Lösungen

1

```
Remove["Global`*"]

f1[x_] := (x - 1) (x + 1) (x^2 + 1) + 4 x^3 + 3 x^2 + 2 x;
f1[x] // Expand
```

$$-1 + 2\,x + 3\,x^2 + 4\,x^3 + x^4$$

```
f1'[x]
```

$$2 + 6\,x + 12\,x^2 + 2\,(-1 + x)\,x\,(1 + x) + (-1 + x)\,(1 + x^2) + (1 + x)\,(1 + x^2)$$

```
f1'[x] // Expand
```

$$2 + 6\,x + 12\,x^2 + 4\,x^3$$

2

```
Remove["Global`*"]

f2[x_] := Sin[x] E^x Cosh[x];
f2'[x]
```

$$e^x\,\text{Cos}[x]\,\text{Cosh}[x] + e^x\,\text{Cosh}[x]\,\text{Sin}[x] + e^x\,\text{Sin}[x]\,\text{Sinh}[x]$$

```
% /. {Cosh[x] → (E^x + E^-x) / 2, Sinh[x] → (E^x - E^-x) / 2}
```

$$\frac{1}{2}\,e^x\,(e^{-x} + e^x)\,\text{Cos}[x] + \frac{1}{2}\,e^x\,(-e^{-x} + e^x)\,\text{Sin}[x] + \frac{1}{2}\,e^x\,(e^{-x} + e^x)\,\text{Sin}[x]$$

```
% // Simplify
```

$$\frac{1}{2}\,(\text{Cos}[x] + e^{2x}\,\text{Cos}[x] + 2\,e^{2x}\,\text{Sin}[x])$$

3

```
Remove["Global`*"]

f3[x_] := (x - 1) (x + 2) / (x^2 - 1); f3[x] // Simplify
```

$$\frac{2 + x}{1 + x}$$

```
D[%, x]
```

$$\frac{1}{1+x}-\frac{2+x}{(1+x)^2}$$

```
% // Together
```

$$-\frac{1}{(1+x)^2}$$

4

```
Remove["Global`*"]
```

```
f4[x_] := Cos[Cos[x]]; f4'[x]
```

```
Sin[x] Sin[Cos[x]]
```

5

```
Remove["Global`*"]
```

```
f5[x_] := Cos[Cos[Cos[x]]]; f5'[x]
```

```
-Sin[x] Sin[Cos[x]] Sin[Cos[Cos[x]]]
```

6

```
Remove["Global`*"]
```

```
f6[x_] := (2 x)^(3 x); f6'[x]
```

```
3 2^(3 x) x^(3 x) Log[2] + 2^(3 x) x^(3 x) (3 + 3 Log[x])
```

```
% // Simplify
```

```
3 8^x x^(3 x) (1 + Log[2] + Log[x])
```

7

```
Remove["Global`*"]
```

```
Sign[x]
```

```
Sign[x]
```

```
f7[x_] := x^3 Log[Abs[x^3]]; f7'[x] /. Abs'[x] → 0
```

$$3\,x^2\,\text{Log}[\text{Abs}[x]^3]$$

```
f7[x_] := x^3 Log[x^3 Sign[x^3]]; f7'[x] /. Sign'[x] → 0
```

$$3\,x^2 + 3\,x^2\,\text{Log}[x^3\,\text{Sign}[x]^3]$$

```
% // Simplify
```

$$3\,x^2\,(1 + \text{Log}[x^3\,\text{Sign}[x]^3])$$

8

```
Remove["Global`*"]
```

```
f8[x_] := 2 Sin[2 x]; f8'[x]
```

$$4\,\text{Cos}[2\,x]$$

```
Solve[4 Cos[2 x] == Tan[30 Degree], {x}]
```

$$\left\{\left\{x \to -\frac{1}{2}\,\text{ArcCos}\left[\frac{1}{4\sqrt{3}}\right]\right\},\ \left\{x \to \frac{1}{2}\,\text{ArcCos}\left[\frac{1}{4\sqrt{3}}\right]\right\}\right\}$$

```
% // N
```

$$\{\{x \to -0.712976\},\ \{x \to 0.712976\}\}$$

9

```
Remove["Global`*"]
```

```
f9[x_] := E^-x - 2 x^3 + 4; f9'[x]
```

$$-e^{-x} - 6\,x^2$$

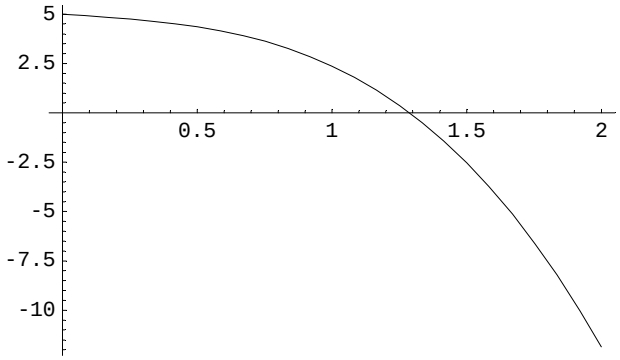
```
f91[x_] := Evaluate[ f9'[x]]
```

```
w[0] = 1;
w[n_] := w[n - 1] - f9[w[n - 1]] / f91[w[n - 1]] // N
```

```
Table[{n, w[n]}, {n, 0, 9}] // TableForm
```

```
0      1
1      1.37185
2      1.29304
3      1.28825
4      1.28823
5      1.28823
6      1.28823
7      1.28823
8      1.28823
9      1.28823
```

```
Plot[f9[x], {x, 0, 2}];
```

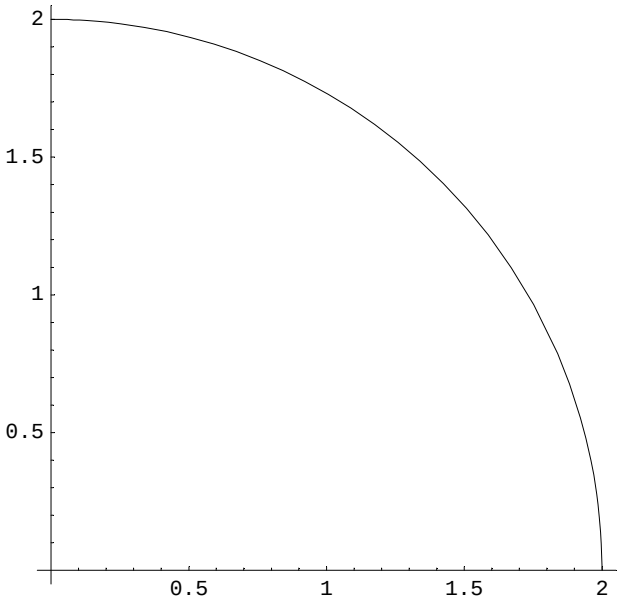


10

```
Remove["Global`*"]
```

```
f10[x_] := Sqrt[4 - x^2];
Dreieck10[x_] := x f10[x] / 2;
```

```
Plot[f10[x], {x, 0, 2}, AspectRatio → Automatic];
```



```
Dreieck10'[x]
```

$$-\frac{x^2}{2\sqrt{4-x^2}}+\frac{\sqrt{4-x^2}}{2}$$

```
Solve[Evaluate[Dreieck10'[x] == 0], {x}]
```

$$\left\{\left\{x\to -\sqrt{2}\right\},\left\{x\to \sqrt{2}\right\}\right\}$$

```
% // N
```

```
{{x → -1.41421}, {x → 1.41421}}
```

```
1.4142135623730951 / 2
```

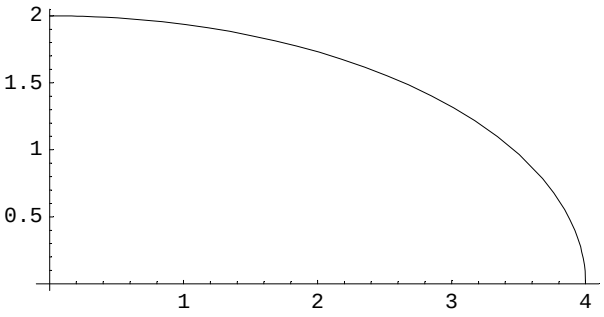
```
0.707107
```

11

```
Remove["Global`*"]
```

```
f11[x_] := Sqrt[4 - (x / 2)^2];
Dreieck11[x_] := x f11[x] / 2;
```

```
Plot[f11[x], {x, 0, 4}, AspectRatio → Automatic];
```




```
Dreieck11'[x]
```

$$-\frac{x^2}{8\sqrt{4-\frac{x^2}{4}}}+\frac{1}{2}\sqrt{4-\frac{x^2}{4}}$$

```
Solve[Evaluate[Dreieck11'[x] == 0], {x}]
```

$$\left\{\left\{x\to -2\sqrt{2}\right\},\left\{x\to 2\sqrt{2}\right\}\right\}$$

```
% // N
```

```
{{x → -2.82843}, {x → 2.82843}}
```

```
2.8284271247461903 / 4
```

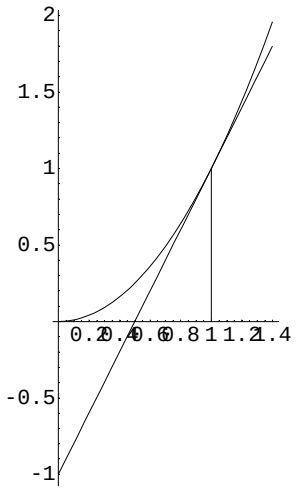
```
0.707107
```

12

```
Remove["Global`*"]

f12[x_] := x^2;
t12[x_, x1_] := Evaluate[f12[y] + f12'[y] (x - y) /. y → x1];

a = 1;
Plot[{f12[x], t12[x, a]}, {x, 0, 1.4},
  AspectRatio → Automatic, Epilog → {Line[{{a, 0}, {a, f12[a]}}]}];
```



13

```
Remove["Global`*"]

f13[x_] := Sin[x];
h13[x_] := (Pi - x - x) f13[x]

h13'[x]
```

$(\pi - 2\,x)\,\text{Cos}[x] - 2\,\text{Sin}[x]$

```
Solve[Evaluate[h13'[x] == 0], {x}]
```

$\text{Solve}[(\pi - 2\,x)\,\text{Cos}[x] - 2\,\text{Sin}[x] == 0,\ \{x\}]$

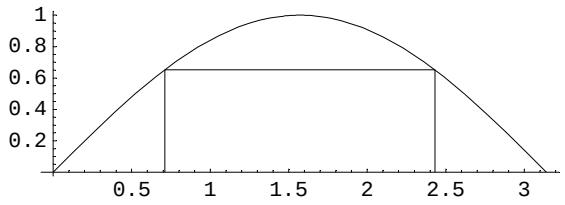
```
fr = FindRoot[Evaluate[h13'[x] == 0], {x, 0.5}]
```

$\{x \to 0.710463\}$

```
x1 = x /. fr
```

0.710463

```
Plot[f13[x], {x, 0, Pi}, AspectRatio → Automatic,
  Epilog → {Line[{{x1, 0}, {x1, f13[x1]}, {Pi - x1, f13[x1]}, {Pi - x1, 0}}]}];
```



14

```
Remove["Global`*"]
```

```
f14[x_] := (x - 1) (x - 2) (x + 3) (x + 5);
f14[x] // Expand
```

$$30 - 29\,x - 7\,x^2 + 5\,x^3 + x^4$$

```
f14''[x] // Simplify
```

$$2\,(-7 + 15\,x + 6\,x^2)$$

```
Solve[Evaluate[f14'[x] == 0], {x}] // N // Chop
```

$$\{\{x \to 1.54619\}, \{x \to -1.1238\}, \{x \to -4.17239\}\}$$

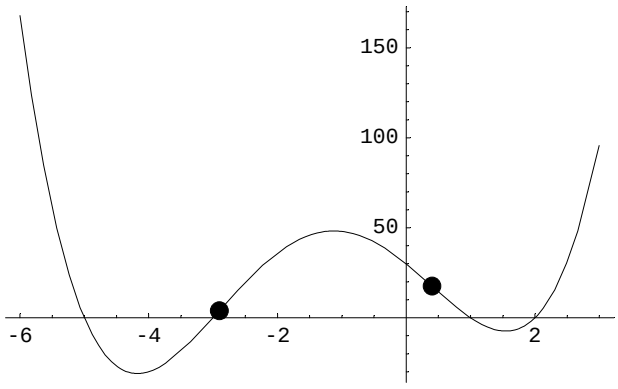
```
sv = Solve[Evaluate[f14''[x] == 0], {x}] // N // Chop
```

$$\{\{x \to -2.90202\}, \{x \to 0.402019\}\}$$

```
{x1 = x /. sv[[1]], x2 = x /. sv[[2]]}
```

$$\{-2.90202, 0.402019\}$$

```
Plot[f14[x], {x, -6, 3},
  Epilog → {PointSize[0.03], Point[{x1, f14[x1]}], Point[{x2, f14[x2]}]}];
```



15

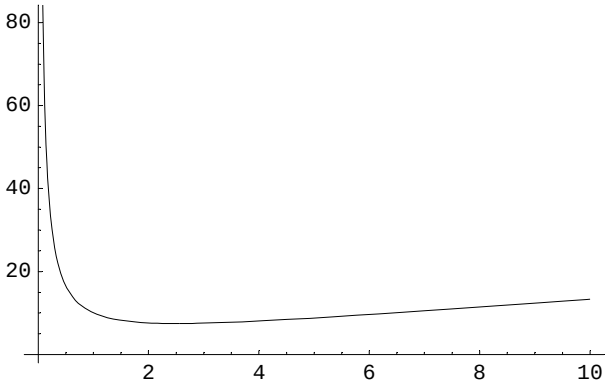
```
Remove["Global`*"]
```

x : 2.3 =(x+3) : y; L^2=(x+3)^2+y^2 = (x^2+2.3^2)/x *(x+3)

```
f15[x_] := Sqrt[x^2 + (2.3)^2] (x + 3) / x; f15[x]
```

$$\frac{(3+x)\sqrt{5.29+x^2}}{x}$$

```
Plot[f15[x],{x,0,10}];
```




```
f15'[x]//Simplify
```

$$\frac{-15.87+0.\,x^2+x^3}{x^2\sqrt{5.29+1.\,x^2}}$$

```
so1 = Solve[Evaluate[f15'[x]==0],{x}]
```

```
{{x → -1.2565 - 2.17632 i}, {x → -1.2565 + 2.17632 i}, {x → 2.513}}
```

```
xMax = x /. so1[[3]]
```

```
2.513
```

```
Laenge = Sqrt[(xMax^2 + 2.3^2)] / xMax * (xMax + 3)
```

```
7.47345
```

```
WinkelInRad = ArcTan[2.3 / xMax]
```

```
0.741172
```

```
WinkelInGrad = ArcTan[2.3 / xMax] / Degree
```

```
42.466
```

Das ist fast 45 Grad, jedoch nicht exakt 45 Grad!