

Lösungen

1

```
Remove["Global`*"]
```

Plot von fFourier[t]:

```
<<Calculus`FourierTransform`
```

```
?*FourierTrig*
```

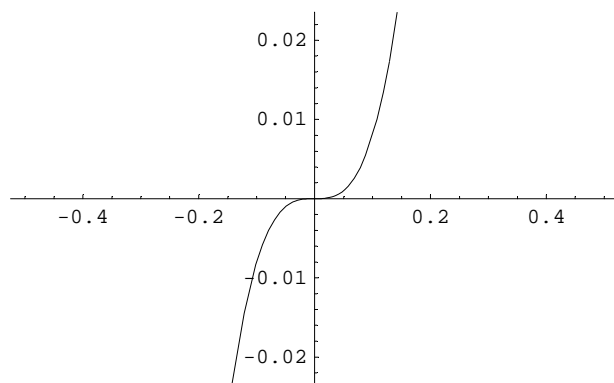
Calculus`FourierTransform`

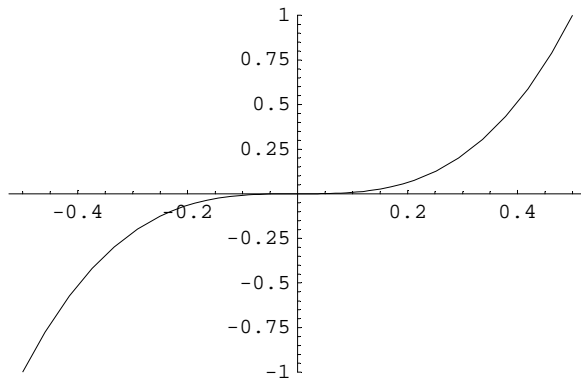
FourierTrigSeries NFourierTrigSeries

```
?FourierTrigSeries
```

FourierTrigSeries[expr, t, k] gives the kth order Fourier trigonometric series approximation to the periodic function of t that is equal to expr for $-1/2 \leq t \leq 1/2$, and has a period of 1. FourierTrigSeries[expr, t, k, FourierParameters -> {a, b}] gives the kth order Fourier trigonometric series approximation to the periodic function of t that is equal to expr for $-1/(2 \text{Abs}[b]) \leq t \leq 1/(2 \text{Abs}[b])$, and has a period of $1/\text{Abs}[b]$. Mehr...

```
f[t_]:= (2 t)^3;  
Plot[f[t],{t,-1/2,1/2}];  
Plot[f[t],{t,-1/2,1/2},PlotRange->{-1,1}];
```

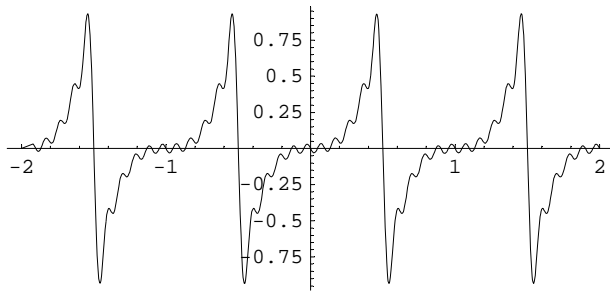




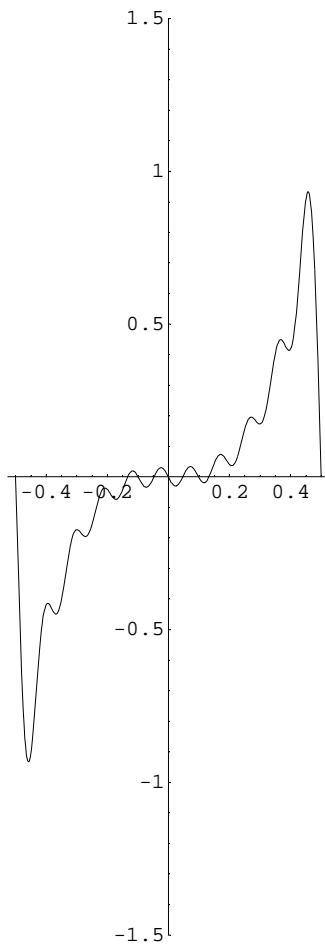
```
FourierTrigSeries[f[t], t, 10]
```

$$\begin{aligned} & \frac{2(-6 + \pi^2) \sin[2\pi t]}{\pi^3} + \frac{(3 - 2\pi^2) \sin[4\pi t]}{2\pi^3} + \\ & \frac{2(-2 + 3\pi^2) \sin[6\pi t]}{9\pi^3} + \frac{(3 - 8\pi^2) \sin[8\pi t]}{16\pi^3} + \frac{2(-6 + 25\pi^2) \sin[10\pi t]}{125\pi^3} + \\ & \frac{(1 - 6\pi^2) \sin[12\pi t]}{18\pi^3} + \frac{2(-6 + 49\pi^2) \sin[14\pi t]}{343\pi^3} + \\ & \frac{(3 - 32\pi^2) \sin[16\pi t]}{128\pi^3} + \frac{2(-2 + 27\pi^2) \sin[18\pi t]}{243\pi^3} + \frac{(3 - 50\pi^2) \sin[20\pi t]}{250\pi^3} \end{aligned}$$

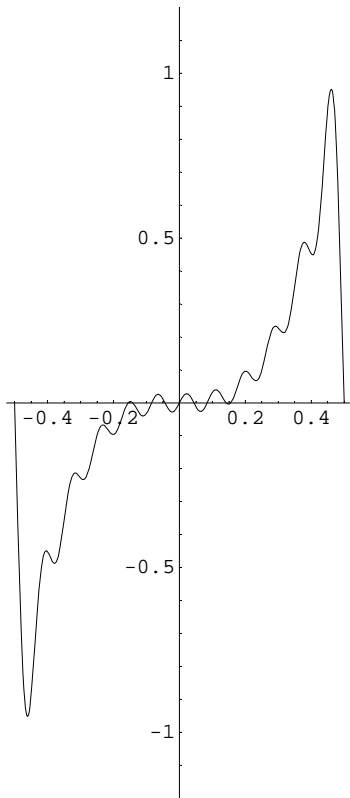
```
Plot[Evaluate[FourierTrigSeries[f[t], t, 10]], {t, -2, 2}, AspectRatio->Automatic];
```



```
Plot[Evaluate[FourierTrigSeries[f[t], t,  
10]],{t,-1/2,1/2},AspectRatio->Automatic,PlotRange->{-1.2,1.5}];
```

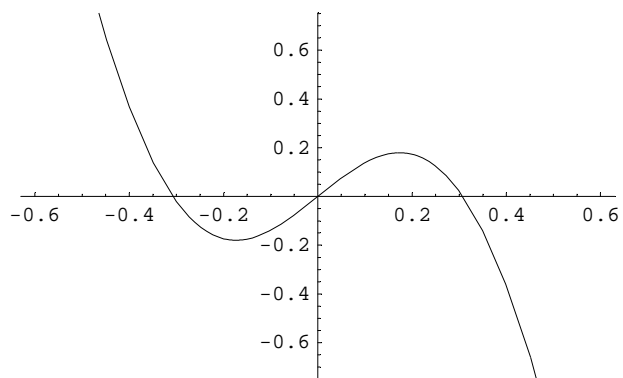


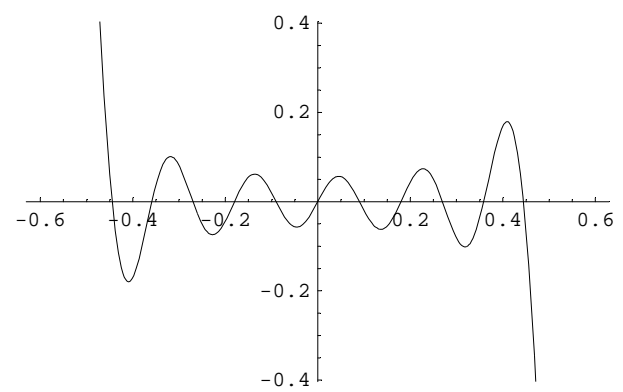
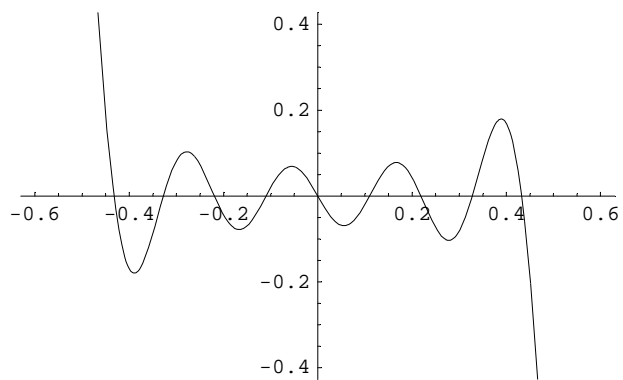
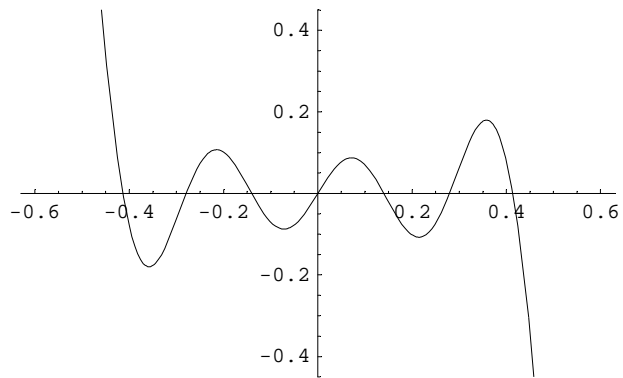
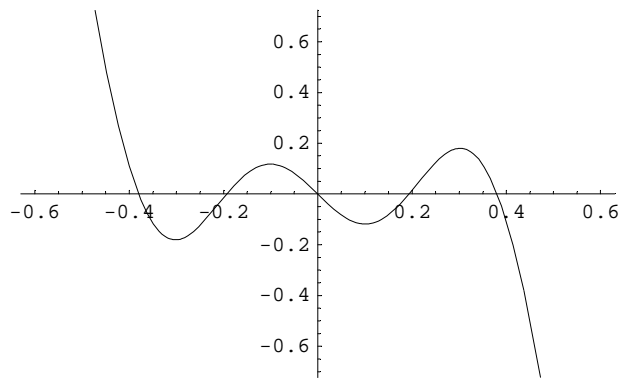
```
Plot[Evaluate[FourierTrigSeries[f[t], t,  
11]],{t,-1/2,1/2},AspectRatio->Automatic,PlotRange->{-1.2,1.2}];
```

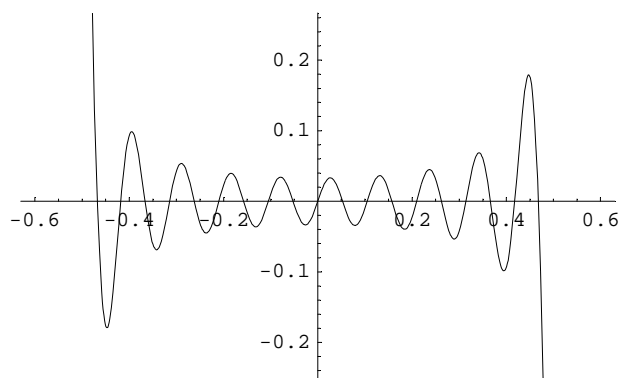
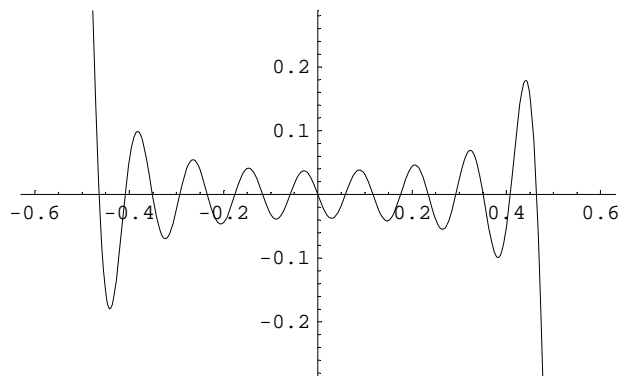
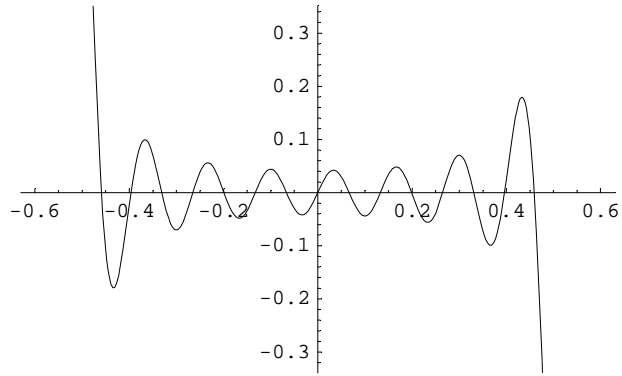
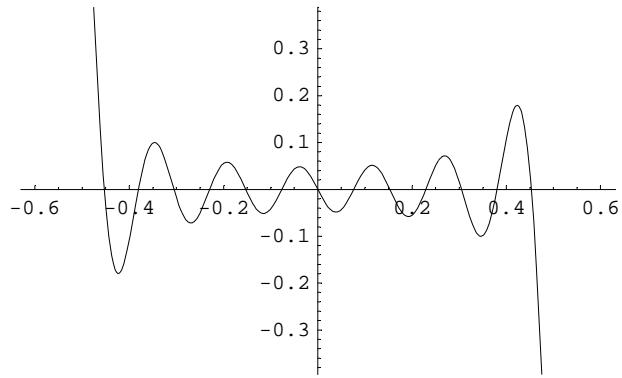


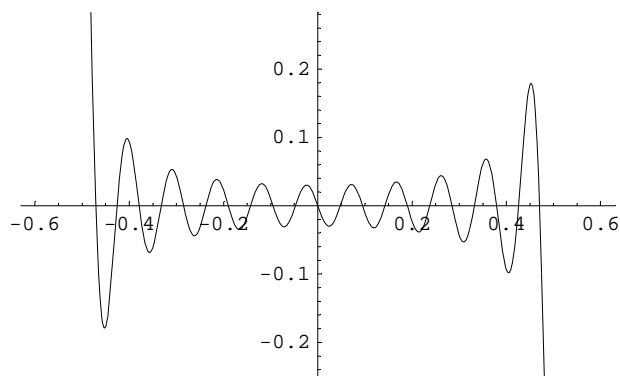
Plot von $f_{\text{Fourier}}[t]-f[t]$:

```
Table[Plot[Evaluate[FourierTrigSeries[f[t], t, n]-f[t]],{t,-0.5,0.5}],{n,1,10}];
```









Overshoot heuristisch (Grösse von $|f_{\text{Fourier}}[t]-f[t]|$):

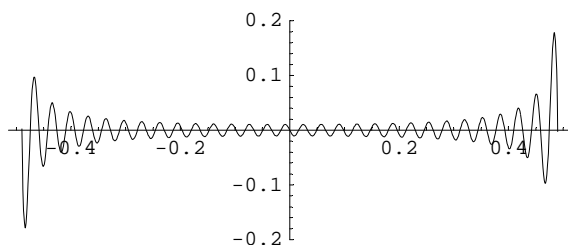
```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 10]-f[t]/.t->u)],{u,0.45}]
{0.179089, {u → 0.452411}}
```

Ca. 17.9 % Overshoot

```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 20]-f[t]/.t->u)],{u,0.47}]
{0.179009, {u → 0.475614}}
```

```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 30]-f[t]/.t->u)],{u,0.48}]
{0.178993, {u → 0.483608}}
```

```
Plot[Evaluate[FourierTrigSeries[f[t], t,
30]-f[t]],{t,-1/2+0.01,1/2-0.01},AspectRatio->Automatic,PlotRange->{-0.2,0.2}];
```



Da hier Die Funktion f über die zulässigen Grenzen hinaus definiert ist, kommt es exakt auf den Grenzen beim Plot zu einem Fehler. Man darf daher nicht bis exakt an die Grenzen plotten.

?*Max*

System`

ConstrainedMax	MaxPlotPoints	\$MaxExtraPrecision
FindMaximum	MaxPoints	\$MaxLicenseProcesses
IndentMaxFraction	MaxRecursion	\$MaxMachineNumber
Max	MaxStepFraction	\$MaxNumber
MaxBend	MaxSteps	\$MaxPiecewiseCases
Maximize	MaxStepSize	\$MaxPrecision
MaxIterations	NMaximize	\$MaxRootDegree
MaxMemoryUsed	SpanMaxSize	

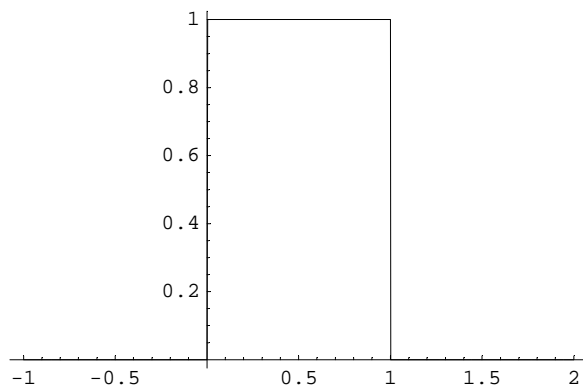
FindMaximum[f, {x, x0}] searches for a local maximum in f, starting from the point x = x0. FindMaximum[f, {{x, x0}, {y, y0}, ...}] searches for a local maximum in a function of several variables. Mehr...

2

```
fFour[Ω_, f_] := 1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)], {λ, -Infinity, Infinity}]
```

a

```
f[λ_ /; λ < 0] := 0;
f[λ_ /; 0 <= λ && λ < 1] := 1;
f[λ_ /; λ >= 1] := 0;
Plot[f[λ], {λ, -1, 2}];
fFour[Ω, f]
```



Integrate::idiv : Integral of $e^{-i\lambda\Omega} \lambda^3$ does not converge on $\{-\infty, \infty\}$. Mehr...

$$\frac{\int_{-\infty}^{\infty} 8 e^{-i\lambda\Omega} \lambda^3 d\lambda}{2\pi}$$

Das System kann nicht mit uneigentlichen komplexen Integralen dieser Art problemlos umgehen. Es bedarf weiterer Interventionen! Problem: Sinus und Cosinus sind an der Stelle unendlich nicht definiert!

```
1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)], {λ, 0, 1}]
```

$$\frac{4 (6 + e^{-i\Omega} (-6 - 6 i \Omega + 3 \Omega^2 + i \Omega^3))}{\pi \Omega^4}$$

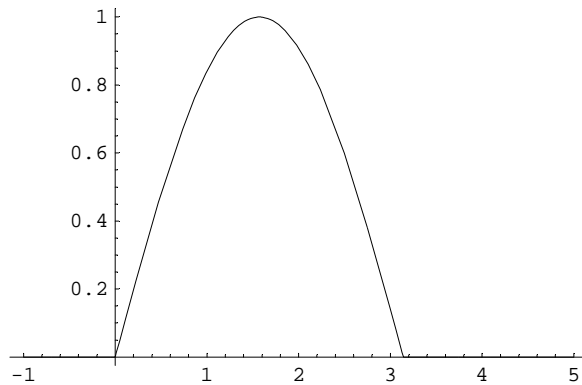

```
Limit[1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,-n,n}],{n->Infinity}]
```

$$\left\{ \text{Limit} \left[\frac{8 i (n \Omega (-6 + n^2 \Omega^2) \text{Cos}[n \Omega] - 3 (-2 + n^2 \Omega^2) \text{Sin}[n \Omega])}{\pi \Omega^4}, n \rightarrow \infty \right] \right\}$$

b

```
Remove["Global`*"]
```

```
f[λ_ /; λ<0]:=0;
f[λ_ /; 0<=λ && λ<Pi]:= Sin[λ];
f[λ_ /; λ>=Pi]:=0;
Plot[f[λ],{λ,-1,5}];
```



```
1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,0,Pi}]
```

$$\frac{\int_0^{\pi} e^{-i \lambda \Omega} f[\lambda] d\lambda}{2 \pi}$$

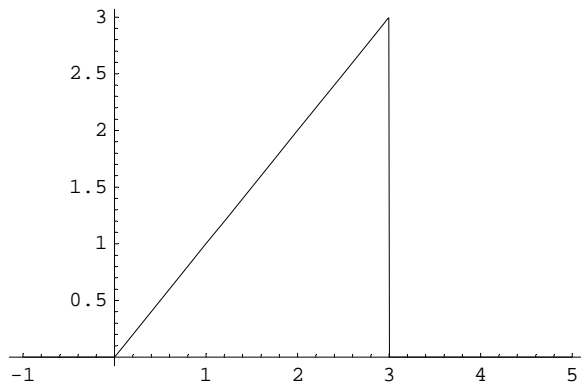
```
1/(2 π) Integrate[Evaluate[Sin[λ] (Cos[λ Ω]+I Sin[λ])],{λ,0,Pi}]
```

$$\frac{\frac{i \pi}{2} + \frac{1 + \text{Cos}[\pi \Omega]}{1 - \Omega^2}}{2 \pi}$$

c

```
Remove["Global`*"]
```

```
f[λ_ /; λ<0]:=0;  
f[λ_ /; 0<=λ && λ<3]:= λ;  
f[λ_ /; λ>=3]:=0;  
Plot[f[λ],{λ,-1,5}];
```



```
1/(2 π) Integrate[Evaluate[λ E^(-I λ Ω)],{λ,0,3}]
```

$$\frac{-1 + e^{-3 i \Omega} (1 + 3 i \Omega)}{2 \pi \Omega^2}$$