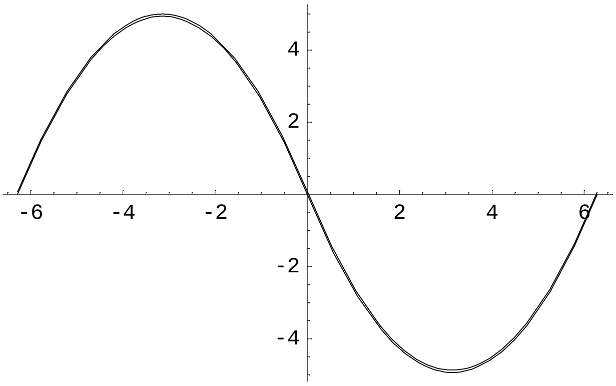
```
hInt[x_,c_]:= NIntegrate[h[t]-a[0]/2,{t,-Pi,x}]+c;
p1=Plot[hInt[t,5],{t,-2Pi,2Pi}];
```



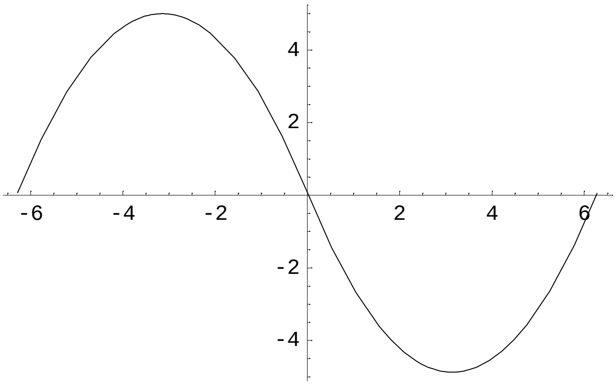

```
mInt[t_,c_]= c+Sum[Evaluate[a[k] Sin[k t/2] 2/k],{k,1,10}];
mInt[t,c]
```

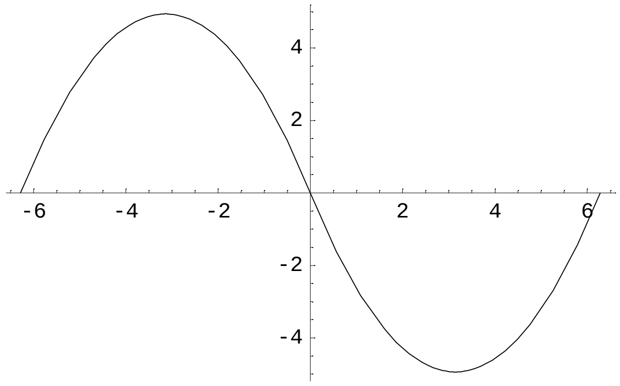
$$c - 5.09296\,\mathrm{Sin}\left[\frac{t}{2}\right] - 3.00386\times10^{-16}\,\mathrm{Sin}[t] - 0.188628\,\mathrm{Sin}\left[\frac{3\,t}{2}\right] - 4.74046\times10^{-16}\,\mathrm{Sin}[2\,t] - 0.0407437\,\mathrm{Sin}\left[\frac{5\,t}{2}\right] - 1.91422\times10^{-16}\,\mathrm{Sin}[3\,t] - 0.0148483\,\mathrm{Sin}\left[\frac{7\,t}{2}\right] - 1.87741\times10^{-16}\,\mathrm{Sin}[4\,t] - 0.00698623\,\mathrm{Sin}\left[\frac{9\,t}{2}\right] + 1.19271\times10^{-17}\,\mathrm{Sin}[5\,t]$$

```
Plot[{hInt[t,5],mInt[t,0]},{t,-2Pi,2Pi}];
```




```
Plot[hInt[t,5],{t,-2Pi,2Pi}];
Plot[mInt[t,0],{t,-2Pi,2Pi}];
```





Kommentar

Damit zwei ungefähr gleiche Graphen erscheinen, muss
" hInt[x_,c_]:= NIntegrate[h[t]-a[0]/2,{t,-Pi,x}]+c; " verwebdet werden. Durch h[t]-a[0]/2 wird das konstante Glied eliminiert, sodass dann beid er Integration kein linearer Anteil erscheint.