Lösungen

1

1a

```
pA={x,0};
pB={0,1};
pC={0,0};
pE= 1/2 {-1,1};
pD= 1/2 {x,-x};
pF1=pA+{1,x};
pF=pB+ 1/2 (pF1-pB) //Simplify
```

$\left\{\frac{1+x}{2}, \frac{1+x}{2}\right\}$

```
pE-pD
```

$\left\{-\frac{1}{2}-\frac{x}{2}, \frac{1}{2}+\frac{x}{2}\right\}$

```
(pE-pD).(pF-pC)//Expand
```

0

Rechtwinklig

1b

```
Norm[pE-pD]//Simplify
```

$\frac{\text{Abs}[1+x]}{\sqrt{2}}$

```
Norm[pF-pC]
```

$\frac{\text{Abs}[1+x]}{\sqrt{2}}$

```
(Norm[pE-pD]//Simplify) == (Norm[pF-pC]//Simplify)
```

True

2

2 a

```
A={{1,2,3},{3,2,1},{1,3,2}}; A//MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

```
Det[A]
```

12

2 b

```
B={{0,0,1,2,3},{1,0,u,v,w},{0,0,3,2,1},{0,2,g,h,j},{0,0,1,3,2}}; B//MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 3 \\ 1 & 0 & u & v & w \\ 0 & 0 & 3 & 2 & 1 \\ 0 & 2 & g & h & j \\ 0 & 0 & 1 & 3 & 2 \end{pmatrix}$$

```
Det[B]
```

−24

2 c

```
Remove[a,b,c]

v={r,s,t}; w={a,b,c};
M={w,w+v,w+2v}; Det[M]
```

0

Zeilen linear abhängig, da Differenz von Zeilen linear abhängig, nach den Elementarsubstitutionen!

3

```
pA={12,10,0};
pB={9,7,12};
pC={-2,2,8};
```

3 a

```
(pB-pA).(pB-pC)
```

0

==> rechtwinklig

```
Norm[(pB-pA)]
```

$9\sqrt{2}$

```
Norm[(pB-pA)]//N
```

12.7279

```
Norm[(pB-pA)]==Norm[(pB-pC)]
```

True

3 b

```
pD = pA+(pC-pB)
```

{1, 5, -4}

3 c

```
pM = pA+1/2 (pC-pA)
```

{5, 6, 4}

```
g[t_]:= pM+t Cross[pB-pA,pB-pC]; g[t]
```

{5 - 72 t, 6 + 144 t, 4 + 18 t}

```
solv1=Solve[Det[{pB-pA,pB-pC,g[t]-pM}]/3==1944,{t}]//Flatten
```

$\left\{t \to \frac{2}{9}\right\}$

```
%//N
```

{t → 0.222222}

```
pS1=g[t]/.solv1
```

{-11, 38, 8}

```
solv2=Solve[Det[{pB-pA,pB-pC,g[t]-pM}]/3==-1944,{t}]//Flatten
```

$\left\{t \to -\frac{2}{9}\right\}$

```
%//N
```

{t → -0.222222}

```
pS1=g[t]/.solv2
```

{21, -26, 0}

4

```
pA = {-1,9,8}; pB = {1,10,10}; pC = {-5,5,8};

k[x_,y_,z_]:= x^2+y^2+z^2-2z-8;
k[{x_,y_,z_}]:= k[x,y,z];
ϕ[λ_,μ_]:= pA+ λ (pB-pA) + μ (pC-pA);
pM = {0,0,1};
r=3;

k[x,y,z] == (x-0)^2 + (y-0)^2 + (z-1)^2 -r^2//ExpandAll
```

True

```
gϕ[t_]:= pM + t Cross[(pB-pA),(pC-pA)]
```

4 a

```
solv1 = Solve[ϕ[λ,μ]==gϕ[t],{λ,μ,t}]//Flatten
```

$\left\{\lambda \to -2,\ \mu \to \frac{1}{4},\ t \to -\frac{3}{4}\right\}$

```
pS0 = gϕ[t]/.solv1
```

{-6, 6, 4}

```
%//N
```

{-6., 6., 4.}

```
k[gϕ[t]]==0
```

$-8 - 2\,(1 - 4\,t) + (1 - 4\,t)^2 + 128\,t^2 == 0$

```
solv2=Solve[k[gϕ[t]]==0,{t}]//Flatten
```

$\left\{t \to -\frac{1}{4},\ t \to \frac{1}{4}\right\}$

```
%//N
```

{t → -0.25, t → 0.25}

```
pS1 = gϕ[t]/.solv2[[1]]
```

{-2, 2, 2}

```
pS2 = gϕ[t]/.solv2[[2]]
```

{2, -2, 0}

```
%//N
```

$\{2., -2., 0.\}$

4 b

```
Norm[pS0-pS1]
```

6

```
%//N
```

6.

```
Norm[pS0-pS2]
```

12

```
%//N
```

12.

4 c

```
gMAB[t_]:= pM + t r (pB-pA)/Norm[(pB-pA)]
```

```
gMAB[1]
```

$\{2, 1, 3\}$

```
gMAB[-1]
```

$\{-2, -1, -1\}$

```
gAB[t_]:= pA + t (pB-pA)
```

```
Simplify[(gAB[t]-gMAB[1]).(gMAB[1]-gMAB[-1])]==0
```

$6\,(4+3\,t) == 0$

```
solv1=Solve[(gAB[t]-gMAB[1]).(gMAB[1]-gMAB[-1])==0,{t}]//Flatten
```

$\left\{t \to -\frac{4}{3}\right\}$

```
%//N
```

$\{t \to -1.33333\}$

```
pP1 = gAB[t]/.solv1
```

$\left\{-\frac{11}{3}, \frac{23}{3}, \frac{16}{3}\right\}$

```
%//N
```

$\{-3.66667, 7.66667, 5.33333\}$

```
solv2=Solve[(gAB[t]-gMAB[-1]).(gMAB[1]-gMAB[-1])==0,{t}]//Flatten
```

$\left\{t \to -\frac{10}{3}\right\}$

```
%//N
```

{t → -3.33333}

```
pP2 = gAB[t]/.solv2
```

$\left\{-\frac{23}{3}, \frac{17}{3}, \frac{4}{3}\right\}$

```
%//N
```

{-7.66667, 5.66667, 1.33333}

4 d

```
pM = {0,0,1};
r=3;
```

4 e

```
Norm[Cross[pA,pM]]/2
```

$\sqrt{\frac{41}{2}}$

```
Norm[Cross[pA,pM]]/2 //N
```

4.52769