Lösungen

1

a

```
Integrate[(x^3-2x^2) Log[x],x]
```

$$\frac{2\,x^3}{9} - \frac{x^4}{16} - \frac{2}{3}\,x^3\,\mathrm{Log}[x] + \frac{1}{4}\,x^4\,\mathrm{Log}[x]$$

b

```
Integrate[a/((x+1)(x+3)),{x,0,1}]
```

$$\frac{1}{2}\,a\,\mathrm{Log}\left[\frac{3}{2}\right]$$

$$\mathbf{Solve}\left[\frac{1}{2}\,a\,\mathrm{Log}\left[\frac{3}{2}\right] == 1,\ \{a\}\right]$$

$$\left\{\left\{a \to \frac{2}{\mathrm{Log}\left[\frac{3}{2}\right]}\right\}\right\}$$

```
N[%]
```

$\{\{a \to 4.93261\}\}$

c

```
Integrate[(x+2)x(x-3)+c x,{x,-2,3}]
```

$$-\frac{125}{12} + \frac{5\,c}{2}$$

$$\mathbf{Solve}\left[-\frac{125}{12} + \frac{5\,c}{2} == 0,\ \{c\}\right]$$

$$\left\{\left\{c \to \frac{25}{6}\right\}\right\}$$

```
N[%]
```

$\{\{c \to 4.16667\}\}$

d

```
Integrate[Cos[x] E^Sin[x]-1/(2 Sqrt[x]),{x,1,3}]
```

$1 - \sqrt{3} - e^{\text{Sin}[1]} + e^{\text{Sin}[3]}$

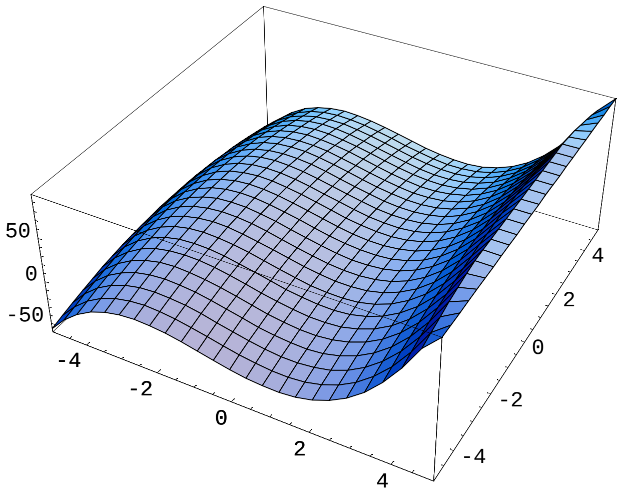
```
N[%]
```

-1.90026

```
D[E^(Sin[x])-Sqrt[x],x]
```

$-\frac{1}{2\sqrt{x}} + e^{\text{Sin}[x]}\,\text{Cos}[x]$

```
Plot3D[2+x(x-2)(x+4)-(y^2-1),{x,-5,5},{y,-5,5}];
```



2

```
Remove["Global`*"]
```

```
Normal[Series[E^(t^2)-1,{t,0,10}]]
```

$t^2 + \frac{t^4}{2} + \frac{t^6}{6} + \frac{t^8}{24} + \frac{t^{10}}{120}$

```
Integrate[Normal[Series[E^(t^2)-1,{t,0,10}]],{t,0,x}]
```

$\frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \frac{x^9}{216} + \frac{x^{11}}{1320}$

3

```
Remove["Global`*"]
```

```
Solve[(8/9)^3 x==1,{x}]
```

$\{\{x \to \frac{729}{512}\}\}$

```
N[%]
```

$\{\{x \to 1.42383\}\}$

|x| < 1.42383

4

```
Remove["Global`*"]
```

a

Tangentensteigung in diese Richtung

b

Vektor in Richtung der grössten Steigung, Betrag = grösste Steigung.

c

```
f[x_,y_]:=2+x(x-2)(x+4)-(y^2-1);
OP0={1,2}; OQ0={4,-4};
e=(OQ0-OP0) 1/Norm[OQ0-OP0]
```

$\{\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\}$

```
N[%]
```

$\{0.447214, -0.894427\}$

```
f[x,y]//Expand
```

$3 - 8 x + 2 x^2 + x^3 - y^2$

```
grad[f_]:={D[f,x],D[f,y]};
grad[f[x,y]]
```

$\{(-2 + x) x + (-2 + x) (4 + x) + x (4 + x), -2 y\}$

```
%//ExpandAll
```

$\{-8 + 4 x + 3 x^2, -2 y\}$

```
grad[f[x,y]].e /.{x->1,y->2}
```

$\frac{7}{\sqrt{5}}$

```
N[%]
```

3.1305

d

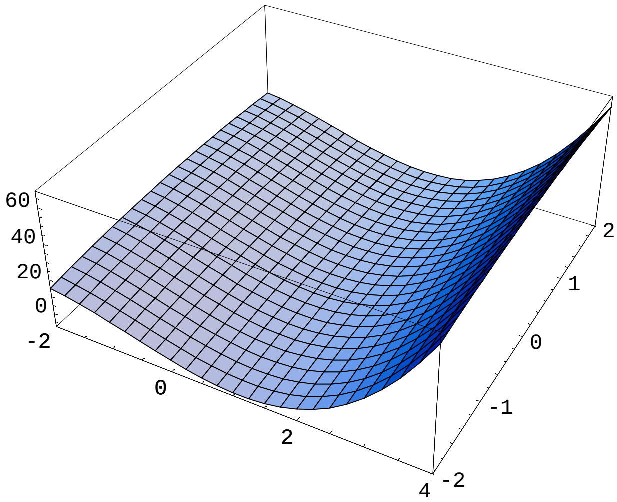
```
Solve[grad[f[x,y]]=={0,0},{x,y}]
```

$\{\{y \to 0, x \to \frac{2}{3}(-1-\sqrt{7})\}, \{y \to 0, x \to \frac{2}{3}(-1+\sqrt{7})\}\}$

```
N[%]
```

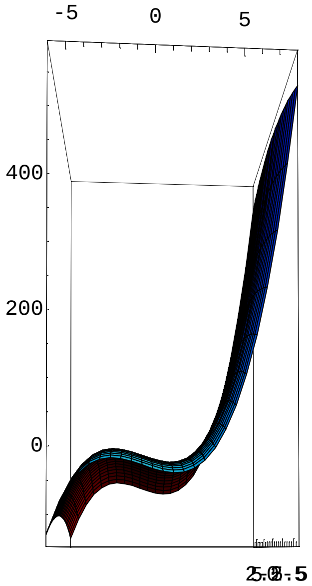
$\{\{y \to 0., x \to -2.4305\}, \{y \to 0., x \to 1.09717\}\}$

```
Plot3D[f[x,y],{x,-2,4},{y,-2,2}];
```



Kein Minimum, Maximum und Sattel.
In y-Richtung Parabel nach oben geöffnet, also mit Maximum.
In x-Richtung Parabel 3. Ordnung mit Maximum uns Minimum

```
Plot3D[f[x,y],{x,-6,8},{y,-6,6},ViewPoint->{-0.155, -3.096,
-0.202},AspectRatio->2];
```



5

```
Remove["Global`*"]
```

a

```
DSolve[{y''[x] +y[x]==0,y[0]==2,y'[0]==1},y[x],x]
```

{{y[x] → 2 Cos[x] + Sin[x]}}

b

```
DSolve[y'[x] (x+3) == (x)/((y[x])^3),y[x],x]
```

$\{\{y[x] \to -\sqrt{2}\ (x + C[1] - 3\ \text{Log}[3 + x])^{1/4}\},\ \{y[x] \to -i\ \sqrt{2}\ (x + C[1] - 3\ \text{Log}[3 + x])^{1/4}\},$
$\{y[x] \to i\ \sqrt{2}\ (x + C[1] - 3\ \text{Log}[3 + x])^{1/4}\},\ \{y[x] \to \sqrt{2}\ (x + C[1] - 3\ \text{Log}[3 + x])^{1/4}\}\}$

```
N[%]
```

$\{\{y[x] \to -1.41421\ (x + C[1] - 3.\ \text{Log}[3. + x])^{1/4}\},$
$\{y[x] \to (0. - 1.41421\ i)\ (x + C[1] - 3.\ \text{Log}[3. + x])^{1/4}\},$
$\{y[x] \to (0. + 1.41421\ i)\ (x + C[1] - 3.\ \text{Log}[3. + x])^{1/4}\},$
$\{y[x] \to 1.41421\ (x + C[1] - 3.\ \text{Log}[3. + x])^{1/4}\}\}$

Nur reelle Lösungen nehmen

6

```
Remove["Global`*"]
```

a

```
f[x_]:=8-x^3; a=0; b=2;
```

```
Iy = Integrate[x^2 f[x],{x,a,b}]
```

$\frac{32}{3}$

```
N[%]
```

10.6667

b

```
Ix = 1/3 Integrate[f[x]^3,{x,a,b}]
```

$\frac{6912}{35}$

```
N[%]
```

197.486

c

```
Ip=Iy+Ix
```

$\frac{21856}{105}$

```
N[%]
```

208.152

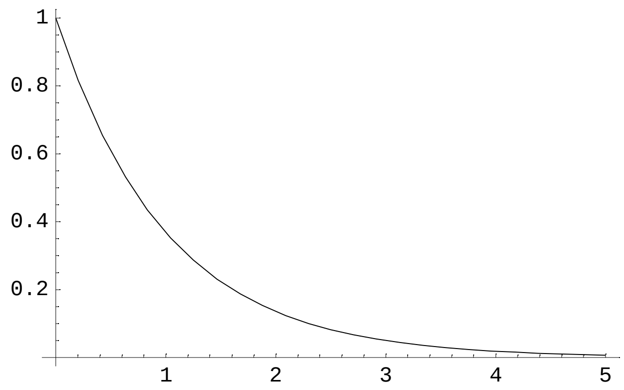
d

Das Trägheitsmoment I ist die Entsprechung der Masse bei der Drehbewegung, wo die kinetische Energie gleich 1/2 mal Winklegeschwindigkeit im Quadrat mal Trägheitsmoment ist.
Man berechnet I als Integral über r (Abstand eines Masseelementes) im Quadrat mal dm. Es ist also die Summe der Momente r mal (r dm).

7

```
Remove["Global`*"]
```

```
f[x_]:=E^(-x); a=0; b=5;
Plot[f[x],{x,a,b}];
```



a

```
Integrate[Evaluate[Sqrt[1+D[f[x],x]^2]],{x,a,b}]
```

$$\sqrt{2} - \text{ArcSinh}[1] + \text{ArcSinh}[e^5] - \frac{2\,\text{Cosh}[5]}{\sqrt{1+e^{10}}}$$

```
NIntegrate[Evaluate[Sqrt[1+D[f[x],x]^2]],{x,a,b}]
```

5.22598

Vergleich: Diagonallänge Rechteck um Anfangs- und Endpunkt:

```
Sqrt[5^2+1^2]//N
```

5.09902

b

```
2 Pi Integrate[Evaluate[f[x] Sqrt[1+D[f[x],x]^2]],{x,a,b}]
```

$$\frac{1}{2}\pi\left(10 + 2\sqrt{2} - \frac{2}{\sqrt{1+e^{10}}} - \frac{2}{e^{10}\sqrt{1+e^{10}}} + \text{Log}\left[3+2\sqrt{2}\right] - \text{Log}\left[2+e^{10}+2\sqrt{1+e^{10}}\right]\right)$$

```
2 Pi NIntegrate[Evaluate[f[x] Sqrt[1+D[f[x],x]^2]],{x,a,b}]
```

7.16946

c

```
Pi Integrate[f[x]^2,{x,a,b}]
```

$$\left(\frac{1}{2} - \frac{1}{2\,e^{10}}\right)\pi$$

```
N[%]
```

1.57073

```
Pi NIntegrate[f[x]^2,{x,a,b}]
```

```
1.57073
```

d

Nein. Die im Kurs in solchen Fällen erhaltene Biegelinen werden durch einfache Polynome beschrieben. Hier ist eine Exponentialfunktion gegeben.

8

```
Remove["Global`*"]
```

a

```
m=1; t1=1;
a[t_]:= Cos[t];
v[t_]:= Integrate[a[tt],{tt,0,t}];
v[t]
```

```
Sin[t]
```

```
v[t1]
```

```
Sin[1]
```

```
N[%]
```

```
0.841471
```

```
p[t1] = m v[t1]
```

```
Sin[1]
```

```
N[%]
```

```
0.841471
```

b

Impuls ist Masse mal Geschwindigkeit (bei vielen massen wird das Produkt aufsummiert resp. integriert).
Die Kraft ist die Ableitung des Impulses nach der Zeit bei konstanter Masse.

c

```
s[t_]:= Integrate[v[tt],{tt,0,t}];
s[t]
```

```
1 - Cos[t]
```

```
s[t1]
```

$1 - \mathrm{Cos}[1]$

```
N[%]
```

0.459698

d

```
Solve[s[t] == sI,{t}]
```

$\{\{t \to -\mathrm{ArcCos}[1 - sI]\}, \{t \to \mathrm{ArcCos}[1 - sI]\}\}$

```
Integrate[m a[ArcCos[1-sI]],{sI,0,s1}]
```

$s1 - \frac{s1^2}{2}$

```
(%/.s1->s[t1])//Simplify
```

$\frac{\mathrm{Sin}[1]^2}{2}$

Oder

```
W[t_]:= 1/2 m v[t]^2;
W[t1]
```

$\frac{\mathrm{Sin}[1]^2}{2}$

```
N[%]
```

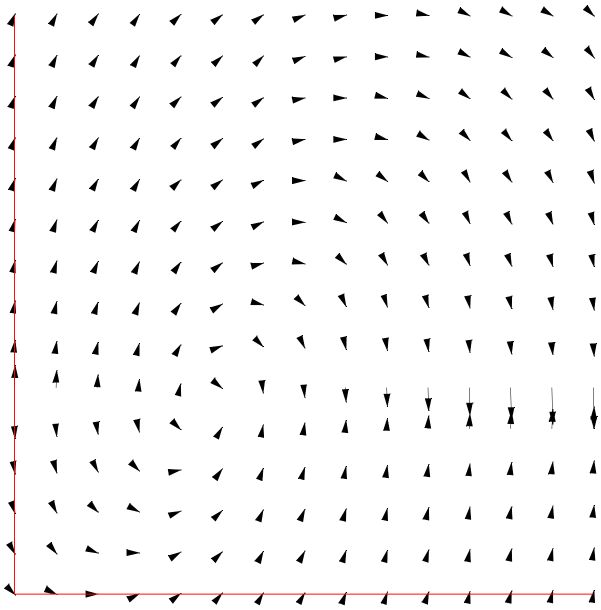
0.354037

9

```
Remove["Global`*"]
```

```
<<Graphics`PlotField`
```

```
PlotVectorField[{1,(y-2x+1)/(y-1)},{x,0,3},{y,0,3},
Epilog->
{Hue[1],Line[{{0,0},{3,0}}],Line[{{0,0},{0,3}}]},
AspectRatio->Automatic];
```



```
PlotVectorField[{1,(y-2x+1)/(y-1)},{x,-1,3},{y,-1,3},
Epilog->
{Hue[1],Line[{{-1,0},{3,0}}],Line[{{0,-1},{0,3}}]},
AspectRatio->Automatic];
```