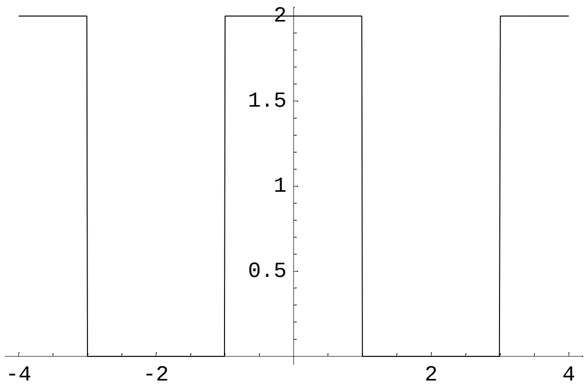
Lösungen

1

a

```
Remove["Global`*"]

f[x_ /; -5<=x && x<-3]:= 2;
f[x_ /; -3<=x && x<-1]:=0;
f[x_ /; -1<=x && x<1]:= 2;
f[x_ /; 1<=x && x<3]:=0;
f[x_ /; 3<=x && x<5]:=2;
Plot[f[t],{t,-4,4}];
```



b

```
a[k_]:=1/2 Integrate[2 Cos[Pi/2 k t],{t,-1,1}];
a[k]
```

$$\frac{4\,\mathrm{Sin}\left[\frac{k\pi}{2}\right]}{k\,\pi}$$

```
b[k_]:=0;
```

```
a[0]
```

2

```
Table[{k, " ", a[k]} // N // Chop, {k, 0, 50}] // TableForm
```

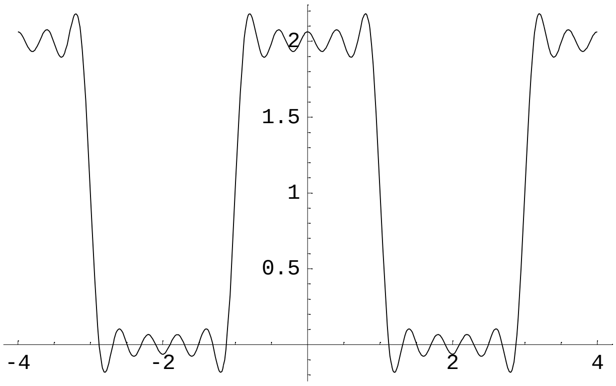
0	2.
1.	1.27324
2.	0
3.	−0.424413
4.	0
5.	0.254648
6.	0
7.	−0.181891
8.	0
9.	0.141471
10.	0
11.	−0.115749
12.	0
13.	0.0979415
14.	0
15.	−0.0848826
16.	0
17.	0.0748964
18.	0
19.	−0.0670126
20.	0
21.	0.0606305
22.	0
23.	−0.0553582
24.	0
25.	0.0509296
26.	0
27.	−0.047157
28.	0
29.	0.0439048
30.	0
31.	−0.0410722
32.	0
33.	0.038583
34.	0
35.	−0.0363783
36.	0
37.	0.0344119
38.	0
39.	−0.0326472
40.	0
41.	0.0310546
42.	0
43.	−0.0296102
44.	0
45.	0.0282942
46.	0
47.	−0.0270902
48.	0
49.	0.0259845
50.	0

```
r[t_,n_]=a[0]/2+Sum[a[k] Cos[Pi/2 k t]+b[k] Sin[Pi/2 k t], {k,1,n}];
r[t,n]//Simplify
```

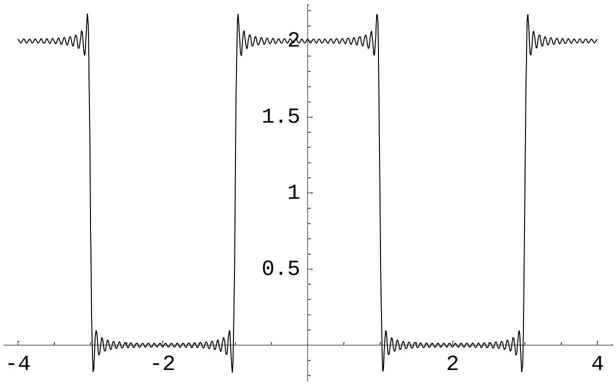
$$\frac{1}{(1+n)\,\pi}\Big(e^{-\frac{1}{2}i\pi t}\Big(-\big(-i\,e^{-\frac{1}{2}i\pi t}\big)^n \text{Hypergeometric2F1}\big[1+n,1,2+n,-i\,e^{-\frac{1}{2}i\pi t}\big]-\big(i\,e^{-\frac{1}{2}i\pi t}\big)^n \text{Hypergeometric2F1}\big[1+n,1,2+n,i\,e^{-\frac{1}{2}i\pi t}\big]+e^{\frac{i\pi t}{2}}\Big(-i\big(-i\,e^{\frac{i\pi t}{2}}\big)^{1+n}\text{Hypergeometric2F1}\big[1+n,1,2+n,-i\,e^{\frac{i\pi t}{2}}\big]+i\big(i\,e^{\frac{i\pi t}{2}}\big)^{1+n}\text{Hypergeometric2F1}\big[1+n,1,2+n,i\,e^{\frac{i\pi t}{2}}\big]+(1+n)\big(\pi+i\,\text{Log}\big[1-i\,e^{-\frac{1}{2}i\pi t}\big]-i\,\text{Log}\big[1+i\,e^{-\frac{1}{2}i\pi t}\big]+i\,\text{Log}\big[1-i\,e^{\frac{i\pi t}{2}}\big]-i\,\text{Log}\big[1+i\,e^{\frac{i\pi t}{2}}\big]\big)\Big)\Big)\Big)$$

c

```
Plot[Evaluate[r[t,10]],{t,-4,4}];
```




```
Plot[Evaluate[r[t,50]],{t,-4,4}];
```



d

Setzt man t = 0, so wird der Cosinus in der Fourierreihe gleich 1. man hat dann

$$\texttt{a[0] / 2 + Sum}\Big[\frac{4\,\text{Sin}[\frac{k\pi}{2}]}{k\,\pi},\ \{k, 1, \text{Infinity}\}\Big]\ \texttt{// N}$$

2. + 0. i

```
(2 - a[0] / 2) Pi / 4
```

$\frac{\pi}{4}$

```
Sum[Sin[k π / 2] / k, {k, 1, n}] // Simplify
```

$\frac{1}{2(1+n)}$ (ⅈ (ⅈ (-ⅈ)^n Hypergeometric2F1[1 + n, 1, 2 + n, -ⅈ] + ⅈ ⅈ^n Hypergeometric2F1[1 + n, 1, 2 + n, ⅈ] + (1 + n) (Log[1 - ⅈ] - Log[1 + ⅈ])))

```
(2 - a[0] / 2) Pi / 4 == (Sum[Sin[k π / 2] / k, {k, 1, Infinity}] // Simplify)
```

$\frac{\pi}{4} == \frac{1}{2}$ ⅈ (Log[1 - ⅈ] - Log[1 + ⅈ])

Damit kann man approximieren.

e

```
Integrate[f[t]^2, {t, -3, 1}] / 4
```

$\frac{1}{4}\int_{-3}^{1} f[t]^2 \, dt$

```
Integrate[2^2, {t, -1, 1}] / 4
```

2

```
(a[0]^2) / 4 + 1 / 2 Sum[a[k]^2, {k, 1, Infinity}]
```

2

f

```
Integrate[f[t]^2, {t, -3, 1}] / 4
```

$\frac{1}{4}\int_{-3}^{1} f[t]^2 \, dt$

```
Integrate[2^2, {t, -1, 1}] / 4
```

2

```
(a[0]^2) / 4 + 1 / 2 Sum[(4 Sin[k π / 2] / (k π))^2, {k, 1, n}]
```

$1 + \frac{1}{\pi^2}\left(8\left(\frac{\pi^2}{8} + \frac{1}{8}\left((-1)^n \text{PolyGamma}\left[1, 1 + \frac{n}{2}\right] - (-1)^n \text{PolyGamma}\left[1, \frac{1+n}{2}\right] - 4\,\text{PolyGamma}[1, 1 + n]\right)\right)\right)$

```
(2 - (a[0]^2) / 4) * 2 * π^2 / 4^2
```

$\frac{\pi^2}{8}$

```
(2 - (a[0]^2) / 4) * 2 * π^2 / 4^2 == Sum[(Sin[kπ/2]/k)^2, {k, 1, n}]
```

$$\frac{\pi^2}{8} == \frac{\pi^2}{8} + \frac{1}{8}\left((-1)^n \text{PolyGamma}\left[1, 1+\frac{n}{2}\right] - (-1)^n \text{PolyGamma}\left[1, \frac{1+n}{2}\right] - 4\,\text{PolyGamma}[1, 1+n]\right)$$

```
Sqrt[8 Sum[(Sin[kπ/2]/k)^2, {k, 1, 1000}]] // N
```

3.14096

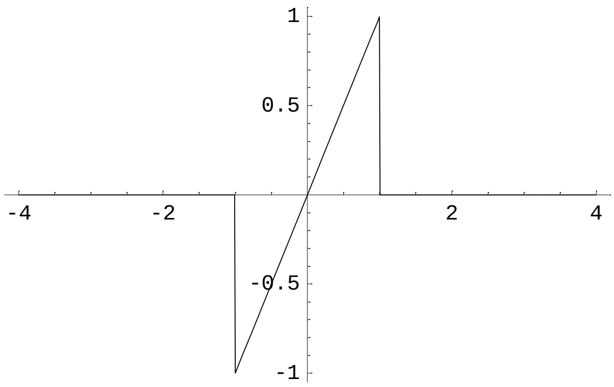
2

a

```
Remove["Global`*"]

f[x_ /; -1<x && x<=1]:= x;
f[x_ /; x<=-1]:=0;
f[x_ /; 1<x]:= 0;

Plot[f[t],{t,-4,4}];
```



b

```
F[Ω_] := 1 / (2 Pi) Integrate[λ E^(-I λ Ω), {λ, -1, 1}]; F[Ω]
```

$$\frac{i\,(\Omega\,\text{Cos}[\Omega] - \text{Sin}[\Omega])}{\pi\,\Omega^2}$$

c

```
Integrate[(Pi / I) F[x] Sin[x], {x, -Infinity, Infinity}]
```

$$-\frac{\pi}{2}$$

```
Integrate[(x Cos[x] - Sin[x])/x^2 Sin[x], {x, -Infinity, Infinity}]
```

$-\frac{\pi}{2}$

```
? *Trig*
```

System`

```
ExpToTrig  TrigExpand  TrigFactorList  TrigToExp
Trig       TrigFactor  TrigReduce
```

```
ExpToTrig[F[Ω] E^(I x Ω) Pi]
```

$\frac{i\,(\Omega\,\mathrm{Cos}[\Omega] - \mathrm{Sin}[\Omega])\,(\mathrm{Cos}[x\,\Omega] + i\,\mathrm{Sin}[x\,\Omega])}{\Omega^2}$

```
-Re[ExpToTrig[F[Ω] E^(I 1 Ω)] Pi] // Simplify
```

$\mathrm{Im}\left[\frac{(\Omega\,\mathrm{Cos}[\Omega] - \mathrm{Sin}[\Omega])\,(\mathrm{Cos}[\Omega] + i\,\mathrm{Sin}[\Omega])}{\Omega^2}\right]$

```
-Integrate[Re[F[Ω] E^(I 1 Ω)] Pi, {Ω, -Infinity, Infinity}]
```

$-\frac{\pi}{2}$

3 Kleinprojekt

Zu einer Kleinprojekt gibt es keine Vorauslösung. Verwende dabei:

```
Integrate[E^(-x^2), {x, -Infinity, Infinity}]
```

$\sqrt{\pi}$