Lösungen

1

1a

```
Integrate[x E^(-x), x]
```

$$e^{-x} (-1 - x)$$

Achtung: Konstante C anfügen!

1b

```
Integrate[(x + 1) / (x + 2) / x, x]
```

$$\frac{\text{Log}[x]}{2} + \frac{1}{2} \text{Log}[2 + x]$$

```
Integrate[(x + 1) / (x + 2) / x, {x, 1, 2}]
```

$$\text{Log}\left[\frac{4}{3}\right] + \frac{1}{2} \text{Log}\left[\frac{3}{2}\right]$$

```
(% // Together) /. (c_ Log[a_] + Log[b_]) → Log[a^c * b]
```

$$\frac{\text{Log}[24]}{2}$$

```
N[%]
```

```
1.58903
```

2

2a

```
p1 = Normal[Series[Sin[x], {x, 0, 2}]]
```

```
x
```

```
p11 = Normal[Series[Sin[x], {x, 0, 5}]]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120}$$

```
Normal[Series[E^z, {z, 0, 2}]]
```

$$1 + z + \frac{z^2}{2}$$

```
p2 = Normal[Series[E^z, {z, 0, 2}]] /. z → -x^2
```

$$1 - x^2 + \frac{x^4}{2}$$

```
1 + p1 p2 // Expand
```

$$1 + x - x^3 + \frac{x^5}{2}$$

```
1 + p11 p2 // Expand
```

$$1 + x - \frac{7\,x^3}{6} + \frac{27\,x^5}{40} - \frac{11\,x^7}{120} + \frac{x^9}{240}$$

```
Normal[Series[1 + Sin[x] E^(-x^2), {x, 0, 5}]]
```

$$1 + x - \frac{7\,x^3}{6} + \frac{27\,x^5}{40}$$

(* Näherung max. Grad 2 *) x

2b

```
r1 = NIntegrate[1 + x, {x, 0, 0.2}]
```

```
0.22
```

```
r2 = NIntegrate[1 + Sin[x] E^(-x^2), {x, 0, 0.2}]
```

```
0.21954
```

```
AbweichungProzent = (r1 - r2) / r1 100
```

```
0.208886
```

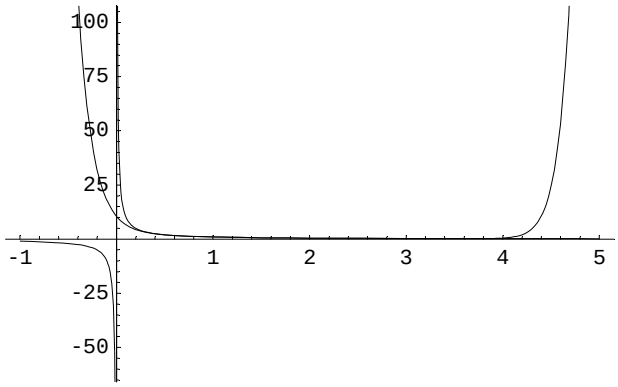
Brauchbar

3

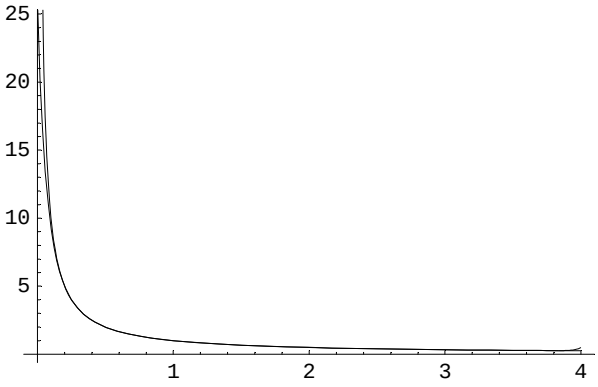
```
Normal[Series[1 / x, {x, 2, 20}]]
```

$$\frac{1}{2} + \frac{2 - x}{4} + \frac{1}{8}(-2 + x)^2 - \frac{1}{16}(-2 + x)^3 + \frac{1}{32}(-2 + x)^4 - \frac{1}{64}(-2 + x)^5 + \frac{1}{128}(-2 + x)^6 - \frac{1}{256}(-2 + x)^7 + \frac{1}{512}(-2 + x)^8 - \frac{(-2 + x)^9}{1024} + \frac{(-2 + x)^{10}}{2048} - \frac{(-2 + x)^{11}}{4096} + \frac{(-2 + x)^{12}}{8192} - \frac{(-2 + x)^{13}}{16384} + \frac{(-2 + x)^{14}}{32768} - \frac{(-2 + x)^{15}}{65536} + \frac{(-2 + x)^{16}}{131072} - \frac{(-2 + x)^{17}}{262144} + \frac{(-2 + x)^{18}}{524288} - \frac{(-2 + x)^{19}}{1048576} + \frac{(-2 + x)^{20}}{2097152}$$

```
Plot[Evaluate[{Normal[Series[1 / x, {x, 2, 20}]], 1 / x}], {x, -1, 5}];
```




```
Plot[Evaluate[{Normal[Series[1 / x, {x, 2, 50}]], 1 / x}], {x, 0, 4}];
```




```
k[n_] := 1 / 2^(n + 1)
```

```
r = k[n] / k[n + 1]
```

```
2
```

```
Limit[k[n] / k[n + 1], n → Infinity]
```

```
2
```

4

```
Remove["Global`*"]
```

```
v = {2, -1};  P = {2, 1};  f[x_, y_] := Sin[x] + Cos[y];
grad[f_] := {D[f, x], D[f, y]};  grad[f[x, y]]
```

```
{Cos[x], -Sin[y]}
```

```
richtAbleit = grad[f[x, y]].v / Norm[v] /. {x → P[[1]], y → P[[2]]}
```

$$\frac{2\,\text{Cos}[2] + \text{Sin}[1]}{\sqrt{5}}$$

```
N[%]
```

```
0.00410422
```

```
ArcTan[richtAbleit]
```

$$\text{ArcTan}\left[\frac{2\,\text{Cos}[2]+\text{Sin}[1]}{\sqrt{5}}\right]$$

```
ArcTan[richtAbleit] // N
```

```
0.0041042
```

```
ArcTan[richtAbleit] / Degree // N
```

```
0.235153
```

5

```
Remove["Global`*"]
```

```
DSolve[y'[x] == x^2 / E^y[x], y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \text{Log}\left[\frac{x^3}{3}+C[1]\right]\right]\right\}\right\}$$

```
DSolve[{y'[x] == x^2 / E^y[x], y[1] == 1}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \text{Log}\left[\frac{1}{3}\,(-1+3\,e)+\frac{x^3}{3}\right]\right]\right\}\right\}$$

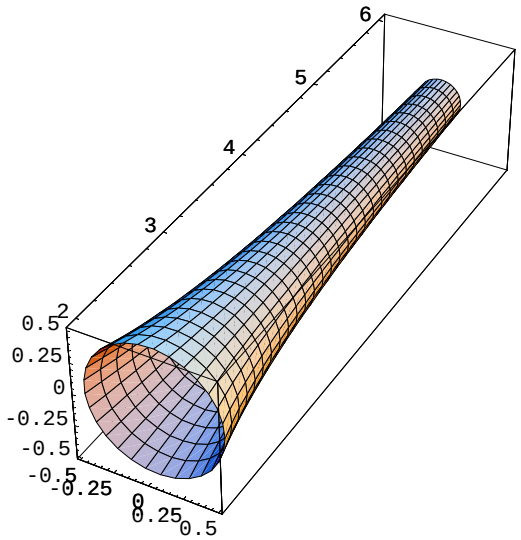
```
% // N
```

```
{{y → Function[{x}, Log[0.333333 (-1. + 3. 2.71828) + 0.333333 x^3]]}}
```

6

```
Remove["Global`*"]
```

```
f[x] := 1 / x; a = 2; b = 6;
ParametricPlot3D[{Cos[s] / x, x, Sin[s] / x}, {x, 2, 6}, {s, 0, 2 π}];
```



6a

```
Integrate[Evaluate[Sqrt[(D[f[x], {x, 1}])^2 + 1]], {x, a, b}]
```

$$\frac{1}{4}\,(-1)^{1/4}\left(\text{Beta}\left[-1296, -\frac{1}{4}, \frac{3}{2}\right] - \text{Beta}\left[-16, -\frac{1}{4}, \frac{3}{2}\right]\right)$$

```
Mantellaenge = NIntegrate[Evaluate[Sqrt[(D[f[x], {x, 1}])^2 + 1]], {x, a, b}]
```

```
4.01992
```

```
(*Vergleich*) Sqrt[4^2 + 0.5^2] // N
```

```
4.03113
```

6b

```
2 Pi Integrate[Evaluate[f[x] Sqrt[(D[f[x], {x, 1}])^2 + 1]], {x, a, b}]
```

$$\frac{1}{36}\,\pi\left(9\sqrt{17} - \sqrt{1297} - 36\,\text{ArcSinh}[4] + 36\,\text{ArcSinh}[36]\right)$$

```
Mantelflaeche = 2 Pi NIntegrate[Evaluate[f[x] Sqrt[(D[f[x], {x, 1}])^2 + 1]], {x, a, b}]
```

```
6.95089
```

6c

```
Pi Integrate[f[x] ^2, {x, a, b}]
```

$$\frac{\pi}{3}$$

```
Rotationsvolumen = Pi NIntegrate[f[x] ^2, {x, a, b}]
```

```
1.0472
```