Fourier mit *Mathematica*, Grundversion

A Einführung

```
?*Fourier*
```

System`

```
Fourier              InverseFourier
FourierCosTransform  InverseFourierCosTransform
FourierParameters    InverseFourierSinTransform
FourierSinTransform  InverseFourierTransform
FourierTransform
```

Calculus`FourierTransform`

```
DTFourierTransform           NFourierCosSeriesCoefficient
FourierCoefficient           NFourierCosTransform
FourierCosCoefficient        NFourierExpSeries
FourierCosSeriesCoefficient  NFourierExpSeriesCoefficient
FourierExpSeries             NFourierSeries
FourierExpSeriesCoefficient  NFourierSinCoefficient
FourierFrequencyConstant     NFourierSinSeriesCoefficient
FourierOverallConstant       NFourierSinTransform
FourierSample                NFourierTransform
FourierSeries                NFourierTrigSeries
FourierSinCoefficient        NInverseDTFourierTransform
FourierSinSeriesCoefficient  NInverseFourierCoefficient
FourierTrigSeries            NInverseFourierCosTransform
InverseDTFourierTransform    NInverseFourierSinTransform
InverseFourierCoefficient    NInverseFourierTransform
NDTFourierTransform          $FourierFrequencyConstant
NFourierCoefficient          $FourierOverallConstant
NFourierCosCoefficient
```

Fourier[list] finds the discrete Fourier transform of a list of complex numbers. Mehr…

FourierCosTransform[expr, t, ω] gives the symbolic Fourier cosine transform of expr. FourierCosTransform[expr, {t1, t2, ... }, {ω1, ω2, ... }] gives the multidimensional Fourier cosine transform of expr. Mehr…

"FourierParameters is an option to Fourier transform functions that specifies the convention to follow for the overall constant and the frequency constant. For FourierParameters -> {a, b}, FourierTransform[expr, t, w] is equivalent to Sqrt[Abs[b]/((2 Pi)^(1-a))] Integrate[expr Exp[I b w t], {t,-Infinity,Infinity}]."

FourierSinTransform[expr, t, ω] gives the symbolic Fourier sine transform of expr. FourierSinTransform[expr, {t1, t2, ... }, {ω1, ω2, ... }] gives the multidimensional Fourier sine transform of expr. Mehr…

FourierTransform[expr, t, ω] gives the symbolic Fourier transform of expr. FourierTransform[expr, {t1, t2, ... }, {ω1, ω2, ... }] gives the multidimensional Fourier transform of expr. Mehr...

InverseFourier[list] finds the discrete inverse Fourier transform of a list of complex numbers. Mehr...

InverseFourierCosTransform[expr, ω, t] gives the symbolic inverse Fourier cosine transform of expr. InverseFourierCosTransform[expr, {ω1, ω2, ... }, {t1, t2, ... }] gives the multidimensional inverse Fourier cosine transform of expr. Mehr...

InverseFourierSinTransform[expr, ω, t] gives the symbolic inverse Fourier sine transform of expr. InverseFourierSinTransform[expr, {ω1, ω2, ... }, {t1, t2, ... }] gives the multidimensional inverse Fourier sine transform of expr. Mehr...

InverseFourierTransform[expr, ω, t] gives the symbolic inverse Fourier transform of expr. InverseFourierTransform[expr, {ω1, ω2, ... }, {t1, t2, ... }] gives the multidimensional inverse Fourier transform of expr. Mehr...

```
<< Calculus`FourierTransform`
```

```
?Calculus`FourierTransform`N*
```

Calculus`FourierTransform`

NDTFourierTransform	NFourierSinSeriesCoefficient
NFourierCoefficient	NFourierSinTransform
NFourierCosCoefficient	NFourierTransform
NFourierCosSeriesCoefficient	NFourierTrigSeries
NFourierCosTransform	NInverseDTFourierTransform
NFourierExpSeries	NInverseFourierCoefficient
NFourierExpSeriesCoefficient	NInverseFourierCosTransform
NFourierSeries	NInverseFourierSinTransform
NFourierSinCoefficient	NInverseFourierTransform

```
Remove["Global`*"]
```

```
FourierTransform[y[x], x, ω]
```

FourierTransform[y[x], x, ω]

```
FourierTransform[y'[x], x, ω]
```

-ⅈ ω FourierTransform[y[x], x, ω]

```
FourierTransform[y''[x], x, ω]
```

$-\omega^2$ FourierTransform[y[x], x, ω]

```
FourierTransform[y'''[x], x, ω]
```

ⅈ ω^3 FourierTransform[y[x], x, ω]

```
FourierTransform[y''''[x], x, ω]
```

```
ω^4 FourierTransform[y[x], x, ω]
```

```
FourierTransform[y[x] + D[y[x], x], x, ω] // Simplify
```

```
FourierTransform[y[x] + y′[x], x, ω]
```

```
fourierTransform[y[x], x, ω] := FourierTransform[y[x], x, ω]
```

```
fourierTransform[y1_ + y2__, x, ω] :=
 FourierTransform[y1, x, ω] + FourierTransform[y2, x, ω]
```

```
fourierTransform[y1_ + y2_ + y3_, x, ω] :=
 FourierTransform[y1, x, ω] + FourierTransform[y2, x, ω]
```

```
fourierTransform[y[x] + D[y[x], x], x, ω] // Factor
```

```
(1 - ⅈ ω) FourierTransform[y[x], x, ω]
```

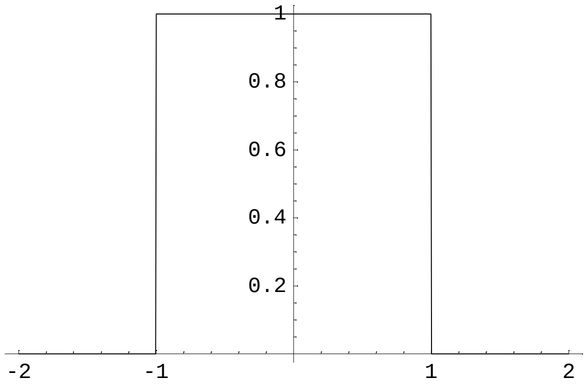
```
fourierTransform[y[x] + 2 D[y[x], x] + D[y[x], {x, 2}], x, ω] // Factor
```

```
(1 - 2 ⅈ ω) FourierTransform[y[x], x, ω]
```

```
? UnitStep
```

```
UnitStep[x] represents the unit step function, equal to 0 for x <
   0 and 1 for x ≥ 0. UnitStep[x1, x2, ... ] represents the multidimensional
   unit step function which is 1 only if none of the xi are negative. Mehr…
```

```
f[t_] := UnitStep[t + 1] - UnitStep[t - 1];
Plot[f[t], {t, -2, 2}];
```




```
FourierTransform[f[x], x, ω] // Factor
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[\omega]}{\omega}$$

B Lösungen

1

```
FourierTransform[y'[x], x, ω]
```

$-i\,\omega$ FourierTransform[y[x], x, ω]

```
FourierTransform[y''[x], x, ω]
```

$-\omega^2$ FourierTransform[y[x], x, ω]

```
FourierTransform[y'''[x], x, ω]
```

$i\,\omega^3$ FourierTransform[y[x], x, ω]

```
FourierTransform[y''''[x], x, ω]
```

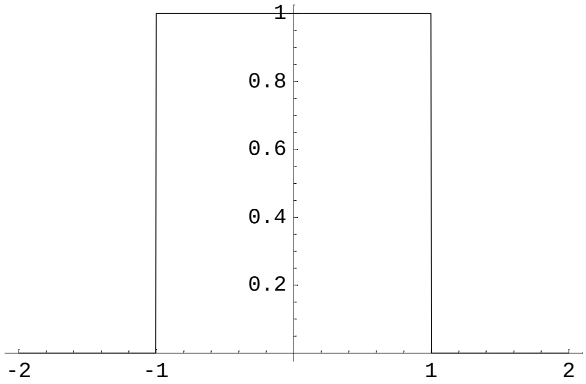
ω^4 FourierTransform[y[x], x, ω]

2

a

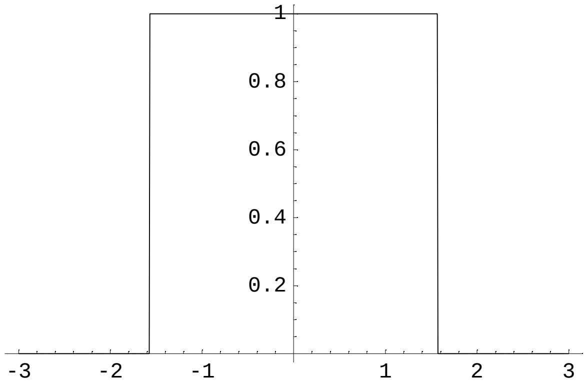
Sei $c = 1$

```
f1[t_] := UnitStep[t + 1] - UnitStep[t - 1];
Plot[f1[t], {t, -2, 2}];
```



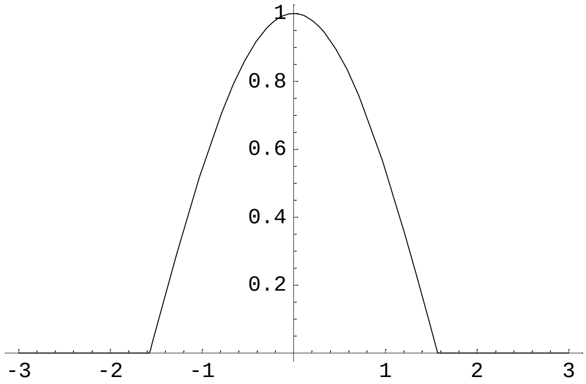
b

```
f2[t_] := f1[2 t / Pi];
Plot[f2[t], {t, -3, 3}];
```

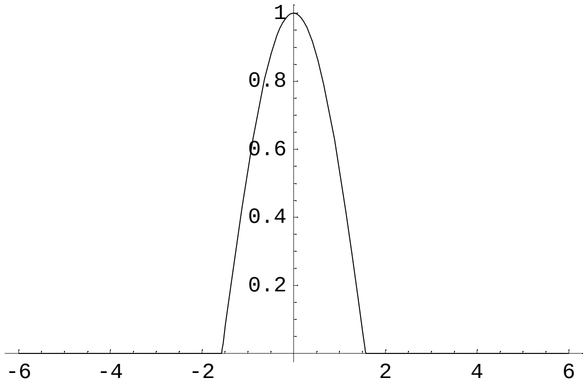


c

```
f3[t_] := f2[t] Cos[t];
Plot[f3[t], {t, -3, 3}];
```



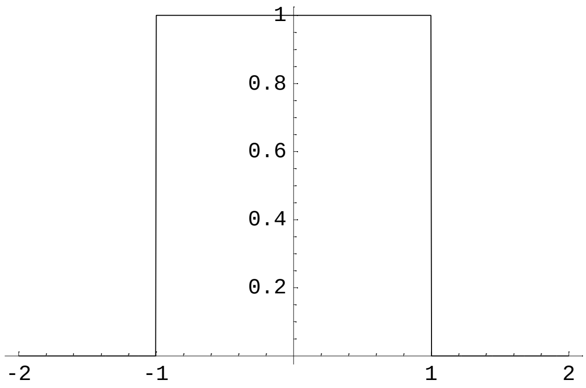

```
Plot[f3[t], {t, -6, 6}];
```



3

Sei c = 1

```
f1[t_] := UnitStep[t + 1] - UnitStep[t - 1];
Plot[f1[t], {t, -2, 2}];
```



a

```
fourierTransform[1 * y'[x] + 2 * y[x], x, ω]
```

$-i\,\omega$ FourierTransform[y[x], x, ω] + FourierTransform[2 y[x], x, ω]

```
links = (-i ω FourierTransform[y[x], x, ω] + 2 FourierTransform[ y[x], x, ω] // Factor) /.
  FourierTransform[ y[x], x, ω] → Y[ω]
```

$(2 - i\,\omega)\, Y[\omega]$

```
rechts = FourierTransform[f1[x], x, ω]
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[\omega]}{\omega}$$

```
solve1 = Solve[links == rechts, {Y[ω]}] // Flatten
```

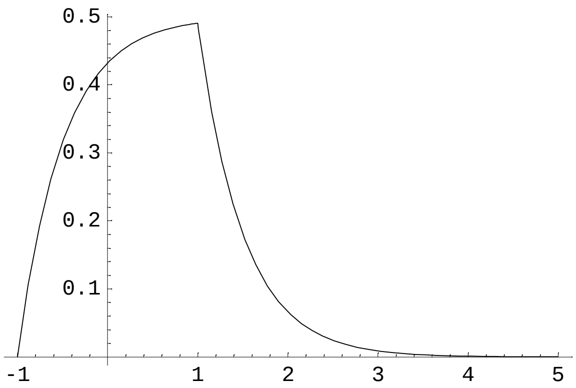
$$\left\{Y[\omega] \to \frac{i\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[\omega]}{\omega\,(2\,i + \omega)}\right\}$$

```
Y1[ω] := Y[ω] /. solve1
```

```
I1[x_] := InverseFourierTransform[Y1[ω], ω, x] // Simplify;
I1[x]
```

$$-\frac{1}{4}\,(\mathrm{Cosh}[2\,(1+x)] - \mathrm{Sinh}[2\,(1+x)])\,(\mathrm{Sign}[-1+x]\,(\mathrm{Cosh}[2\,(1+x)] + \mathrm{Sinh}[2\,(1+x)]) - \mathrm{Sign}[1+x]\,(\mathrm{Cosh}[2\,(1+x)] + \mathrm{Sinh}[2\,(1+x)]) - 2\,e^4\,\mathrm{UnitStep}[-1+x] + 2\,\mathrm{UnitStep}[1+x])$$

```
Plot[I1[x], {x, -1, 5}];
```



b

```
fourierTransform[1 * y'[x] + 1 / 2 * y[x], x, ω]
```

FourierTransform$\left[\frac{y[x]}{2}, x, \omega\right] - i\,\omega$ FourierTransform$[y[x], x, \omega]$

```
links =
 (-i ω FourierTransform[y[x], x, ω] + 1 / 2 FourierTransform[ y[x], x, ω] // Factor) /.
  FourierTransform[ y[x], x, ω] → Y[ω]
```

$\frac{1}{2}\,(1 - 2\,i\,\omega)\,Y[\omega]$

```
rechts = FourierTransform[f1[x] / 2, x, ω]
```

$\frac{\text{Sin}[\omega]}{\sqrt{2\pi}\,\omega}$

```
solve2 = Solve[links == rechts, {Y[ω]}] // Flatten
```

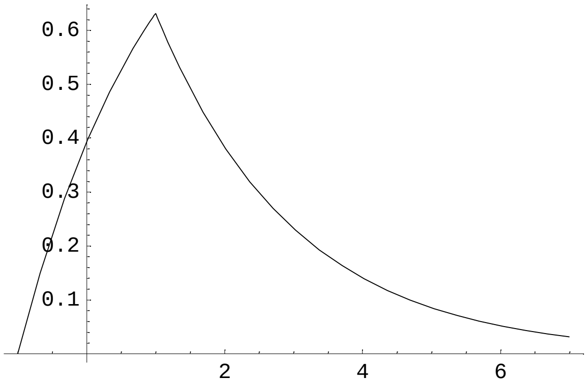
$\left\{Y[\omega] \to \frac{\sqrt{\frac{2}{\pi}}\,\omega\,\text{Sin}[\omega] + 2\,i\,\sqrt{\frac{2}{\pi}}\,\omega^2\,\text{Sin}[\omega]}{\omega^2 + 4\,\omega^4}\right\}$

```
Y2[ω] := Y[ω] /. solve2
```

```
I2[x_] := InverseFourierTransform[Y2[ω], ω, x] // Simplify;
I2[x]
```

$$\frac{1}{2}\Big(\text{Cosh}\left[\tfrac{1}{2}\,\text{Abs}[-1+x]\right] - \text{Cosh}\left[\tfrac{1}{2}\,\text{Abs}[1+x]\right] + \text{Sign}[1-x] + \text{Sign}[1+x] - \text{Sinh}\left[\tfrac{1}{2}\,\text{Abs}[-1+x]\right] + \text{Sinh}\left[\tfrac{1}{2}\,\text{Abs}[1+x]\right] + \text{Cosh}\left[\tfrac{1+x}{2}\right]\text{UnitStep}[-1-x] + \text{Sinh}\left[\tfrac{1+x}{2}\right]\text{UnitStep}[-1-x] - \text{Cosh}\left[\tfrac{1}{2}\,(-1+x)\right]\text{UnitStep}[1-x] - \text{Sinh}\left[\tfrac{1}{2}\,(-1+x)\right]\text{UnitStep}[1-x] + \text{Cosh}\left[\tfrac{1}{2}-\tfrac{x}{2}\right]\text{UnitStep}[-1+x] + \text{Sinh}\left[\tfrac{1}{2}-\tfrac{x}{2}\right]\text{UnitStep}[-1+x] - \text{Cosh}\left[\tfrac{1+x}{2}\right]\text{UnitStep}[1+x] + \text{Sinh}\left[\tfrac{1+x}{2}\right]\text{UnitStep}[1+x]\Big)$$

```
Plot[I2[x], {x, -1, 7}];
```



c

```
fourierTransform[1 * y'[x] - 1 * y[x], x, ω]
```

FourierTransform[-y[x], x, ω] - ⅈ ω FourierTransform[y[x], x, ω]

```
links = (-ⅈ ω FourierTransform[y[x], x, ω] - FourierTransform[ y[x], x, ω] // Factor) /.
  FourierTransform[ y[x], x, ω] → Y[ω]
```

-ⅈ (-ⅈ + ω) Y[ω]

```
rechts = FourierTransform[f1[x], x, ω]
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \text{Sin}[\omega]}{\omega}$$

```
solve3 = Solve[links == rechts, {Y[ω]}] // Flatten
```

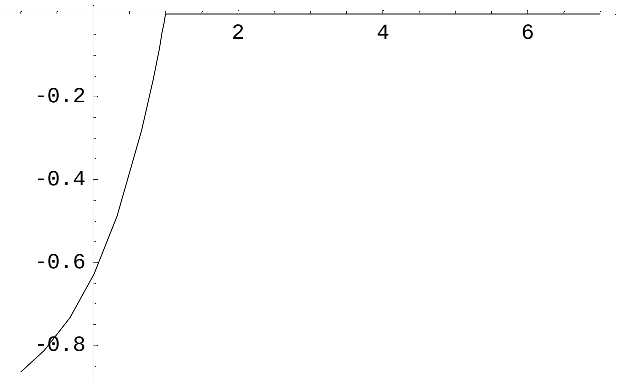
$$\left\{Y[\omega] \to \frac{i \sqrt{\frac{2}{\pi}}\ \text{Sin}[\omega]}{\omega\ (-i + \omega)}\right\}$$

```
Y3[ω] := Y[ω] /. solve3
```

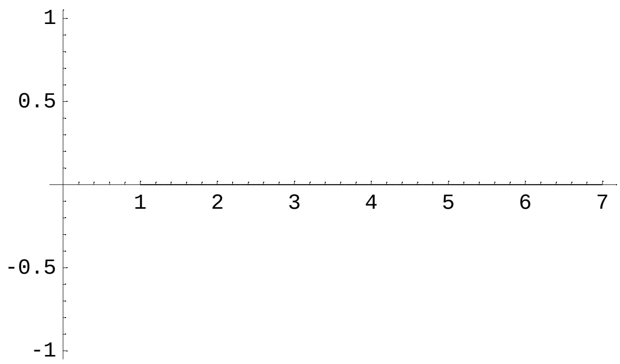
```
I3[x_] := InverseFourierTransform[Y3[ω], ω, x] // Simplify;
I3[x]
```

$$\frac{1}{2} \left(-\text{Sign}[1-x] - \frac{1}{e} \left(e\ \text{Sign}[1+x] + 2\ (\text{Cosh}[x] + \text{Sinh}[x])\ (e^2\ \text{UnitStep}[-1-x] - \text{UnitStep}[1-x])\right)\right)$$

```
Plot[I3[x], {x, -1, 7}];
```




```
Plot[I3[x], {x, 1, 7}];
```



d

```
FourierTransform[y''[x], x, ω] +
 2 FourierTransform[y'[x], x, ω] + FourierTransform[y[x], x, ω]
```

```
(FourierTransform[y''[x], x, ω] +
   2 FourierTransform[y'[x], x, ω] + FourierTransform[y[x], x, ω]) // Factor
```

$-(i+\omega)^2$ FourierTransform[y[x], x, ω]

```
links = (-(i + ω)^2 FourierTransform[y[x], x, ω]) /. FourierTransform[ y[x], x, ω] → Y[ω]
```

$-(i+\omega)^2\, Y[\omega]$

```
rechts = FourierTransform[f1[x], x, ω]
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[\omega]}{\omega}$$

```
solve4 = Solve[links == rechts, {Y[ω]}] // Flatten
```

$$\left\{Y[\omega] \to -\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[\omega]}{\omega\ (i+\omega)^2}\right\}$$

```
Y4[ω] := Y[ω] /. solve4
```

```
I4[x_] := InverseFourierTransform[Y4[ω], ω, x] // Simplify;
I4[x]
```

$$-\frac{1}{2}(\text{Cosh}[1+x]-\text{Sinh}[1+x])\,(e\,\text{Sign}[-1+x](\text{Cosh}[x]+\text{Sinh}[x])-e\,\text{Sign}[1+x](\text{Cosh}[x]+\text{Sinh}[x])-2\,e^2\,x\,\text{UnitStep}[-1+x]+4\,\text{UnitStep}[1+x]+2\,x\,\text{UnitStep}[1+x])$$

```
Plot[I4[x], {x, -1, 7}];
```