Lösungen

1

a

```
D[5 x^5+4 x^4 +3 x^3+2 x^2+x+1+c+x^(-1),x]
```

$1 - \frac{1}{x^2} + 4\,x + 9\,x^2 + 16\,x^3 + 25\,x^4$

b

```
Sgn[x] = const
```

const

```
D[Tan[x]+E^x+Log[x Sgn[x]],x]
```

$e^x + \frac{1}{x} + \mathrm{Sec}[x]^2$

c

```
D[Sin[x]+Log[pi x] x^2,x]
```

$x + \mathrm{Cos}[x] + 2\,x\,\mathrm{Log}[\mathrm{pi}\,x]$

d

```
D[E^x/x-Cos[x] Log[x],x]
```

$-\frac{e^x}{x^2} + \frac{e^x}{x} - \frac{\mathrm{Cos}[x]}{x} + \mathrm{Log}[x]\,\mathrm{Sin}[x]$

e

```
D[Sin[3 E^x]+2 E^(-x^3),x]
```

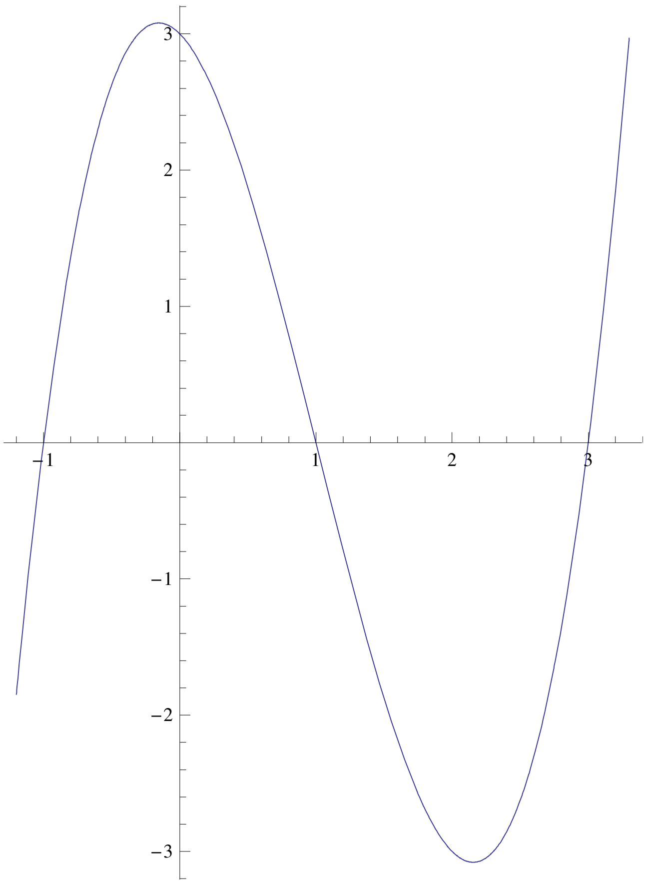
$-6\,e^{-x^3}\,x^2 + 3\,e^x\,\mathrm{Cos}[3\,e^x]$

2

```
f[x_]:=(x-3)(x-1)(x+1)//Expand; f[x]
```

$3 - x - 3\,x^2 + x^3$

```
Plot[f[x], {x, -1.2, 3.3}, AspectRatio → Automatic]
```



a

```
Solve[f[x] == 0, {x}]
```

```
{{x → -1}, {x → 1}, {x → 3}}
```

b

```
Factor[f[x]]
```

```
(-3 + x) (-1 + x) (1 + x)
```

c

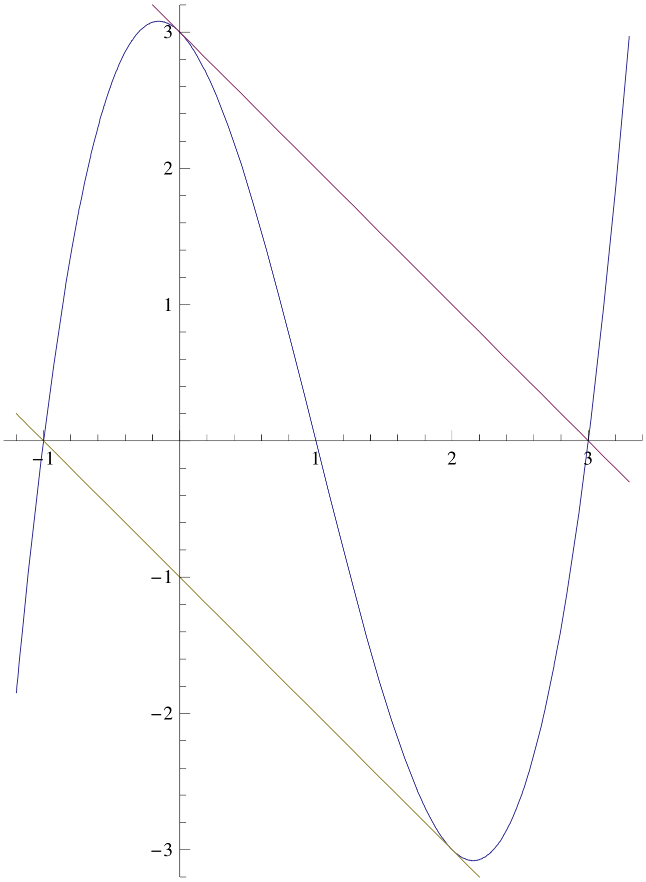
```
winkelGrad[x_] := N[ArcTan[D[f[u], u] /. u → x] / Degree]
winkelGrad[0]
```

```
-45.
```

```
winkelGrad[2]
```

```
-45.
```

```
plot1 = Plot[{f[x], (x - 0) Evaluate[(D[f[u], u] /. u → 0)] + f[0],
   (x - 2) Evaluate[(D[f[u], u] /. u → 2)] + f[2]},
  {x, -1.2, 3.3}, AspectRatio → Automatic, PlotRange → {-3.2, 3.2}]
```



■ d Minimum bei x1, Minimum bei x2

```
f'[x]
```

$-1 - 6x + 3x^2$

```
solv = Solve[Evaluate[f'[x] == 0], {x}] // Flatten
```

$\left\{x \to \frac{1}{3}\left(3 - 2\sqrt{3}\right),\ x \to \frac{1}{3}\left(3 + 2\sqrt{3}\right)\right\}$

```
N[%]
```

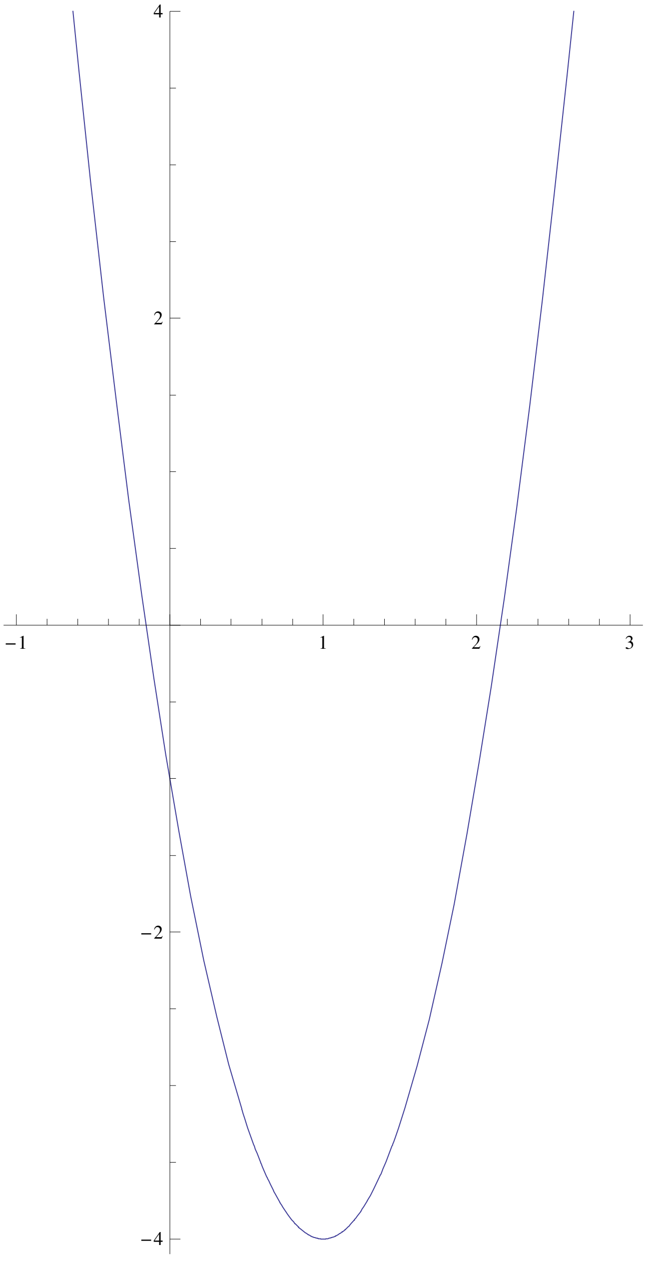
{x → -0.154701, x → 2.1547}

```
x1 = x /. solv[[1]]; x2 = x /. solv[[2]]; Print["x1 = ", x1 // N, " // ", "x2 = ", x2 // N]
```

x1 = -0.154701 // x2 = 2.1547

■ e Monotoniebereiche

```
plot2 = Plot[Evaluate[f'[u] /. u -> x],
  {x, -1, 3}, AspectRatio → Automatic, PlotRange → {-4.1, 4}]
```



Monoton wachsend bis x1 ungef. -0.154701 (Ableitung positiv), dann fallend bis x2 ungef 2.1547 (Ableitungnegativ), dann wieder wachsend (Ableitung positiv).

■ f Wendepunkt

```
f''[x]
```

```
-6 + 6 x
```

```
solv1 = Solve[Evaluate[f''[x] == 0], {x}] // Flatten;  x3 = x /. solv1
```

```
1
```

```
Print["x3 = ", x3]
```

x3 = 1

```
{x2, f[x3]}
```

$\left\{\frac{1}{3}\left(3+2\sqrt{3}\right),\ 0\right\}$

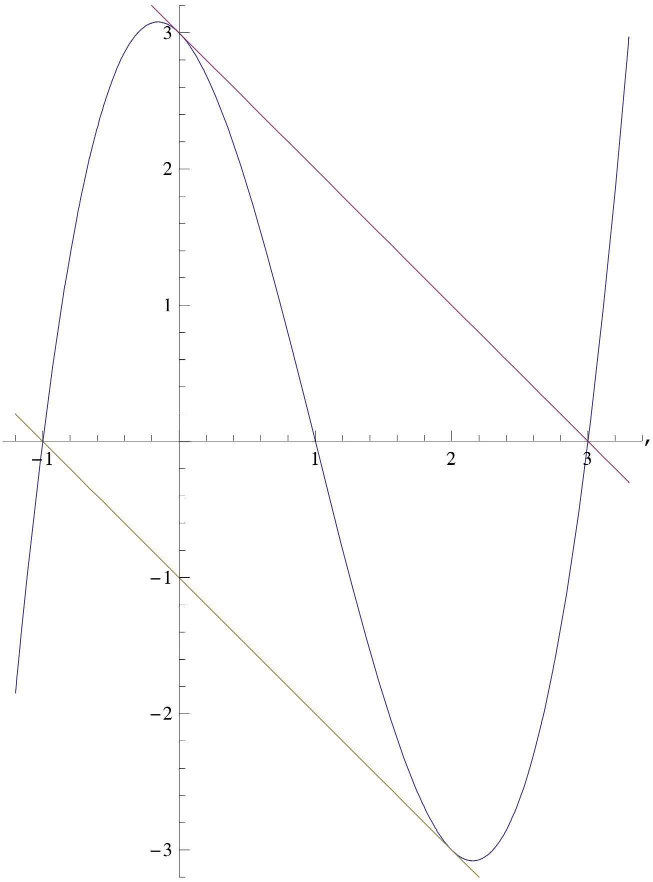
```
N[%]
```

{2.1547, 0.}

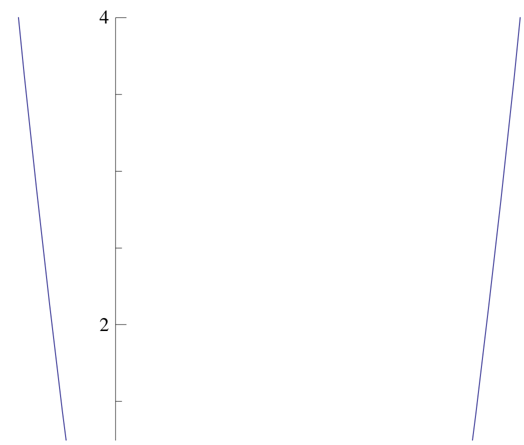
■ e Graphen

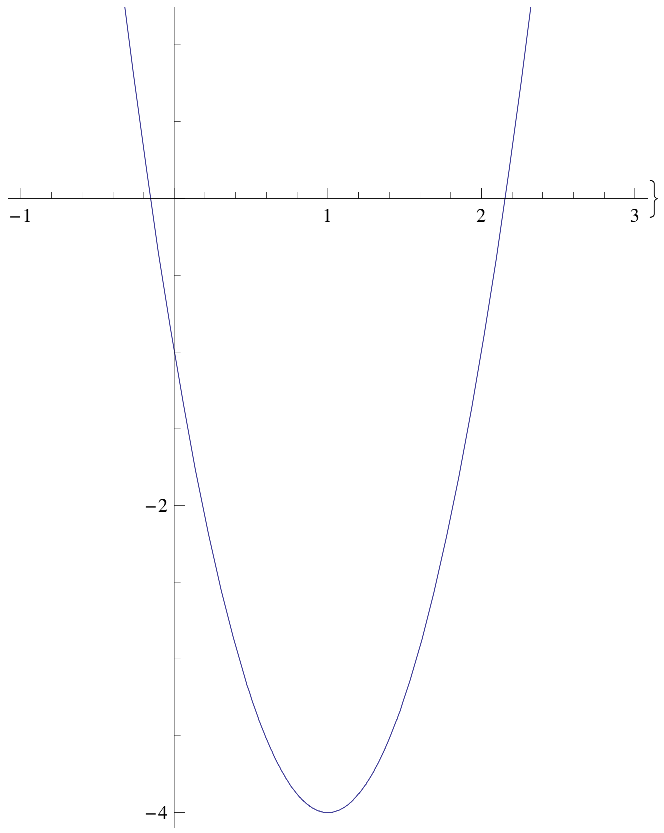
```
{plot1, plot2}
```

{

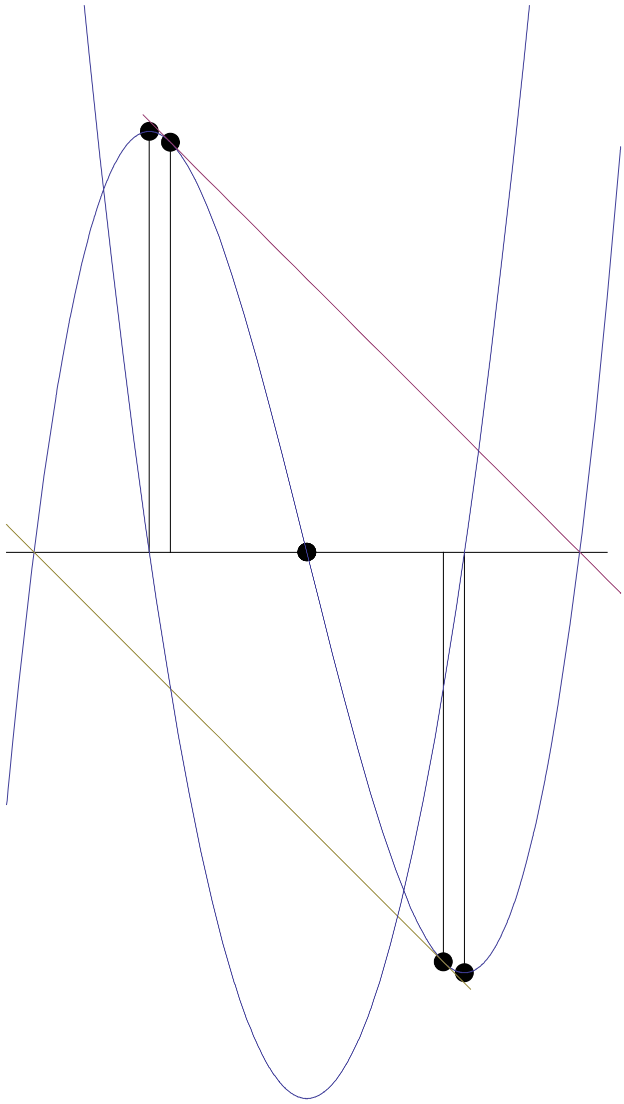


,






```
Show[Graphics[{PointSize[0.03], Point[{x3, f[x3]}], Point[{x1, f[x1]}],
   Point[{x2, f[x2]}], Point[{0, f[0]}], Point[{2, f[2]}], Line[{{x1, 0}, {x1, f[x1]}}],
   Line[{{x2, 0}, {x2, f[x2]}}], Line[{{-1.2, 0}, {3.2, 0}}], Line[{{2, 0}, {2, f[2]}}],
   Line[{{0, f[0]}, {0, 0}}]}], plot1, plot2, PlotRange → {-4.1, 4}]
```



3

```
Remove["Global`*"]
```

a

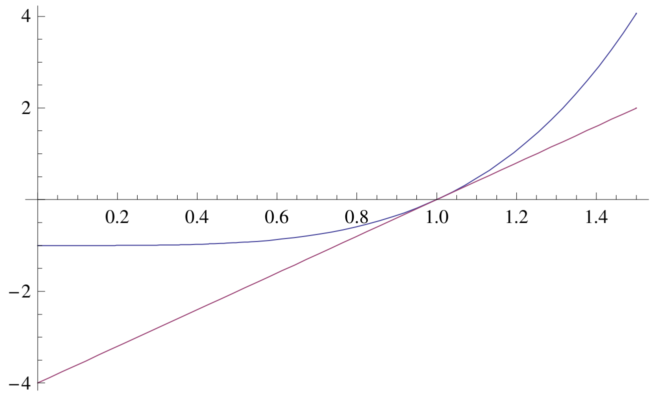
```
f[x_] := x^4 - 1; f[x]
```

$-1 + x^4$

```
x0 = 1;
fLin[x_] := f[x0] + Evaluate[f'[u] /. u → x0] (x - x0);  fLin[x] // Expand
```

$-4 + 4 x$

```
Plot[{f[x], fLin[x]}, {x, 0, 1.5}]
```



■ b

```
FehlerProzent = f[1.1] - fLin[1.1]
```

```
0.0641
```

4

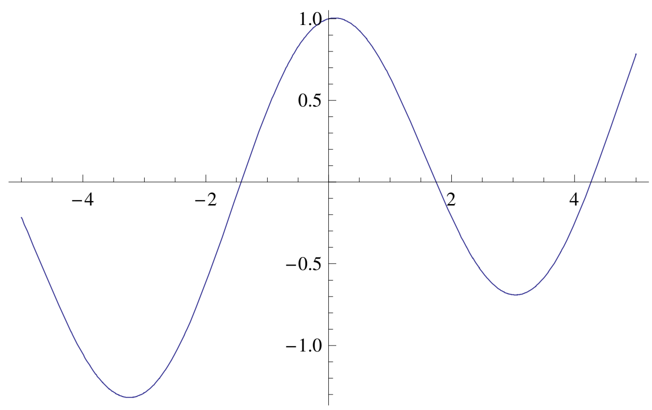
```
Remove["Global`*"]
```

■ a

```
f[x_] := 0.1 x + Cos[x]; f[x]
```

```
0.1 x + Cos[x]
```

```
Plot[f[x], {x, -5, 5}]
```




```
FindRoot[f[x]==0,{x,1.5}]
```

```
{x → 1.74633}
```

■ b

```
x[1] = 1.5;
x[n_] := x[n - 1] - f[x[n - 1]] / Evaluate[D[f[x], x] /. x -> x[n - 1]];
```

```
x[3]
```

```
1.74633
```

```
1.7457848640344924`
```

```
1.74578
```

```
Table[{"x", n, " = ", x[n]}, {n, 1, 5}] // MatrixForm
```

x	1	=	1.5
x	2	=	1.74595
x	3	=	1.74633
x	4	=	1.74633
x	5	=	1.74633

```
(* Ein Klick auf den Output ergibt weitere Ziffern: *)
```

"x"	1	" = "	1.5`
"x"	2	" = "	1.7459481166607174`
"x"	3	" = "	1.7463292679341147`
"x"	4	" = "	1.7463292822528527`
"x"	5	" = "	1.7463292822528529`